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Surface and Guided Waves in Composites with Nematic Coatings

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***Abstract*—**The theoretical prediction of the acoustical polarization effects in the heterogeneous composites, made of thick elastic solids with thin nematic films, is presented. The numerical-analytical solution to the problem of the different wave propagation exhibits some new physical effects in the low frequency domain: the appearance of the critical frequency and the existence of the narrow transition zone where the wave rapidly changes its speed. The associated wave attenuation is highly perturbed in this zone. We also show the possible appearance of the critical frequencies where the attenuation changes the sign. The numerical results of parametrical analysis are presented and discussed.

***Keywords*—**Surface wave, guided wave, heterogeneous composite, nematic coating.

I. Introduction

Being relatively new materials the nematic elastomers exhibit some attractive physical properties, caused by the local orientational symmetry breaking and the entropic rubber elasticity. This class of materials is often called nematic liquid-crystalline rubber-like elastomers. They possess both the strong anisotropy of viscoelastic properties and the additional internal degree of freedom [1-4]. The latter is related to the possible motion of molecular chains having the main orientation (director) at the cross link points. In contrast to the ordinary solids, where the deformations are created by relative movement of the same atoms or molecules forming the bonded low-symmetry lattice, in polymer networks the macroscopic elasticity arises from the entropy change of chains on relative movement of their cross-linked end points, which are relatively far apart. In nematic elastomers, the local director can rotate, in principle, independently of deformation of the cross-linking points. So, by their properties the nematic can be classified as the nontraditional material where the relative movement of cross-linking points provides elastic strains and forces, while the director rotation causes local torques and couple stresses (Cosserat medium). They also can be subdivided into the so-called ideally soft elastomers and non-ideal nematics due to their shear moduli at quasistatic limit. Despite the high interest towards during the last two decades, especially in context of bio-applications and of the possible use as matrix for nano-composites, the acoustic properties of these soft matters have been poorly investigated.

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As it has been recently shown in [5, 6], the characteristics of the acoustical waves in the infinite nematic media strongly depend on the wave polarization and on the direction of propagation. It results in the opportunity of using nematic as acoustical polarizer or filter. Our interest is focused on the problem how the property of nematics may influence the wave propagation in the heterogeneous composites, combining traditional solids with thin nematic coatings. It concerns primarily the analogues with the classical Rayleigh and Love waves in the coated half-space, and the guided fundamental modes in the coated plates.

The analytical and numerical analysis presented below is based on the study of the dispersion relations obtained and shows some exciting phenomena in the low-frequency domain.

II. Modeling Nematic Medium

The constitutive equations have been obtained in [5, 6] by using variation principle applied to the Lagrangian and the dissipation function [4]. They result in the following form

$$\rho\partial_t^2\mathbf{u} = \nabla\cdot\underline{\underline{\boldsymbol{\sigma}}}^{symm}, \quad 0 = \mathbf{n}\times\left[D_1\mathbf{n}\times\boldsymbol{\theta} + D_2\mathbf{n}\times\underline{\underline{\boldsymbol{\varepsilon}}}\right], \qquad (1)$$

where $\mathbf{u}$ is a displacement vector, $\underline{\underline{\boldsymbol{\varepsilon}}}$ is strain tensor and $\underline{\underline{\boldsymbol{\sigma}}}^{symm}$ is a symmetrical part of stress tensor, $\mathbf{n}$ is director and $\boldsymbol{\theta}$ is an independent rotational variable. Here ρ denotes mass density, D_1 and D_2 are rotational stiffnesses. The second equation for torques is quasistatic since the reduced radius of inertia of microstructure is very small. The elastic part of stresses is related with strain by Hooke's low for the transversal anisotropy. Assuming that director is parallel to the axis x_3 of the Cartesian coordinates the stress components acquire the form $(\alpha\alpha = 11,22,33)$

$$\sigma_{\alpha\alpha} = \left(1+\tau_R\partial_t\right)\left(c_{\alpha1}\varepsilon_{11} + c_{\alpha2}\varepsilon_{22} + c_{\alpha3}\varepsilon_{33}\right), \qquad (2)$$

$$\sigma_{12} = \sigma_{21} = \left(1+\tau_R\partial_t\right) c_{66}\gamma_{12}, \qquad (3)$$

$$\sigma_{\alpha3} = c_{44}\left(1+\tau_R\partial_t\right)\gamma_{\alpha3} + (-1)^{\alpha}/2\; D_2\left(1+\tau_2\partial_t\right)\theta_\alpha, \qquad (4)$$

with familiar notations of strains $\varepsilon_{\alpha\alpha}$, $\gamma_{\alpha\beta}$ and elastic moduli $c_{\alpha\beta}$ and c_α:

$$c_{11} = c_{22} = \tfrac{2}{9}c_1 - \tfrac{4}{3}c_2 + 2c_3 + \tfrac{20}{9}c_4,$$

$$c_{12} = \tfrac{2}{9}c_1 - \tfrac{4}{3}c_2 + 2c_3 - \tfrac{16}{9}c_4,$$

$$c_{13} = c_{23} = -\tfrac{4}{9}c_1 + \tfrac{2}{3}c_2 + 2c_3 - \tfrac{4}{9}c_4,$$

$$c_{33} = \tfrac{8}{9}c_1 + \tfrac{8}{3}c_2 + 2c_3 + \tfrac{8}{9}c_4,\ c_{44} = c_{55} = 2c_5,\ c_{66} = 2c_4.$$

The time of relaxation $\tau_R = 10^{-6} \div 10^{-5} s$ is the Rouse time for the corresponding polymer chains, the typical relaxation times for the director are $\tau_1 = 10^{-1} \div 10^{-2}$ [7, 8] and $\tau_2 \sim 10^{-4}$, the latter satisfies the inequality[4]

$$D_2^2\tau_2^2 \le 8c_5 D_1 \tau_R \tau_1.$$

While considering the time harmonic processes $\tau_2 \sim 10^{-4}$ the quasistatic nature of torques yields

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \frac{D_2}{D_1}\frac{1-i\omega\tau_2}{1-i\omega\tau_1}\begin{bmatrix} -\varepsilon_{23} \\ \varepsilon_{13} \end{bmatrix}, \quad (5)$$

and in (2), (3) we set

$$1+\tau_R\partial_t \to 1-i\omega\tau_R \ \text{ with } \ \sigma_{\alpha 3} = (1-i\omega\tau_R)c_{44}^R\gamma_{\alpha 3}, \quad (6)$$

using renormalized shear modulus

$$c_{44}^R = 2c_5 - \frac{D_2^2}{4D_1}\frac{(1-i\omega\tau_2)^2}{(1-i\omega\tau_1)(1-i\omega\tau_R)}. \quad (7)$$

The phenomenological model (1)-(7) permits us to consider the effective medium, which has complex moduli, similarly to the anisotropic viscoelatic solid for all stresses except the transverse shear ones, where the renormalized modulus (7) is introduced.

III. Layered Composite

Let us now consider the combination of elastic solid and thin nematic coating as shown in Fig. 1 ($z = x_3$). Now the director might have an arbitrary position, but the qualitative difference is expected when it is directed vertically or horizontally. Assume that the surface "+" of the coating is stress free, and on the surface "-" we have full contact, i.e., the displacements and stresses on the horizontal area are continuous. In the case of coated plate the lower part of plate is symmetrical to the upper part shown in Fig. 1 with respect to the middle plane where the origin is placed.

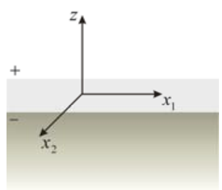


Fig. 1 Geometry of coating

The following cases are considered.

A. Quasi Rayleigh wave in a coated half space

B. Love wave in a coated half space

C. Fundamental A0 and S0 modes in a coated plate with a pure elastic core

For each case A, B or C the isotropic elastic core is considered, the wave propagation is assumed in the direction x_1 and three variants of the director orientation along each of the Cartesian axes are investigated. To this end, in each solid we seek for the solution in the form of linear combination of partial waves (the factor $e^{-i\omega t}$ is omitted)

$$\mathbf{u}(x_1, x_2, \omega) = \mathbf{U}(x_3)e^{ikx_1}, \quad (8)$$

satisfying the equations of motion, the conditions on the interface and the conditions on the faces (if any). The respective relations are obtained by substitution of expression (8) into formulas (1)-(7). For the isotropic solid the relations can be obtained in a standard way [9]. All this yields the matrix relation for the unknown coefficients of partial waves resulting in the dispersion equation (determinant of matrix equals zero). The desired parameters of waves must generally be complex by virtue of damping in the material of coating.

IV. Parameters of Media

Two nematic elastomer are considered as ideal and non ideal for which

$$c_{44}^R(\omega)\Big|_{\omega=0} = 0 \ \text{or}\ c_{44}^R(\omega)\Big|_{\omega=0} \ne 0,$$

respectively. Other parameters of nematic media are taken in the form $\rho = 10^3$ kg/m^3, $c_1 = 2c_2 = 2c_4 = \mu_0 = 10^5$ N/m^2, $c_3 = 10^9$, $c_5 = \mu_0(r+1)^2/8r$, $D_1 = \mu_0(r-1)^2/r + A_0\mu_0 Q$, $D_2 = \mu_0(1-r^2)/r$ (with the parameter of the chain anisotropy $r \approx 3$), $\tau_1 = 0.01$, $\tau_2 = 0.5\times 10^{-4}$, $\tau_R = 10^{-6}$. The coefficient of non ideality is $A_0 = 0.01$ (and $A_0 = 0$ for the ideal case). All parameters are given for the vertical orientation of director, with respective of notations for another orientation. The core can be metal (Aluminum with $\rho = 2700$, the Young modulus $E = 0.688618\times 10^{11}$ and Poisson's ratio $\nu = 0.3442$) or plastic (Polystyrene $\rho = 1060$, $E = 0.376413\times 10^{10}$, $\nu = 0.34256$).

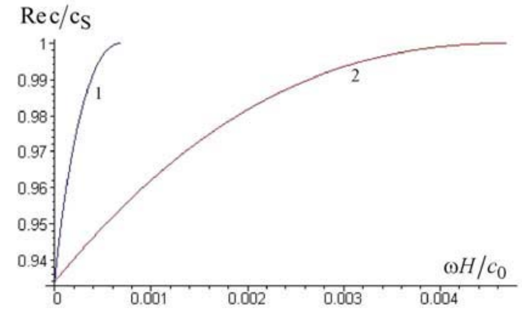
V. Results of Analysis

The algorithm begins with the approximate limit value of the wave speed (or wave number) at low frequency. Note that this value has a very small imaginary part and it is real at zero frequency. This value is substituted into the dispersion equation and then it is corrected by Newton's method [10] to evaluate complex roots of transcendental equation. After obtaining the roots for a few low frequencies we approximate the root for next frequency using the approximation of dispersion curve by the Lagrange polynomials [10]. The procedure uses a changeable step of searching roots with permanent accuracy control.

A. Quasi Rayleigh wave in a coated half space

At low frequency the wave speed is naturally close to the real value of the classical Rayleigh wave of a pure elastic half-space, since the relative thickness of a coating is infinitesimal. The imaginary part of speed appears while increasing the frequency, it growths first and then begins to decrease. Our analysis has shown the existence of critical frequency at which this imaginary part acquires zero value and for greater frequency the surface wave does not exist. The effect takes place both for the ideal and non ideal nematic regardless the type of core, whose parameters may numerically change the critical frequency but the remains qualitatively the same. In Fig. 2 the case of non ideal coating with vertical director is presented; the curve 1 corresponds to the metal and the curve 2 – to the plastic. The speed of surface wave is normalized over the speed of shear wave, the thickness of coating is denoted by $H = 2h$ and the scale of wave speed for nematic medium is $c_0 = \sqrt{\mu_0/\rho}$. The result is also qualitatively stable with respect to changing the director orientation.

Similar effect can be seen in some crystals [10] whose particular anisotropy orientation leads to the changing of the wave type and may cause the appearance of the adjacent waves.



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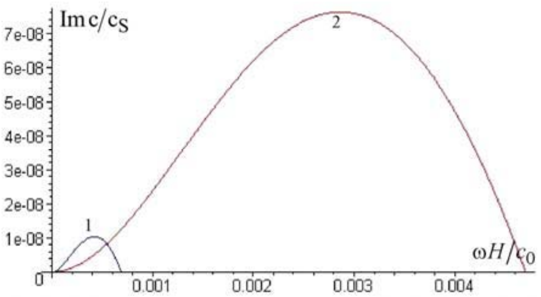
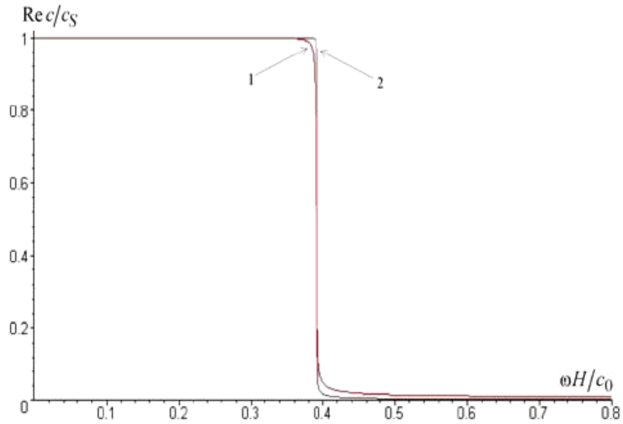


(b)

Fig. 2 Complex wave speed $c = \omega/k$ of the quasi Rayleigh wave

B. Love wave in a coated half space

The existence of similar critical frequency can be seen for Love wave only for the vertical director orientation. In this case the result looks similarly to that shown in Fig. 2. When the director is situated in the horizontal plane the result is similar to the classical Love wave for the half-space with an effective quasi elastic coating (viscosity can be neglected) with a very small attenuation. While the frequency increases we approach the narrow transition zone where the wave speed rapidly changes. The imaginary part behaves by differential low with respect to the real part (see Fig. 3). The attenuation growths and we deal with the highly oscillating and highly attenuating wave. The phase speed and the attenuation are shown in Fig. 4.



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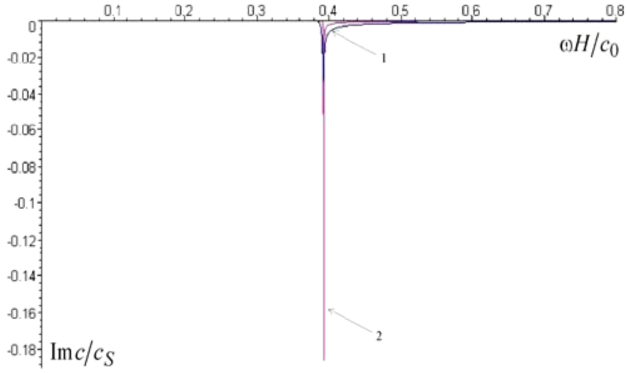
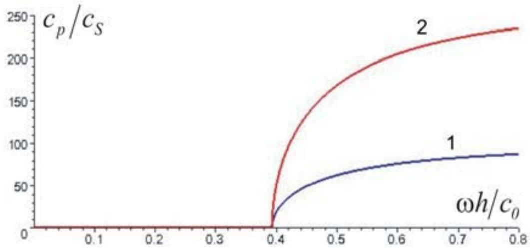


(b)

Fig. 3 Complex wave speed of $c = \omega/k$ Love wave



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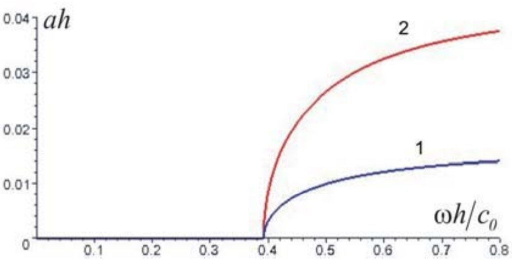

(b)

Fig. 4 Phase speed $c_p = \mathrm{Re}(\omega/k)$ and attenuation of Love wave

The curves 1 in Figs. 3-4 correspond to the combination of plastic core with nematic director along the axis x_1, the curves 2 correspond to the orientation along x_2.

C. Fundamental A0 and S0 modes in a coated plate with a pure elastic core

Let us now consider the coated plate of symmetrical structure across the thickness. The low frequency limit behavior of the A0 wave number is given by the Kirchhoff-Love approximated theory for bending of thin layered plate. The approximation for S0 mode is obtained bye the approximate dynamic plane stress state. Having these limit values [8] we apply the above algorithm to evaluate the exact wave numbers.

The results for different combinations of the core materials and nematic media are qualitatively similar. The typical for the aluminum plate coated by the non ideal nematic material is presented in Fig. 5. As one can see in this scale the graphs have two interesting properties: both A0 and S0 modes have the same high frequency behavior, and there is a critical frequency where the wave attenuation changes its sign. It means that since this frequency the correct wave must have a wave number of the opposite sign, i.e., the process of wave propagation should have a transition character in the vicinity of such critical frequency.

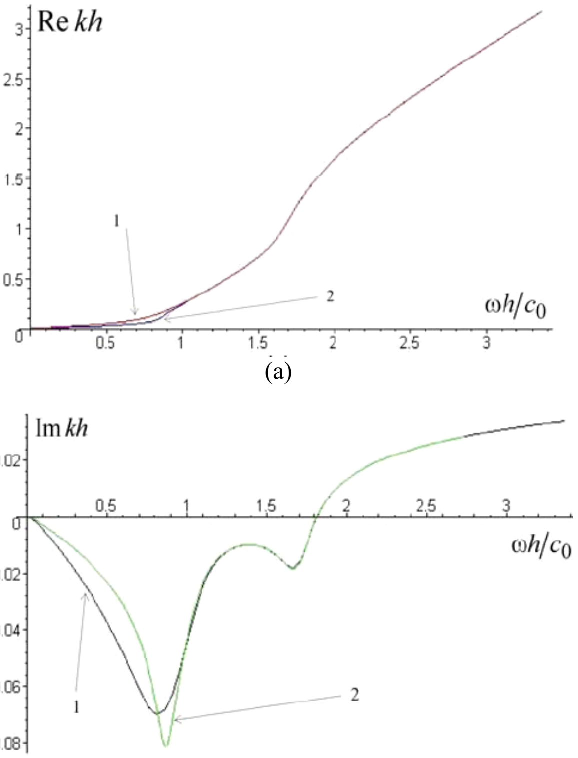

Fig. 5 Phase speed and attenuation of fundamental modes: A0 mode (curve 1) and S0 mode (curve 2)

VI. CONCLUSION

The implementation of the transfer matrices and the impedance matrices techniques, permitting us to describe the wave propagation in isotropic elastic medium with nematic coating have been deduced and investigated. The numerical-analytical solution to the problem of surface wave propagation in such heterogeneous composite has been obtained for exact formulation. New physical effects have been revealed in the low frequency domain: the appearance of the critical frequency above which the quasi Rayleigh wave does not exist; the existence of the narrow transition zone for quasi Love wave where it rapidly changes its characteristics. The numerical-analytical solution to the problem of quasi Lamb wave propagation in the layered plate has been obtained for exact formulation. For the fundamental modes S0 and A0 the unusual effects have been revealed: the appearance of the transition zones of dispersion curves with essential curvatures and the appearance of critical frequencies where the attenuation changes the sign. Thus, we may conclude that the acoustical polarization effects known for body waves in nematic elastomers can be also seen in heterogeneous structures. The new quality of wave propagation can be achieved using rather thin nematic coating at low frequency.

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