Optimal Image Compression Based on Sign and Magnitude Coding of Wavelet Coefficients

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Abstract—Wavelet transforms is a very powerful tools for image compression. One of its advantage is the provision of both spatial and frequency localization of image energy. However, wavelet transform coefficients are defined by both a magnitude and sign. While algorithms exist for efficiently coding the magnitude of the transform coefficients, they are not efficient for the coding of their sign. It is generally assumed that there is no compression gain to be obtained from the coding of the sign. Only recently have some authors begun to investigate the sign of wavelet coefficients in image coding. Some authors have assumed that the sign information bit of wavelet coefficients may be encoded with the estimated probability of 0.5; the same assumption concerns the refinement information bit. In this paper, we propose a new method for Separate Sign Coding (SSC) of wavelet image coefficients. The sign and the magnitude of wavelet image coefficients are examined to obtain their online probabilities. We use the scalar quantization in which the information of the wavelet coefficient to belong to the lower or to the upper sub-interval in the uncertainly interval is also examined. We show that the sign information and the refinement information may be encoded by the probability of approximately 0.5 only after about five bit planes. Two maps are separately entropy encoded: the sign map and the magnitude map. The refinement information of the wavelet coefficient to belong to the lower or to the upper sub-interval in the uncertainly interval is also entropy encoded. An algorithm is developed and simulations are performed on three standard images in grey scale: Lena, Barbara and Cameraman. Five scales are performed using the biorthogonal wavelet transform 9/7 filter bank. The obtained results are compared to JPEG2000 standard in terms of peak signal to noise ration (PSNR) for the three images and in terms of subjective quality (visual quality). It is shown that the proposed method outperforms the JPEG2000. The proposed method is also compared to other codec in the literature. It is shown that the proposed method is very successful and shows its performance in term of PSNR.

Keywords—Image Compression, Wavelet Transform, Sign Coding, Magnitude Coding.

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I. INTRODUCTION

IMAGE compression is used for storage and transmission in multimedia applications. For still image compression, JPEG (based on Discrete Cosine Transform DCT.) [1]-[3] and the recent JPEG2000 [20] (based on Wavelet Transform technology.) are the standards used today. In DCT, image is split in blocks of 8 x 8 pixels and the transform is applied to each block as an independent sub-image. Variable Length Coding (VLC) is used to compress the quantized coefficients. Even JPEG is used until today, its main drawback is the blocking artifacts at low bit rate despite some attempts to overcome this drawback [4], [5]. However, in JPEG2000, the image is decomposed in wavelet domain and the wavelet coefficients are entropy encoded. The image is not split in blocks before the application of wavelet transform. Only in the case where the image dimensions are large (such us the case of JPEG2000 image tests.), the standard allows to split image in tiles to efficiently manage the memory in wavelet transform computation.

Recently, wavelet transform is considered as an alternative to overcome the drawbacks of DCT JPEG. Moreover wavelet-based coding schemes are better matched to Human Visual System (HVS) characteristics [6] due to the multi-resolution analysis properties of wavelet transform. Since, many developments have been done in wavelet based image compression. J. Shapiro has proposed the embedded image coding using Zerotrees of Wavelet coefficients, called EZW codec [7]; since many other papers have been published in this field [8]-[19]. One of the advantage of the wavelet transform is the provision of both spatial and frequency localization of image energy. The energy of wavelet transform is restricted to non negative real numbers. However, the wavelet transform coefficients themselves are not restricted to as such and are defined by both a magnitude and a sign. In all most current wavelet image coding systems, the inefficient coding of the sign of coefficients is accepted as a trade-off for gain obtained through energy compaction. The energy compaction capability says nothing about the nature of the sign of wavelet coefficients. In [7], the author states that a quantized coefficient is equally likely to be positive or negative. So, in early wavelet image coding methods, compression of the sign information of wavelet coefficients was considered impossible with common explanation that the high frequency subbands are zero mean subbands, and therefore are equally likely positive as negative. Only recently, have some authors begun

to investigate the sign of wavelet coefficients in image coding [24]-[28]. In [28], the authors have assumed that the sign information bit of wavelet coefficients may be encoded with an estimated probability of 0.5 and the same assumption concerns the refinement information bit.

In our preview works [29]-[32], we have proposed a codec approach based on probability distribution of EZW symbols. The probability distributions of EZW symbols are estimated. Binary codes are defined based on these probability distributions and used for encoding the significance map. The sign and magnitude of the wavelet coefficients are together encoded using a single binary code word.

In this paper, we propose a new method for Separate Sign Coding (SSC) of wavelet image coefficients. The sign and the magnitude of wavelet image coefficients which are the main information are examined to obtain their online probabilities. We use the scalar quantization in which the information of the wavelet coefficient to be set in the lower or in the upper sub-interval in the uncertainly interval is also examined. The remaining of this paper is organized as follows: In section 2, we present some basics of wavelet transform. A short description of EZW codec is presented in section 3. Our approach is described in section 4. The experimental results and discussions are presented in section 5. Finally, conclusion and perspectives of this work are presented in section 6.

II. BASICS OF WAVELET TRANSFORM

Subband based coding systems has been receiving increased interest as an alternative to block DCT based coding, as they overcome the blocking effect problem and produce images of superior subjective quality. Subband schemes in particular that implement the Discrete Wavelet Transform (DWT) have been receiving significant attention in the field of image compression. The basic idea behind the DWT is to represent any arbitrary function f as a weighted sum of a set of basis functions which are scaled and shifted version of a single mother wavelet ψ. The wavelet decomposition is defined by equation 1:

$$f(t) = \sum_a \sum_b C(a,b)\psi_{a,b}(t) \qquad (1)$$

where $\psi_{a,b}(t) = \dfrac{1}{\sqrt{a}}\psi\left(\dfrac{t-b}{a}\right)$

There exist special choices of ψ such that the set $\{\psi_{a,b}\}$ form an orthogonal basis of $L^2(R)$. In that case:

$$C(a,b) = \langle \psi_{a,b} \rangle = \int \psi_{a,b}(t)f(t)dt \qquad (2)$$

The DWT can be computed via an octave band subband decomposition, where the filter coefficients are derived from the wavelet ψ and satisfy some regularity requirements. The Figs. 1 and 2 show one scale of 2D-DWT decomposition and reconstruction schemes respectively where HPF and LPF are the High Pass Filter and the Low Pass Filter respectively.
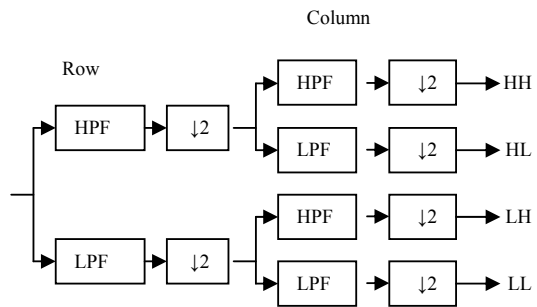

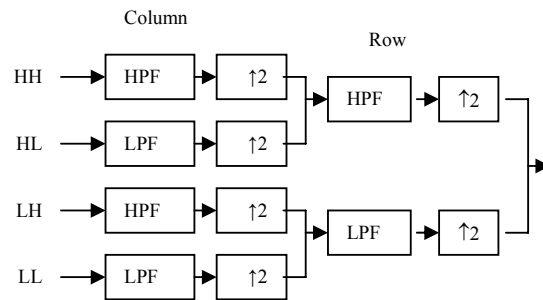Fig. 1 One scale 2D-DWT decomposition scheme

Fig. 2 One scale 2D-DWT reconstruction scheme

However, orthogonal wavelet filters cannot have linear phase, which is desirable in image processing applications. Therefore, biorthogonal wavelet basis [18] is more suitable for image processing applications. In that case, the reconstruction formula is defined by the equation 3:

$$f(t) = \sum_a \sum_b C(a,b)\widetilde{\psi}_{a,b}(t) \qquad (3)$$

where ψ et $\widetilde{\psi}$ are orthogonal to each other. While a set of biorthogonal wavelet filters creates a subband coding scheme with perfect reconstruction in the presence of quantization noise, the choice of the filters can play an important role in the performance of system. A comparative study to determine the role of various characteristics of the wavelet filters such as regularity, filter length have been done by several authors and especially in [21] and in [22].

III. EMBEDDED ZEROTREE WAVELET CODEC

The EZW encoder encodes images in embedded fashion from their dyadic wavelet representations. The goal of embedded coding is to generate a single encoded bit stream that allows achieving any desired bit rate while giving the best reconstructed quality at this rate. In wavelet domain, image is represented by approximation subband (called DC or LL subband) and detail subbands (called AC or HL_i, LH_i, and HH_i, subbands at scale i.) as illustrated in the Fig. 3 (for two scales). The EZW encoder encodes wavelet coefficients by using a sequence of thresholds T. The initial value of threshold T_0 is defined such that $T_0 \succ \dfrac{C}{2}$ where C is the

maximum wavelet coefficient. A coefficient X_i is considered as significant if $|X_i| \geq T$. Significance map, which consists of scanning the wavelet coefficient array to decide if a wavelet coefficient is significant, is generated at each bit plane. Two passes are performed for each threshold value: the dominant pass and the subordinate pass. All significant coefficients found in dominant pass are encoded by four symbols. The tree is structured according to a rule such that a parent coefficient in AC subband is linked with four children in the next finer subband. Only the parent coefficient in DC subband is linked with three children, one in each of the three coarse AC subbands (see figure 3). The four symbols used for encoding the coefficients are ZTR (ZeroTree Root.), IZ (Isolated Zero.), POS (significant positive.) and NEG (significant negative.). ZTR symbol is generated for an insignificant coefficient, which has no significant child. IZ symbol is generated for an insignificant coefficient, which has at least one significant child. POS and NEG are generated for significant coefficients which are positive and negative respectively. In finer AC subbands (HL_1, LH_1 and HH_1.) where the coefficients have no child, the IZ and ZTR symbols are merged to form the Z (zero.) symbol. The subordinate pass refines the quantized coefficients to obtain the best approximation of original wavelet coefficients. The particularity of embedded coding is that it can terminate the encoding at any point and thereby allowing a target rate or target distortion metric to be met exactly. This is interesting for rate and quality scalability applications.
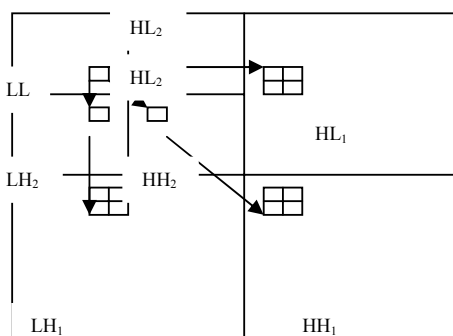

Fig. 3 Wavelet coefficients representation in EZW

IV. CODING OF SIGN AND MAGNITUDE OF WAVELET COEFFICIENTS

In all most current wavelet image coding systems, the inefficient coding of the sign of coefficients is accepted as a trade-off for gain obtained through energy compaction [27]. The energy compaction capability says nothing about the nature of the sign of wavelet coefficients. In [7], the author states that a quantized coefficient is equally likely to be positive or negative. So, in early wavelet image coding, compression of the sign information of wavelet coefficients was considered impossible with common explanation that the high frequency subbands are zero mean subbands, and

therefore are equally likely positive as negative. Only recently, some authors have begun to investigate the sign of wavelet coefficients in image coding [24]-[28]. In [27], the authors have combined sign and coefficient extrapolation in their approach. They have proposed the estimation of wavelet coefficient with probability of the sign being positive or negative. Additionally, they assumed that the distribution wavelet coefficient is independently uniform in the positive and negative domains (see [27] for details). Based on their approach, image compression results that they have obtained outperform the SPIHT codec. In [28], the authors have used the Tarp Filter technique combined with coefficient classification. They assumed that the sign information bit of coefficient may be encoded with an estimated probability of 0.5; the same assumption concerns the refinement information bit (see [28] for details); so, image compression results that they have obtained outperform the JPEG2000 for some standard test images. In this paper, we propose a new approach for separate encoding sign and magnitude information of image wavelet coefficients. Once the image is decomposed in wavelet domain, we consider the coefficient as the data which gives two kind of information: the sign and the magnitude, similar as in the [27], [28]. The magnitude is considered as important if its absolute value is greater or equal to a predefined threshold T, similar to EZW codec. In EZW, such coefficient is encoded as POS or NEG symbol if it is positive or negative respectively. In our approach, a single symbol which we call Significant (S.) is used to encode the magnitude which is greater or equal to the threshold. We use two other symbols ZT and UZT to encode the ZeroTreeroot and the UnZeroTreeroot respectively. ZT and UZT symbols may be considered as ZTR and IZ symbols in EZW codec. ZT describes a tree of zeros where all coefficients which belong to this tree are inferior to the threshold. UZT describes the tree where at least one coefficient is greater or equal to the threshold and belongs to this tree. So, if the root symbol is ZT, the decoder set to zero all coefficients which belong to this tree. However, if the root symbol is UZT, only the root coefficient is set to zero; the coefficients which belong to this tree are set to zero in the decoder if they are inferior to the threshold. In the finest subband HL_1, LH1, and HH_1 where the coefficients have no child, another symbol ZC (ZeroCoefficient.) encodes the coefficients which are inferior to the threshold.

Three kinds of information are considered in our approach:

1) The magnitude information: a magnitude map is generated and contains the S symbol. The presence of the symbol S is described by the symbol '1' and its absence is described by the symbol '0'. These two symbols (1 and 0) are entropy encoded. We propose an algorithm to encode the magnitude information and the probability of the wavelet coefficient to be significant is calculated bit plane by bit plane.

2) The sign information of wavelet coefficients: in our approach, the probability of the quantized wavelet coefficients to be positive or to be negative is calculated

bit plane by bit plane. According to our preview works and other authors, it is known that the probability to find a negative coefficient in LL subband is equal to zero. So, only the HL_k, LH_k and HH_k subbands at scale k are likely to give negative coefficients. We propose an algorithm to encode the sign map which indicates the presence of a negative or a positive coefficient in HL_k, LHk, and HH_k subbands at scale k. A sign map is generated and is described by the symbol '0' for a positive coefficient and by the symbol '1' for a negative coefficient. These symbols are then entropy encoded.

3) The third information is the refinement of the quantized coefficients. Since we have used the scalar quantizer, the quantized wavelet coefficient may be set in the lower or in the upper sub-interval in the uncertainly interval $[T, 2T]$ where T is the threshold. An uncertainly interval $[T, 2T]$ is generated progressively. The probability of the quantized coefficient to belong to the lower sub-interval $[T, (3/2)T[$ or to the upper sub-interval $[(3/2)T, 2T[$ in the refinement processing is calculated also bit plane by bit plane. If a quantized wavelet coefficient is set in the upper sub-interval, the symbol '1' is generated; if a quantized wavelet coefficient is set in the lower sub-interval, the symbol '0' is generated. These symbols are then entropy encoded.

Fig. 4 shows the algorithm for encoding of the magnitude. Fig. 5 shows the algorithm for encoding the sign, which concerns only the detail subbands since the wavelet coefficients of approximation subband are unsigned.

```
if  (map = 1)
        {do nothing }
  else
        if  (MS  = 'ZT')
             { do nothing }
        else
             if (M  >= T)
                   mmap ← '1'
                   MS ← 'S'
             else
                if (descendants  >= T)
                     MS ← 'UZT'
                else
                     MS ← 'ZT'
             end
        end
                          end
```

Fig. 4 Algorithm for magnitude encoding of detail subbands

```
if  (M(i, j) > 0)
        smap(i ,j) ← '0'
  else
        smap(i, j) ← '1'
  end
```

Fig. 5 Algorithm for sign encoding

It is important to note theses considerations:

1) The magnitude information for a same wavelet coefficient is not unique and may change with the number of bit plane: a same wavelet coefficient, which is not significant in a bit plane, will be significant in the other bit plane.

2) The refinement information is not unique for a same wavelet coefficient and may change: a same wavelet coefficient may be set in the upper sub-interval in a bit plane and may be set in the lower sub-interval in the other bit plane.

3) However, the sign information is unique for a same wavelet coefficient: in fact, a wavelet coefficient is either positive or negative.

V. EXPERIMENTAL RESULTS AND DISCUSSIONS

To evaluate our approach, three standard images: Lena, Barbara both 512 x 512 pixels in grey scale and Cameraman 256 x 256 pixels in grey scale are decomposed in wavelet using the biorthogonal 9/7 filter bank. Five scales decomposition are performed. Magnitude, sign and refinement informations are entropy encoded. We present firstly the results of online observed probabilities of: the magnitude information described by the symbol S, the sign information, and the refinement information. We present secondly the results in terms of Peak to Signal Noise Ratio (PSNR) in dB versus bit rate in bit per pixel (bpp) and the decoded Lena, Barbara and Cameraman at 0.15 bpp.
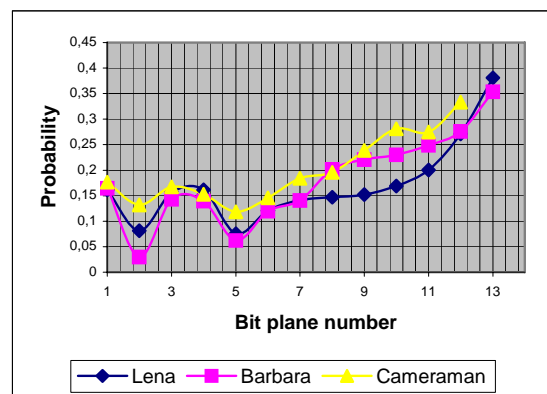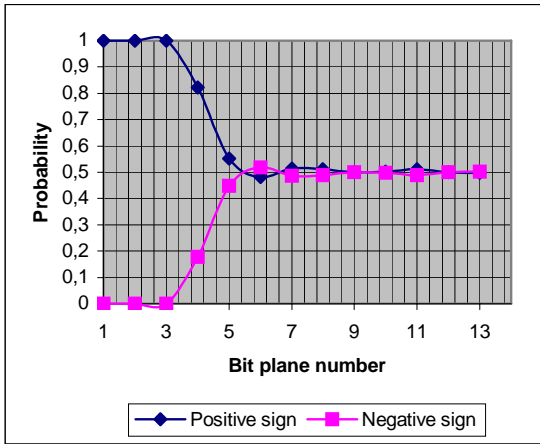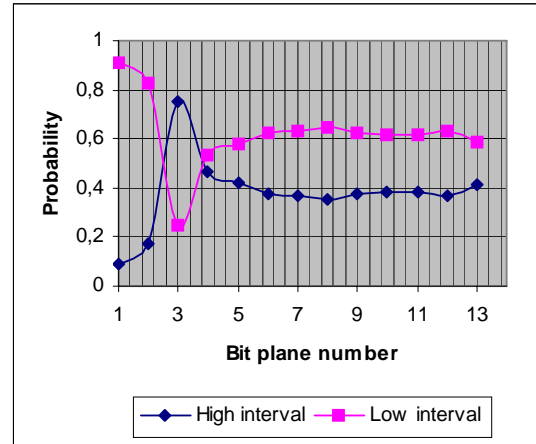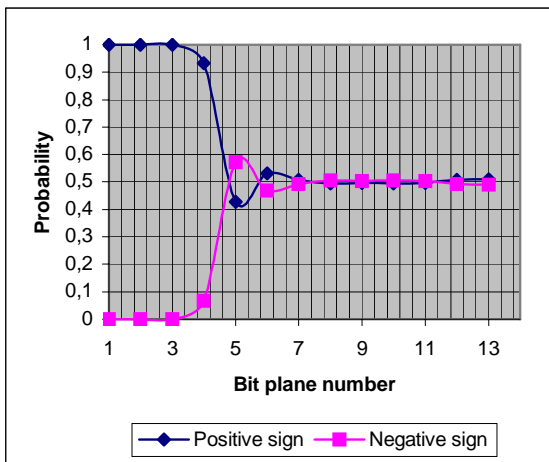

Fig. 6 Observed probabilities of magnitude information for the three standard images versus bit plane number: Lena, Barbara and Cameraman
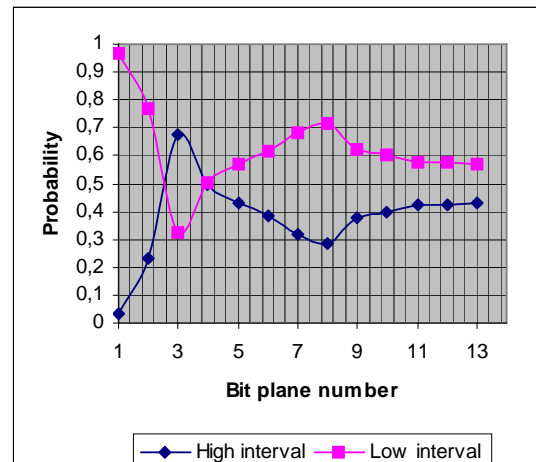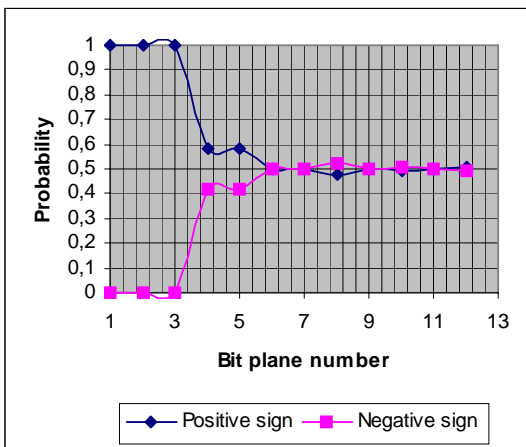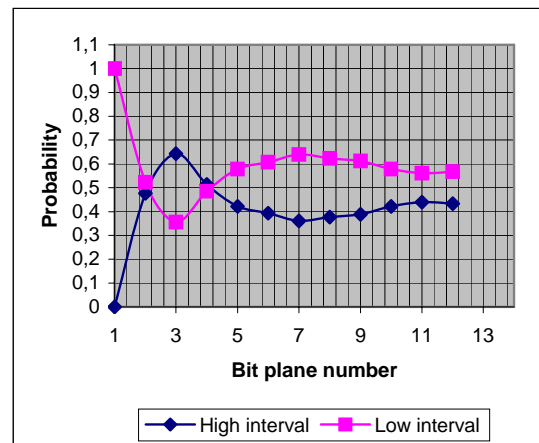
a

b

c

Fig. 7 Observed probabilities of positive and negative sign information versus bit plane number: a)Lena, b) Barbara c) Cameraman

a

b

c

Fig. 8 Observed probabilities of refinement information versus bit plane number: a) Lena, b) Barbara c) Cameraman

The very interesting behaviors observed are the sign information and the refinement information. The figure 7 (a, b and c) shows that after four bit planes, the positive sign information and negative sign information have the equal probability to appear. We may see in this figure that the wavelet coefficients are not all equally distributed in the positive and negative domains. However, some coefficients are equally distributed in positive and negative domains after about five bit planes for Lena, Barbara and Cameraman. So, the estimated probability of 0.5 may not be used to encode the sign information for all bit planes contrarily to the work presented in [28]. The figure 8 (a, b and c) presents the probabilities of the quantized wavelet coefficients to be set in the lower sub-interval or in the upper sub-interval in the refinement processing. We observe in this figure that the probabilities of the quantized wavelet coefficients to be set in the lower sub-interval or in the upper sub-interval in the refinement processing present a symmetry with the probability value of 0.5; we can observe that encoding the refinement information with the probability estimated of 0.5 is not appropriate. So, the refinement information bit may not be encoded with an estimated probability of 0.5 such as in [28].

A fundamental question may arise: these observations are image dependant? The observed probabilities are Lena, Barbara and Cameraman dependant? As the attempt to the response to this question, we have deal with other test images such as Boat, Goldhill and Peppers and we observe the same behaviors.

The figures 9, 10, and 11 present the PSNR in dB versus bit rate in bit per pixel for Lena, Barbara and Cameraman. The JPEG2000 standard [33] is run for the same test images and the results are compared with our results. We may see that our approach outperforms the JPEG2000 standard in PSNR quality and significative gains are obtained for Lena, Barbara and Cameraman images.
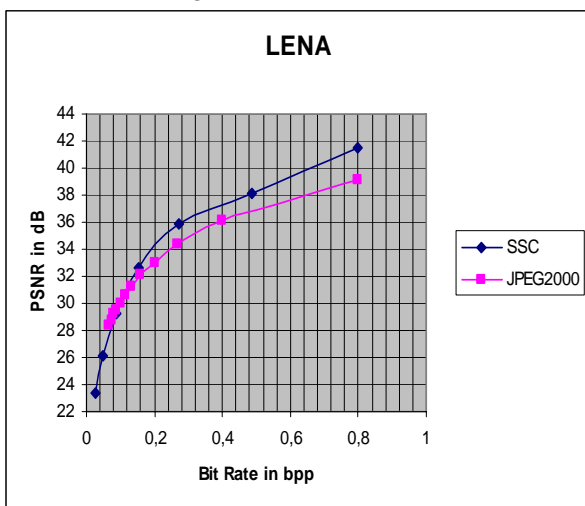

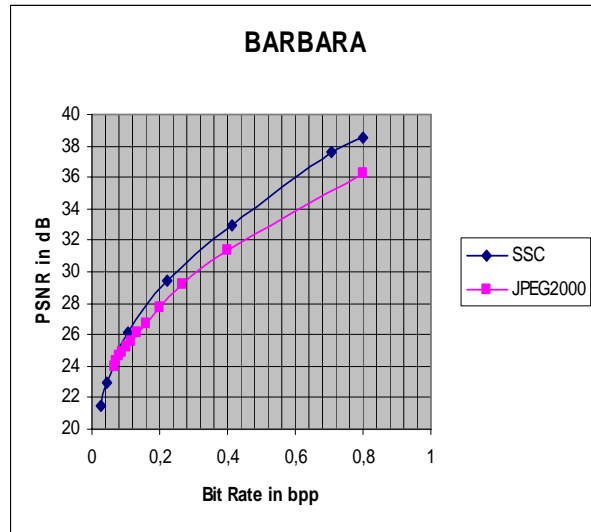
Fig. 10 PSNR versus Bit Rate for BARBARA

Fig. 11 PSNR versus Bit Rate for CAMERAMAN

Fig. 9 PSNR versus Bit Rate for LENA

JPEG2000   0.15 bpp   SSC   0.15 bpp

JPEG2000   0.15 bpp   SSC 0.15 bpp

JPEG2000   0.15 bpp   SSC 0.15 bpp

Fig.12   Comparative subjective qualities of decoded Lena, Barbara and Cameraman at 0.15 bpp
Top left: Lena for JPEG2000 decoded at 0.15 bpp, Top right: Lena for SSC decoded at 0.15bpp.
Center left: Cameraman for JPEG2000 decoded at 0.15 bpp, Center right: Cameraman for SSC decoded at 0.15bpp.
Bottom left: Barbara for JPEG2000 decoded at 0.15 bpp, Bottom right: Barbara for SSC decoded at 0.15bpp.

The figure 12 presents the subjective qualities of Lena, Barbara and Cameraman decoded at 0.15 bpp for our approach and for JPEG2000 standard. We may observe that our approach is competitive with JPEG2000 in subjective quality. Table I, table II and table III compare objective qualities in terms of PSNR in dB for Lena, Barbara and Goldhill respectively decoded at 0.15 bpp, 0.25 bpp and 0.50 bpp with the other codec in the literature. We may see that significant gains are obtained by our approach.

TABLE I
PERFORMANCE COMPARISON OF SSC WITH DIFFERENT CODEC FOR LENA

Rate	JPEG2000 [33]	SPIHT	TCE[28]	SSC
0.15	31.55	31.89	31.97	**32.18**
0.25	33.73	34.11	34.19	**35.63**
0.50	36.45	37.21	37.28	**38.10**

TABLE II
PERFORMANCE COMPARISON OF SSC WITH DIFFERENT CODEC FOR BARBARA

Rate	JPEG2000 [33]	SPIHT	TCE[28]	SSC
0.15	25.58	25.67	25.90	**27.75**
0.25	29.00	27.58	27.88	**30.05**
0.50	32.66	31.40	31.82	**34.00**

TABLE III
PERFORMANCE COMPARISON OF SSC WITH DIFFERENT CODEC FOR GOLDHILL

Rate	JPEG2000 [33]	SPIHT	TCE[28]	SSC
0.15	29.15	28.96	29.04	**29.20**
0.25	30.63	30.56	30.64	**31.50**
0.50	33.07	33.13	33.23	**34.35**

VI. CONCLUSION & PERSPECTIVES

We have proposed a new approach called Separate Sign and magnitude Coding of wavelet image coefficients SSC. An algorithm is developed and the probabilities of magnitude, sign and refinement informations are calculated online, bit plane by bit plane and these informations are entropy encoded. We show that the sign information of wavelet coefficients may not be encoded by an estimated probability of 0.5; the probability estimated of 0.5 for encoding the sign information may be used only after about five bit planes.

We also show that the refinement information may not be encoded by the estimated probability of 0.5. In fact, the probabilities of the quantized wavelet coefficients to be set in the lower sub-interval or in the upper sub-interval in the refinement processing present a symmetry with the probability value of 0.5.

The obtained results are compared to JPEG2000 standard in terms of objective quality (PSNR) for Lena, Barbara,

Cameraman and Goldhill test images. The comparison is also done in terms of subjective quality (visual quality) with JPEG2000 for Lena, Barbara and Cameraman images decoded at 0.15 bpp. We show that the proposed method outperforms the JPEG2000 standard.

The comparison is also done with the other codec in the literature in terms of PSNR. We show that the proposed method outperforms these codec in terms of objective qualities.

In the future we will analyze the influence of wavelet filters. Different filters will be used to analyze their impact on the probability distributions of magnitude, sign and refinement processing. We will also apply this approach to different type of images such as medical images, satellite images, etc. for which the characteristics are different compared to photographic images used in this work.

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