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Robust Disturbance Rejection for Left Invertible Singular Systems with Nonlinear Uncertain Structure

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***Abstract*—**The problem of robust disturbance rejection (RDR) using a proportional state feedback controller is studied for the case of Left Invertible MIMO generalized state space linear systems with nonlinear uncertain structure. Sufficient conditions for the problem to have a solution are established. The set of all proportional feedback controllers solving the problem subject to these conditions is analytically determined.

***Keywords*—**System Theory, Uncertain Systems, Robust Control, Singular Systems.

I. INTRODUCTION

CONSIDER the general class of multivariable, linear singular systems, having nonlinear uncertain structure [1]:

$$\begin{aligned} E(q)\dot{x}(t) &= A(q)x(t) + B(q)u(t) + D(q)z(t) \\ y(t) &= C(q)x(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n\times 1}$, $u \in \mathbb{R}^{m\times 1}, z \in \mathbb{R}^{\zeta\times 1}, y \in \mathbb{R}^{p\times 1}$ denote the vector of the states, the control inputs, the unknown disturbances and the outputs respectively and where $\mathbb{R}$ denotes the set of real numbers. The matrices $E(q) \in [\wp(q)]^{n\times n}$, $A(q) \in [\wp(q)]^{n\times n}$, $B(q) \in [\wp(q)]^{n\times m}$, $D(q) \in [\wp(q)]^{n\times \zeta}$ and $C(q) \in [\wp(q)]^{p\times n}$ are function matrices depending upon the uncertainty vector $q = \begin{bmatrix} q_1 & \cdots & q_l \end{bmatrix} \in \mathbb{Q}$, where $\mathbb{Q}$ is the uncertainty domain and $\wp(q)$ is the set of all functions of q. The domain $\mathbb{Q}$ can be any set, while the values of the functions of $\wp(q)$ are considered to be real. The uncertainties $q_1,\ldots,q_l$ do not depend upon the time. Regarding the structure of $E(q), A(q), B(q), D(q)$ and $C(q)$ no limitation or specification (continuity, boundness, smoothness, etc.) is required, thus covering the cases of nonlinear and distributed system's uncertain structure. It is important to mention that the matrix $E(q)$ may or may not be singular while its singularity may depend upon the values of the uncertainties. The singular system (1) is assumed to be solvable in the robust sense.

The problem of disturbance rejection is one of the most important control design problems ([2]-[4] and the references therein). The problem of disturbance rejection for the case of nonuncertain generalized state space systems has attracted considerable attention (see [5]-[8] as well as the references therein). For the case of left invertible normal linear uncertain systems, the problem of robust disturbance rejection has extensively been solved in [9], where necessary and sufficient conditions for the problem to have a solution have been derived. Other familiar results can be found in [10]-[15]. For the case of generalized state space linear systems having nonlinear uncertain structure, the problem of disturbance rejection has not as yet been studied. The familiar problem of disturbance rejection with simultaneous data sensitivity has been studied in [16]. Also it is important to mention that another robust transfer function design problem for generalized state space linear systems with nonlinear uncertain structure the familiar problem of robust input – output decoupling has been studied in [1].

In the present paper, the problem Robust Disturbance Rejection (RDR) for singular linear systems having nonlinear uncertain structure is solved, using a proportional state feedback controller. Sufficient conditions for the solvability of the problem are established and the set of all proportional state feedback controllers solving the problem under these conditions is derived (Section 3).

II. PRELIMINARY DEFINITIONS AND PROBLEM FORMULATION

To system (1) apply the proportional state feedback controller

$$u(t) = Fx(t) + G\omega(t) \tag{2}$$

where $F \in \mathbb{R}^{m\times n}$ is the feedback matrix, $G \in \mathbb{R}^{m\times m}$ is the

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precompensator matrix and $\omega(t)$ is the vector of external inputs. For the controller to be robust the elements of the feedback matrix and the precompensator matrix. The precompensator is assumed to be invertible in order to ensure the linear independence of the influence of the external inputs.

The resulting closed loop system must be solvable (see [1], [5] and [15]) in the robust sense, i.e. $\det[sE(q)-A(q)-B(q)F]\not\equiv 0 \ \ \forall q\in\mathbb{Q}$ where s is the complex frequency. The RDR problem for the case of generalized state space systems is stated in the following definition:

Definition 2.1: The RDR problem via a proportional feedback controller of the form (2), for generalized state space linear systems having nonlinear uncertain structure is solvable if there exist controller matrices $F\in\mathbb{R}^{m\times n}$, $G\in\mathbb{R}^{m\times m}$ as well as an appropriate matrix, let $H(s,q)$, with elements being rational functions of s with real coefficients being nonlinear functions of q such that the transfer function of the overall closed loop system satisfies the following equality for every $q\in\mathbb{Q}$

$$\begin{aligned}C(q)[sE(q)-A(q)-B(q)F]^{-1}\begin{bmatrix}B(q)G & D\end{bmatrix}&=\\=\begin{bmatrix}H(s,q) & 0_{p\times\zeta}\end{bmatrix}\end{aligned}\tag{3}$$

For the controller to be implementable the controller matrices must be independent from the uncertainties. □

As already mentioned the open loop system is assumed to be robustly solvable i.e. that $\det[sE(q)-A(q)]\not\equiv 0 \ \ \forall q\in\mathbb{Q}$. Hence there exist a real function, let $\mu(q)$ such that $\det[\mu(q)E(q)+A(q)]\not\equiv 0 \ \ \forall q\in\mathbb{Q}$. To facilitate the construction of $\mu(q)$ we mention that its value set can be chosen to be a subset of the integer set $\{0,1,2,\ldots,n\}$. According to [1] $\mu(q)$ can be defined as follows

$$\mu(q)=\begin{cases}0\ , & \forall\, q\in\mathbb{Q}:\det A(q)\neq 0\\ & \forall\, q\in\mathbb{Q}:\\ i\in\{1,2,\ldots,n\}, & \sum_{j=0}^{i-1}\left|\det[jE(q)-A(q)]\right|=0\\ & \wedge\ \det[iE(q)-A(q)]\neq 0\end{cases}$$

Define

$$\begin{aligned}\bar{E}(q)&=[\mu(q)E(q)+A(q)]^{-1}E(q)\\ \bar{B}(q)&=[\mu(q)E(q)+A(q)]^{-1}B(q)\\ \bar{D}(q)&=[\mu(q)E(q)+A(q)]^{-1}D(q)\end{aligned}$$

and $w(s,q)=[s+\mu(q)]^{-1}$. From the robust solvability of the open loop system and the robust solvability of the closed loop system we observe that the matrix

$$I_m+w(s,q)F\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\bar{B}(q)$$

is well defined and invertible, where I_j is the j *-th* dimension unity matrix.

Based on this observation as well as the aforementioned definitions and the results in [5], the equation in (3) can be rewritten as follows

$$\begin{aligned}&C(q)\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\begin{bmatrix}\bar{B}(q) & \bar{D}(q)\end{bmatrix}=\\&=-[w(s,q)]^{-1}H\left(\left([w(s,q)]^{-1}-\mu(q)\right),q\right)G^{-1}\times\\&\left\{\begin{bmatrix}I_m & 0\end{bmatrix}+w(s,q)F\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\begin{bmatrix}\bar{B}(q) & \bar{D}(q)\end{bmatrix}\right\}\end{aligned}\tag{4}$$

As already mentioned the open loop system has been assumed to be left invertible, i.e.

$$\mathrm{Rank}\left[C(q)[sE(q)-A(q)]^{-1}B(q)\right]=m,\forall q\in\mathbb{Q}$$

and consequently that $p\geq m$. Using the definitions just before (4) the above condition can be rewritten as follows

$$\mathrm{Rank}\left[C(q)\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\bar{B}(q)\right]=m,\forall q\in\mathbb{Q}$$

As a direct extension of a similar result in [5] and [9] the following lemma can be presented.

Lemma 2.1: A necessary condition for the solvability of the RDR problem of the left invertible GSS system (1) via the control law (2) is

$$\begin{aligned}&\mathrm{Rank}\left[C(q)\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\begin{bmatrix}\bar{B}(q) & \bar{D}(q)\end{bmatrix}\right]=\\&=\mathrm{Rank}\left[C(q)\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\bar{B}(q)\right]=m\ ,\ \forall q\in\mathbb{Q}\end{aligned}\tag{5}$$

. □

Based on Lemma 2.1, [5] and [9] it can readily be concluded that there exist an invertible for every $q\in\mathbb{Q}$ uncertain matrix, let $J\left(w(s,q),q\right)$, such that

$$\begin{aligned}&J\left(w(s,q),q\right)C(q)\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\begin{bmatrix}\bar{B}(q) & \bar{D}(q)\end{bmatrix}=\\&=\begin{bmatrix}\hat{C}(q)\\ 0_{(p-m)\times m}\end{bmatrix}\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\begin{bmatrix}\bar{B}(q) & \bar{D}(q)\end{bmatrix},\forall q\in\mathbb{Q}\end{aligned}\tag{6}$$

where

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$$\operatorname{rank}\left[\hat{C}(q)\left[\overline{B}(q) \quad \overline{D}(q)\right]\right] = \operatorname{rank}\left[\hat{C}(q)\overline{B}(q)\right] = m, \forall q \in \mathbb{Q} \quad (7)$$

The matrix $J\left(w(s,q),q\right)$ can be constructed following a finite step explicit algorithm presented in [9] and [17].

From (6) and (4) we observe that

$$J\left(w(s,q),q\right) H\left(\left(\left[w(s,q)\right]^{-1} - \mu(q)\right), q\right) = = \begin{bmatrix} \hat{H}\left(\left(\left[w(s,q)\right]^{-1} - \mu(q)\right), q\right) \\ 0_{(p-m)\times m} \end{bmatrix} \quad (8)$$

where

$$\operatorname{Rank}\left[\hat{H}\left(\left(\left[w(s,q)\right]^{-1} - \mu(q)\right), q\right)\right] = m, \forall q \in \mathbb{Q} \quad (9)$$

The above condition is derived by Lemma 2.1 and the robust invertibility of the matrix $I_m + w(s,q)F\left[w(s,q)I_n - \overline{A}(q)\right]^{-1}\overline{B}(q)$.

Based on (6) and (8) the equation (4) can be expressed equivalently as follows

$$w(s,q)P\left(w(s,q),q\right)\,\hat{C}(q)\left[w(s,q)I_n - \overline{A}(q)\right]^{-1}\left[\overline{B}(q) \quad \overline{D}(q)\right] = = \left[I_m + w(s,q)F\left[w(s,q)I_n - \overline{A}(q)\right]^{-1}\overline{B}(q) \;\middle|\; w(s,q)F\left[w(s,q)I_n - \overline{A}(q)\right]^{-1}\overline{D}(q)\right] \quad (10)$$

where

$$P\left(w(s,q),q\right) = -G\left[\hat{H}\left(\left(\left[w(s,q)\right]^{-1} - \mu(q)\right), q\right)\right]^{-1}$$

The robust invertibility of the matrix $P\left(w(s,q),q\right)$ is derived by the invertibility of the precompensator and the condition (9).

Equation (10) involves as unknowns the matrices $P\left(w(s,q),q\right)$ and F. Clearly it must hold that

$$\det\left[P\left(w(s,q),q\right)\right] \not\equiv 0,\; \forall q \in \mathbb{Q} \quad (11)$$

For all matrices $P\left(w(s,q),q\right)$ and F that satisfy (10) with $P\left(w(s,q),q\right)$ robust invertible, the robust invertibility of the matrix $I_m + w(s,q)F\left[w(s,q)I_n - \overline{A}(q)\right]^{-1}\overline{B}(q)$ and subsequently the solvability of the closed loop system is guaranteed for every value of the uncertain parameters.

Assume that the conditions of Lemma 2.1 are satisfied. Then according to the above results the problem has been reduced to that of solving equation (10) with respect to $P\left(w(s,q),q\right)$ and F, subject to the constraint (11).

III. Necessary and Sufficient Conditions

From the first block of the equation (10) and condition (7) we conclude that the rational matrix $P\left(w(s,q),q\right)$ must be proper with respect to w and thus it can be expanded in negative powers of w as follows

$$P(w,q) = P_0(q)w^0 + P_1(q)w^{-1} + \cdots$$

Using the above expansion, expand both sides of equation (10) in negative powers of w, at any particular $q \in \mathbb{Q}$. Equating the coefficients of like powers of w in both sides of the resulting equation, at any particular $q \in \mathbb{Q}$, then using a similar expression in [5], the equation (11) is equivalent to the following set of $2n+1$ algebraic matrix equation

$$-F\left[\left[\overline{E}(q)\right]^0\overline{\Delta}(q) \quad \left[\overline{E}(q)\right]^1\overline{\Delta}(q) \quad \cdots \left[\overline{E}(q)\right]^{2n}\overline{\Delta}(q)\right] + +\left[P_0(q) \quad P_1(q) \quad \cdots \quad P_{2n}(q)\right]\Pi(q) = = \left[\overline{J} \quad 0_{m\times(m+\zeta)} \quad \cdots \quad 0_{m\times(m+\zeta)}\right] \quad (12)$$

where

$$\overline{J} = \left[I_m \quad 0_{m\times\zeta}\right]$$

$$\overline{\Delta}(q) = \left[\overline{B}(q) \quad \overline{D}(q)\right]$$

$$\Pi(q) = \begin{bmatrix} \hat{C}(q)\left[\overline{E}(q)\right]^0\overline{\Delta}(q) & \cdots & \hat{C}(q)\left[\overline{E}(q)\right]^{2n}\overline{\Delta}(q) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \hat{C}(q)\left[\overline{E}(q)\right]^0\overline{\Delta}(q) \end{bmatrix}$$

From condition (7), it can easily be observed that there exist a matrix let $\Pi^{**}(q)$ such that

$$\operatorname{rank}\begin{bmatrix} \hat{C}(q)\left[\overline{E}(q)\right]^0\overline{\Delta}(q) \\ \Pi^{**}(q) \end{bmatrix} = m+\zeta,\; \forall q \in \mathbb{Q} \quad (13)$$

The construction of such a matrix of the form $\Pi^{**}(q)$ can be done after following the steps of the algorithm proposed in [9] for the construction of a respective matrix. Consider the block triangular composite matrix

$$\Pi^*(q)=\begin{bmatrix}\Pi_0^*(q) & \Pi_1^*(q) & \cdots & \Pi_{2n}^*(q)\\ 0 & \Pi_0^*(q) & \cdots & \Pi_{2n-1}^*(q)\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \Pi_0^*(q)\end{bmatrix}$$

where

$$\Pi_0^*(q)=\begin{bmatrix}\hat{C}(q)\left[\bar{E}(q)\right]^0\bar{\Delta}(q)\\ \Pi^{**}(q)\end{bmatrix}$$

$$\Pi_j^*(q)=\begin{bmatrix}\hat{C}(q)\left[\bar{E}(q)\right]^j\bar{\Delta}(q)\\ 0\end{bmatrix}, j=1,2,\ldots,2n$$

According to (13) the matrix $\Pi^*(q)$ is invertible $\forall q\in\mathbb{Q}$. Postmultipication of equation (12) by $\left[\Pi^*(q)\right]^{-1}$, yields

$$FL_D(q)=-N_D(q) \tag{14a}$$

$$\begin{bmatrix}P_0(q) & \cdots & P_{2n}(q)\end{bmatrix}=FL_B(q)+N_B(q) \tag{14b}$$

where

$$L_B(q)=L(q)M_B,\ L_D(q)=L(q)M_D$$

$$N_B(q)=N(q)M_B,\ N_D(q)=N(q)M_D$$

$$L(q)=\left[\left[\bar{E}(q)\right]^0\bar{\Delta}(q)\ \cdots\ \left[\bar{E}(q)\right]^{2n}\bar{\Delta}(q)\right]\left[\Pi^*(q)\right]^{-1}$$

$$N(q)=\left[\bar{J}\ \cdots\ 0_{m\times(m+\zeta)}\right]\left[\Pi^*(q)\right]^{-1}$$

$$M_B=\begin{bmatrix}\bar{J}^T & \cdots & 0_{m\times(m+\zeta)}^T\\ \vdots & \ddots & \vdots\\ 0_{m\times(m+\zeta)}^T & \cdots & \bar{J}^T\end{bmatrix},$$

$$M_D=\begin{bmatrix}\tilde{J} & \cdots & 0_{\zeta\times(m+\zeta)}^T\\ \vdots & \ddots & \vdots\\ 0_{\zeta\times(m+\zeta)}^T & \cdots & \tilde{J}\end{bmatrix}$$

and where

$$\tilde{J}=\begin{bmatrix}0_{m\times\zeta}\\ I_\zeta\end{bmatrix}$$

Note that $\begin{bmatrix}\bar{J}^T & \tilde{J}\end{bmatrix}=I_{m+\zeta}$.

Thus far the problem has been reduced to that of solving the linear uncertain equations in (14a) and (14b) under the constraints (i) *the condition* (11) *is satisfied* and (ii) the *matrix* F *is independent from the uncertainties*.

Before presenting our main result the following definitions are presented

$$R_B(q)=\left[L_D(q)\right]_{\mathbb{R}}^{\perp}L_B(q) \tag{15a}$$

$$S_B(q)=N_B(q)-\left\langle N_D(q)\setminus L_D(q)\right\rangle_{\mathbb{R}}L_B(q) \tag{15b}$$

For the definition of $[\bullet]_{\mathbb{R}}^{\perp}$ and $\left\langle\bullet\setminus\bullet\right\rangle_{\mathbb{R}}$ (where $\bullet$ denotes the argument quantities) see [18]. For their computation a finite step explicit algorithm has been proposed in [17].

Divide $R_B(q)$ and $S_B(q)$ into $2n+1$ groups of columns to yield

$$R_B(q)=\begin{bmatrix}R_0(q) & \cdots & R_{2n}(q)\end{bmatrix} \tag{16a}$$

$$S_B(q)=\begin{bmatrix}S_0(q) & \cdots & S_{2n}(q)\end{bmatrix} \tag{16c}$$

where the submatrices $R_j(q)$ and $S_j(q)$ ($j=0,1,\ldots,2n$) have m columns.

Also define the rational with respect to w matrices

$$\begin{aligned}S(w,q)&=\\ &=\left\{w^{-1}I_m+\left\langle N_D(q)\setminus L_D(q)\right\rangle_{\mathbb{R}}\left[wI_n-\bar{A}(q)\right]^{-1}\bar{B}(q)\right\}\times\\ &\times\left\{\hat{C}(q)\left[wI_n-\bar{A}(q)\right]^{-1}\bar{B}(q)\right\}^{-1}\end{aligned} \tag{17a}$$

$$\begin{aligned}R(w,q)&=\left\{\begin{bmatrix}L_D(q)\\ N_D(q)\end{bmatrix}_{\mathbb{R}}^{\perp}\left[wI_n-\bar{A}(q)\right]^{-1}\bar{B}(q)\right\}\times\\ &\times\left\{\hat{C}(q)\left[wI_n-\bar{A}(q)\right]^{-1}\bar{B}(q)\right\}^{-1}\end{aligned} \tag{17b}$$

Using condition (7) it can readily be observed that the above rational matrices are proper with respect to w.

Define

$$S^*(w,q)=w^{-1}I_m+\left\langle N_D(q)\setminus L_D(q)\right\rangle_{\mathbb{R}}\left[wI_n-\bar{A}(q)\right]^{-1}\bar{B}(q)$$

Using the above definition and the definitions (15)-(17) as well as Lemma 3.1 we are now in position to present the following theorem.

Theorem 3.1: The robust disturbance rejection problem is solvable for left invertible uncertain generalized state space systems via a proportional state feedback controller if the following conditions are satisfied

i) $\mathrm{Rank}\left[\bar{C}(q)\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\left[\bar{B}(q)\quad\bar{D}(q)\right]\right]=$

$$=\mathrm{Rank}\left[\bar{C}(q)\left[w(s,q)I_n-\bar{A}(q)\right]^{-1}\bar{B}(q)\right]=m,\ \forall q\in\mathbb{Q} \tag{18}$$

ii) $$\mathrm{rank}_{\mathbb{R}}\begin{bmatrix} L_D(q) \\ N_D(q) \end{bmatrix} = \mathrm{rank}_{\mathbb{R}}\left[L_D(q)\right] \tag{19}$$

iii) The rational matrix $S^*(w,q)$ is robustly invertible.

Proof: The necessity of (18) comes from Lemma 2.1. According to [18], the equation (14a) is solvable with respect to F, being independent from the uncertainties, if and only if the condition $\mathrm{rank}_{\mathbb{R}}\begin{bmatrix} L_D(q) \\ -N_D(q) \end{bmatrix} = \mathrm{rank}_{\mathbb{R}}\left[L_D(q)\right]$ is satisfied or equivalently if and only if (19) is satisfied. If the condition (19) is satisfied then according to [18], the general solution of equation (14a) with respect to F is

$$F = T\left[L_D(q)\right]_{\mathbb{R}}^{\perp} + \left\langle -N_D(q) \setminus L_D(q) \right\rangle_{\mathbb{R}}$$

where T is an $m \times (n - \mathrm{rank}_{\mathbb{R}}\left[L_D(q)\right])$ arbitrary matrix being independent from the uncertainties. After observing that that

$$\left\langle -N_D(q) \setminus L_D(q) \right\rangle_{\mathbb{R}} = -\left\langle N_D(q) \setminus L_D(q) \right\rangle_{\mathbb{R}}$$

The general solution of the feedback matrix takes on the form

$$F = T\left[L_D(q)\right]_{\mathbb{R}}^{\perp} + \left\langle N_D(q) \setminus L_D(q) \right\rangle_{\mathbb{R}} \tag{20}$$

Substituting (20) into (14b) the following general solution of $\begin{bmatrix} P_0(q) & \cdots & P_{2n}(q) \end{bmatrix}$ is derived

$$\begin{bmatrix} P_0(q) & \cdots & P_{2n}(q) \end{bmatrix} = TR_B(q) + S_B(q) \tag{21}$$

where for the derivation of (21) the definitions in (15) have been used. Using the block matrix forms in (16) the relation (21) can be expressed as follows

$$P_j(q) = TR_j(q) + S_j(q) \quad (j = 0, 1, \ldots, 2n) \tag{22}$$

Substitute the general solution of F, given in (20), to the first block of equation (10). Then the general solution of $P(w,q)$ is derived

$$P(w,q) = TR(w,q) + S(w,q) \tag{23}$$

For the derivation of (23) the definitions in (17) have been used. Clearly, for every T there exist a unique $P(w,q)$ given by (23). Observe that (23) is equivalent to (21) and (22).

Using (23) it can be concluded that a sufficient condition for (11) to be satisfied is the condition

$$\det\left[S(w,q)\right] \not\equiv 0, \; \forall q \in \mathbb{Q} \tag{24}$$

i.e. that the matrix $S(w,q)$ is robustly invertible. Based on the invertibility of the open loop system it is observed that the condition (24) is equivalent to the condition (iii) of the theorem.

□

For the definition of $\mathrm{rank}_{\mathbb{R}}[\bullet]$ see [18]. For its computation a finite step explicit algorithm has been proposed in [17].

Based on the proof of the above theorem the following theorem can be presented.

Corollary 3.1: If the conditions of Theorem 3.1 are satisfied the general class of the controller matrix F solving the problem is

$$F = T\left[L_D(q)\right]_{\mathbb{R}}^{\perp} + \left\langle N_D(q) \setminus L_D(q) \right\rangle_{\mathbb{R}}$$

where T is an $m \times (n - \mathrm{rank}_{\mathbb{R}}\left[L_D(q)\right])$ arbitrary matrix being independent from the uncertainties and it is restricted to satisfy the condition

$$\det\left[TR(s,q) + S(s,q)\right] \not\equiv 0, \; \forall q \in \mathbb{Q} \tag{25}$$

The matrix G can be any arbitrary invertible matrix being independent from the uncertainties. □

IV. Conclusion

The robust disturbance rejection problem for left invertible linear generalized state space systems with nonlinear uncertain structure has been studied for the first time. Sufficient conditions for the solution of the problem have been established (Theorem 3.1). Under these conditions the general expressions of the controller matrices solving the problem have analytically been determined (Corollary 3.1).

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