International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:6, No:9, 2012

Lower Bounds of Some Small Ramsey Numbers

Decha Samana* and Vites Longani

Abstract—For positive integer s and t, the Ramsey number $R(s,t)$ is the least positive integer n such that for every graph G of order n, either G contains K_s as a subgraph or $\overline{G}$ contains K_t as a subgraph. We construct the circulant graphs and use them to obtain lower bounds of some small Ramsey numbers.

Keywords—Lower bound, Ramsey numbers, Graphs, Distance line.

I. Introduction

FOR positive integer s and t, the Ramsey number $R(s,t)$ is the least positive integer n such that for every graph G of order n, either G contains K_s as a subgraph or $\overline{G}$ contains K_t as a subgraph.

The problem of determining Ramsey numbers is known to be very difficult. The few known exact values and several bounds for different graphs are scattered among many technical paper [1]

s \ t	3	4	5	6	7	8	9	10	11
3	6*	9*	14*	18*	23*	28*	36*	40	46
4		18*	25*	35	49	56	73	92	98
5			43	58	80	101	126	144	171
6				102	113	132	169	179	253

* *Exact Ramsey numbers*

Table 1. Known nontrivial values and some lower bounds for Ramsey numbers $R(s,t)$.

For small Ramsey numbers $R(s,t)$, the general method in establishing a lower bound is to construct a graph G which does not contain K_s and the $\overline{G}$ of G does not contain K_t. In this paper, we construct the circulant graphs and use them to obtain lower bounds for some small Ramsey numbers.

definition 1. *Let G be a circulant graph with n vertices and i,j be vertices in G. The line distance of line $\{i,j\}$, denoted by d_{ij}, is defined as*

$$d_{ij} = min\{|i-j|, n-|i-j|\}$$

and a line distance set is a set of the line distances.

*Corresponding author. Email: dechasamana@hotmail.com
D. Samana is with Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand 10520.
V. Longani is with College of Arts, Media and Technology, and Mathematics Department, Faculty of Science, Chiang Mai University, Chiang Mai, Thailand.

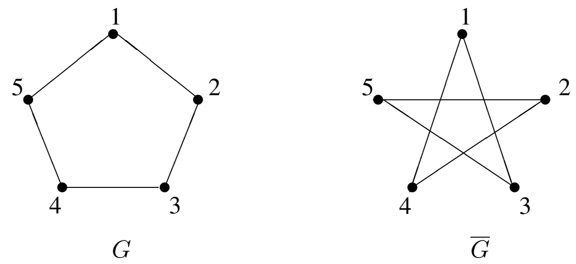


Figure 1

Figure 1, line distance of G is 1 and line distance of $\overline{G}$ is 2. In figure 2, line distances of G are $1, 2$ and 4. This is, line distance set of G is $\{1,2,4\}$ and line distance set of $\overline{G}$ is $\{3\}$.

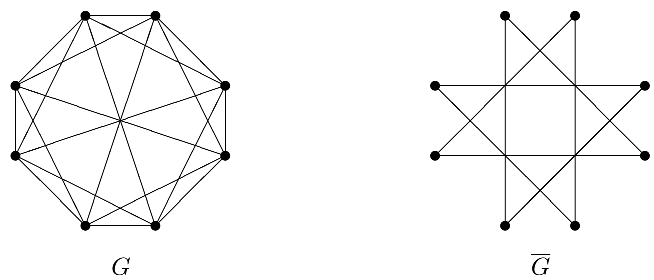


Figure 2

In section II, we construct line distance sets in order to find lower bounds of some Ramsey numbers.

II. The main results

In this section, we find lower bounds of $R(3,10), R(3,11)$, and $R(3,12)$ by constructing line distance sets of G and $\overline{G}$.

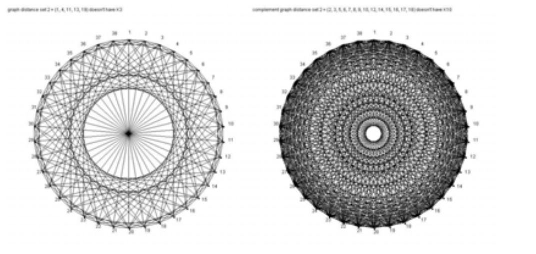
Since G and $\overline{G}$ have symmetric patterns, in verifying that G does not contain K_s and $\overline{G}$ does not contain K_t we can have one vertex fixed and only need to consider other $s-1$ vertices for the case of K_s and other $t-1$ vertices for the case of K_t.

Theorem 1. $R(3,10) \geq 39$.

Proof: The graph G of order 38 in Figure 3a has line distance set as $\{1,4,11,13,19\}$,and the graph $\overline{G}$ in Figure 3b has line distance set as $\{2,3,5,6,7,8,9,10,12,14,15,16,17,18\}$.

It can be verified that G contains no K_3 and $\overline{G}$ contains no K_{10}. According to the definition of Ramsey numbers, we have that $R(3,10) \geq 39$. ∎

Next, we have a lower bound of $R(3,11)$.



(a) (b)

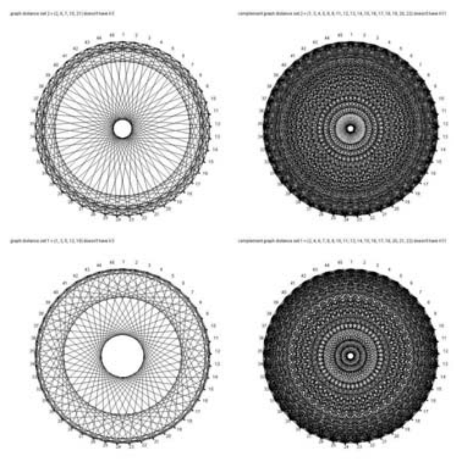
Figure 3. lower bound of Ramsey number $R(3,10) > 38$.

Theorem 2. $R(3,11) \geq 46$.

Proof: We have 6 line distance sets of G and $\overline{G}$ of order 45, see Figure 4 and Figure 5.

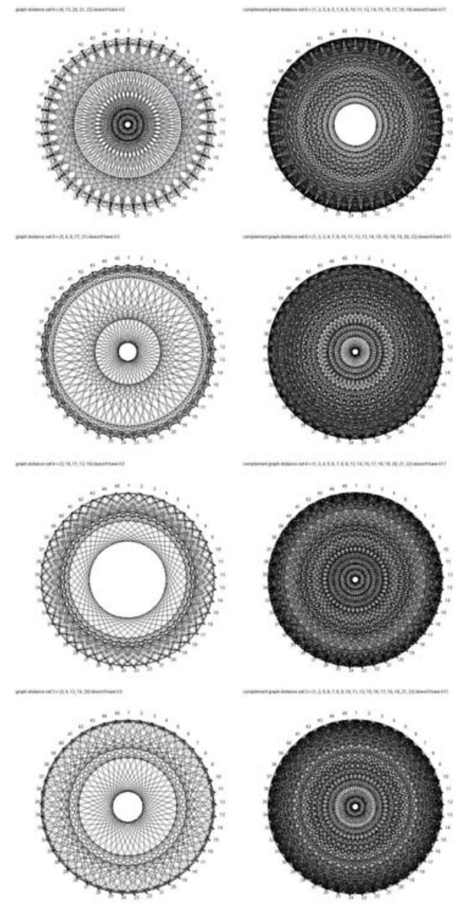
$$\{1,3,5,12,19\},$$
$$\{2,4,6,7,8,9,10,11,13,14,15,16,17,18,20,21,22\};$$
$$\{2,6,7,10,21\},$$
$$\{1,3,4,5,8,9,11,12,13,14,15,16,17,18]19,20,22\};$$
$$\{3,4,12,14,20\},$$
$$\{1,2,5,6,7,8,9,10,11,13,15,16,17,18,19,21,22\};$$
$$\{3,10,11,12,16\},$$
$$\{1,2,4,5,6,7,8,9,13,14,15,17,18,19,20,21,22\};$$
$$\{5,6,8,17,21\},$$
$$\{1,2,3,4,7,9,10,11,12,13,14,15,16,18,19,20,22\};$$
$$\{6,13,20,21,22\},$$
$$\{1,2,3,4,5,7,8,9,10,11,12,14,15,16,17,18,19\}.$$

It can be verified from each G and $\overline{G}$ that G does not contain K_3 and $\overline{G}$ does not contain K_{11}.
Hence $R(3,11) \geq 46$. ■



(a) (b)

Figure 4. lower bound of Ramsey number $R(3,11) > 45$.



(a) (b)

Figure 5. lower bound of Ramsey number $R(3,11) > 45$.

Next, we have a lower bound for $R(3,12)$.

Theorem 3. $R(3,12) \geq 49$

Proof: We have 12 line distance sets of G and $\overline{G}$ of order 48.

$$\{1,3,8,14,18,24\},$$
$$\{2,4,5,6,7,9,10,11,12,13,15,16,17,19,20,21,22,23\};$$
$$\{2,3,8,14,15,24\},$$
$$\{1,4,5,6,7,9,10,11,12,13,16,17,18,19,20,21,22,23\};$$
$$\{2,3,8,17,18,24\},$$
$$\{1,4,5,6,7,9,10,11,12,13,14,15,16,19,20,21,22,23\};$$
$$\{2,7,8,18,21,24\},$$
$$\{1,3,4,5,6,9,10,11,12,13,14,15,16,17,19,20,22,23\};$$
$$\{2,8,9,14,21,24\},$$
$$\{1,3,4,5,6,7,10,11,12,13,15,16,17,18,19,20,22,23\};$$
$$\{3,8,9,10,22,24\},$$
$$\{1,2,4,5,6,7,11,12,13,14,15,16,17,18,19,20,21,23\};$$
$$\{5,6,8,15,22,24\},$$
$$\{1,2,3,4,7,9,10,11,12,13,14,16,17,18,19,20,21,23\};$$
$$\{6,8,9,10,13,24\},$$
$$\{1,2,3,4,5,7,11,12,14,15,16,17,18,19,20,21,22,23\};$$
$$\{6,8,9,19,22,24\},$$

$$\{1,2,3,4,5,7,10,11,12,13,14,15,16,17,18,20,21,23\};$$
$$\{6,8,10,11,15,24\},$$
$$\{1,2,3,4,5,7,9,12,13,14,16,17,18,19,20,21,22,23\};$$
$$\{8,10,15,21,22,24\},$$
$$\{1,2,3,4,5,6,7,9,11,12,13,14,16,17,18,19,20,23\};$$
$$\{8,14,18,21,23,24\},$$
$$\{1,2,3,4,5,6,7,9,10,11,12,13,15,16,17,19,20,22\}.$$

It can be verified from each G and $\overline{G}$ that G does not contain K_3 and $\overline{G}$ does not contain K_{12}.

Hence $$R(3,12) \geq 49.$$ ■

ACKNOWLEDGMENT

The authors would like to thank King Mongkut's Institute of Technology Ladkrabang Research Fund, King Mongkut's Institute of Technology Ladkrabang, Thailand.

REFERENCES

[1] S.P.Radziszowski, Small Ramsey Numbers, *Electronic Journal of Combinatorics*, Dynamic Survey 1, revision♯12, August 2009, http://www.combinatorics.org.
[2] G. Chartrand and L. Lesniak, *Graph&Digraphs*, 4th Edition, Chapman&Hall, 1996.
[3] D. Samana, V. Longani, Lower Bounds of Ramsey Numbers $R(s,t)$, IAENG International Journal of Applied Mathematics, Volume 39, Issue 4, November(2009), http://www.iaeng.org/IJAM.

Decha Samana received B.S. in Mathematics from Naresuan University, M.S. in Applied Mathematics and Ph.D. in Mathematics from Chiang Mai University, Thailand. He is lecturer in Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Thailand. His research interest is Graph Theory and Combinatorics.

Vites Longani received B.S. in Mathematics from Chiang Mai University, M.S. and Ph.D. in Mathematics from University of London, United Kingdom. He is professor in Department of Mathematics, Faculty of Science, Chiang Mai University, Thailand. His research interest is Graph Theory and Combinatorics.