

# Determination of Optimum Length of Frames and Number of Vectors to Compress ECG Signals

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**Abstract**—In this study, to compress ECG signals, KLT (Karhunen-Loeve Transform) method has been used. The purpose of this method is to perform effective ECG coding by a correlation between the length of frames and the number of vectors of ECG signals.

**Keywords**—ECG Compression, EKG Compression.

## I. INTRODUCTION

TODAY, the basic struggle of signal processing groups is to increase the speed of data transmission for different transmission environments which have limited bandwidth. The other struggle in different data storage environments is to work on how the data is stored in the less space as much as possible. The medical signal processing, the speech coding, the speech recognition, the speech classification, the information security etc. for different aims like methods frequency domain, time domain, transform domain, fuzzy logic and neural networks are used [1,2,3,4,5,6,7].

The reason of why biomedical data processing, storing and transmitting important is to use in medical diagnosis and treatment. Because of the needed very large capacity, data storing, transmitting and reconstructing may cause speed and memory problems particularly in biomedical signals (ECG, EEG, EMG etc). The mentioned problems are sorted out by compressing the signals in appropriate ratios by keeping the important parts of the signals [8].

The electrocardiogram (abbreviated ECG or EKG) is a graphic representation of the heart's electrical activity. If we wish to store an ECG recording that spans more than a few minutes, some form of data compression is highly desirable.

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Recordings over quite long periods, as much as 24 hours, maybe needed when a patient has complained of irregular heart rhythms. Doctors may wish to build a database of ECG recordings for their patients so that two ECG traces taken on different dates may be compared.

Compression is also desirable if a doctor wishes to transmit ECG over a telephone line to a cardiac specialist for an expert opinion [9].

Many existing compression algorithms have shown some success in electrocardiogram compression; however, algorithms that produce better compression ratios and less loss of data in the reconstructed data are needed. The Karhunen-Loeve Transform (KLT) appears to be particularly suitable and is often used for ECG compression. A comparison between the ECG compression methods mentioned above. It includes Table 1, that collects reported results from many different sources [9].

## II. METHOD

$x(n)$  discrete signal which has N length can be defined as follows;

$$x(n) = \sum_{i=1}^N x_i \delta_i(n-i) \quad (1)$$

In here,  $\delta_i(n)$  is unit sample sequence and  $x_i$  is amplitude of the signal. By dividing the signal into equal parts;

$$X_{Fk} = \begin{bmatrix} x_{(k-1)L_F+1} \\ x_{(k-1)L_F+2} \\ \vdots \\ x_{kL_F} \end{bmatrix} \quad k = 1, 2, \dots, N_{FT}$$

following the result is taken.

$$X_F = \begin{bmatrix} X_{F1} & X_{F2} & \dots & X_{FN_{FT}} \end{bmatrix} \quad (2)$$

$X_{Fk}$  is the signal of  $k^{th}$  frame,  $N_{FT}$  is the sum of the frames. For each frame, toeplitz matrix is calculated as shown

below.

$$r_k(l+1) = \frac{1}{L_F - l} \sum_{j=[(k-1)L_F+1]}^{[(k \cdot L_F)-1-l]} x_{j+1+l} \quad (3)$$

$$l = 0, 1, 2, \dots, (L_F - 1)$$

as indicated,

$$R_{Fk} = \begin{bmatrix} r_k(1) & r_k(2) & r_k(3) & \cdots & r_k(L_F) \\ r_k(2) & r_k(1) & r_k(2) & \cdots & r_k(L_F-1) \\ r_k(3) & r_k(2) & r_k(1) & \cdots & r_k(L_F-2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_k(L_F) & r_k(L_F-1) & r_k(L_F-2) & \cdots & r_k(1) \end{bmatrix} \quad (4)$$

$k$  is the selected frame number. The calculated  $R_{Fk}$  toeplitz matrix for the  $k^{th}$  frame is a real symmetric and positive matrix as below  $L_F \times L_F$  dimensions.

$$R_{Fk} V_{ik} = \lambda_{ik} V_{ik} \quad (5)$$

$\lambda_{ik}$  and  $V_{ik}$  define eigenvalue and eigenvector for  $R_{Fk}$  toeplitz matrix. The relation between the toeplitz matrix of the  $X_{Fk}$  series and its eigenvectors can be shown as below;

$$X_{Fk} = \sum_{i=1}^{L_F} C_i V_{ik} \quad (6)$$

$$C_i = (X_{Fk})^T V_{ik} \quad (7)$$

By putting  $\lambda_{1k}, \lambda_{2k}, \dots, \lambda_{L_F k}$  eigenvalues in the descending orders,  $V_{1k}, V_{2k}, \dots, V_{L_F k}$  are found with the eigenvectors matching  $\lambda_{1k} \geq \lambda_{2k}, \dots, \lambda_{L_F k}$ .

As a result of using the eigenvectors indicating the signal with the maximum energy for each frame can be calculated with the equation (6).

### III. RECONSTRUCTION OF ECG SIGNAL

According to (6) and (7) equations,  $X_{Fk}$  can be found easily from the eigenvectors for each  $N_{Fk}$  frame. Starting from the first frame of sample ECG signal, the steps below can be performed sequentially. Fig.5 illustrates the flow chart of KLT for ECG signal. Compression ratio changes in line with the length of frame.

- 1.Step:** To divide the ECG signal to appropriate length of frame (4 - 48).
- 2.Step:** To determine the Toeplitz matrix for each frame.

- 3.Step:** To determine the eigenvalues and eigenvectors for each Toeplitz matrix.
- 4.Step:** To determine the  $C_i$  coefficient which shows the linear relation between the ECG signal and eigenvectors.
- 5.Step:** To arrange in decaying sequence the eigenvalues with using KLT method.
- 6.Step:** To specify the maximum eigenvectors (having the maximum energy) equal to maximum eigenvalues.
- 7.Step:** To reconstruct the ECG signal by multiplying the eigenvectors which are having the maximum energy and the coefficient.

$$X_R = C_i V_{ik} \quad (8)$$

### IV. RESULTS

In order to code ECG signal in effective way optimum length of frame and number of vectors are found. 1:3 ratio which is the number of vectors per length of frame may be sufficient for medical diagnosis and treatment.

In this method, firstly for any selected ECG signal toeplitz matrix is found for each frame. And then, the eigenvectors are selected which have maximum energy. Finally, the coding of the ECG signal is performed by using these selected eigenvectors.

Table 2 shows the mean square errors according to the different length of frame and number of vectors.

$$MSE = \|X_{Fk} - C_i X_R\|^2 = \sum_{i=1}^{L_F} (X_{i,Fk} - C_i X_{i,Fk})^2 \quad (9)$$

Figures 1-4 show that the ratios of 1:1, 1:2, 1:3, 1:4 which are between the length of frame and number of vectors. The results of figures and tables search are that to have optimum ECG coding, the length of frame should be as much as minimised, the number of vectors should be as much as maximised.

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TABLE I  
SUMMARY OF ECG DATA COMPRESSION SCHEMES

Method	Compression Ratio	Sampling Rate (No.of bits)	Percent RMS Difference
AZTEC	10,0	500(12)	%28,0
TP	2,0	200(12)	%5,3
CORTES	4,8	200(12)	%7,0
Fan/SAPA	3,0	250	%4,0
Entropy Coding of 2nd Differences	2,8	250(10)	-
Peak Picking (Spline) with Entropy Coding	10,0	500(8)	%14,0
DPCM-Delta Coding with Threshold	4,0	300(8)	-
DPCM-Linear Prediction	2,5	250(12)	-
DPCM-Linear Pred, Interpl..Entropy Coding	7,8	500(8)	%3,5
Orthogonal Transforms-CT, KLT, HT	3,0	250	-
Dual Application of KLT	12,0	250(12)	-
Fourier Descriptors	7,4	250(12)	%7,0

TABLE II  
MEAN SQUARE ERRORS ACCORDING TO THE DIFFERENT LENGTH OF FRAMES AND THE NUMBER OF VECTORS

		The Length of Frames											
		4	8	12	16	20	24	28	32	36	40	44	48
T h e  N u m b e r  o f  V e c t o r s	1	1,4905E-08	3,1043E-08	4,6650E-08	3,3545E-08	4,3175E-08	5,1647E-08	8,4943E-08	8,3023E-08	9,5722E-08	1,2657E-07	1,1947E-07	1,3180E-07
	2	7,5198E-09	1,1849E-08	1,4276E-08	1,7165E-08	1,3723E-08	3,2912E-08	5,1310E-08	3,5322E-08	7,1238E-08	7,6259E-08	7,9500E-08	8,8521E-08
	3	2,8003E-10	1,7806E-09	4,5845E-09	7,8410E-09	7,2620E-09	2,1176E-08	2,8348E-08	2,5356E-08	4,1289E-08	6,0618E-08	6,5069E-08	6,4350E-08
	4	3,4871E-37	1,2025E-09	1,9958E-09	5,7220E-09	6,1577E-09	1,8932E-08	2,6134E-08	2,1353E-08	3,2973E-08	3,6047E-08	4,6542E-08	6,2109E-08
	6		1,3245E-10	7,1976E-10	1,5308E-09	2,6501E-09	4,6459E-09	3,3559E-09	8,8358E-09	1,6611E-08	9,6664E-09	3,3027E-08	2,2613E-08
	8		6,1326E-37	2,9229E-10	6,0482E-10	1,2136E-09	1,5468E-09	2,1588E-09	3,1699E-09	4,6247E-09	4,5812E-09	1,1920E-08	1,0322E-08
	10			5,9019E-11	2,6904E-10	5,5122E-10	7,1990E-10	1,2997E-09	1,1444E-09	1,9324E-09	3,3715E-09	4,1191E-09	4,0666E-09
	12			7,3395E-37	1,1907E-10	2,5988E-10	5,0493E-10	7,1732E-10	8,0277E-10	1,1903E-09	2,3279E-09	1,6904E-09	3,3150E-09
	16				8,5780E-37	8,9471E-11	2,0718E-10	3,0542E-10	4,3918E-10	6,0195E-10	8,4025E-10	7,4741E-10	9,3608E-10
	20					9,2921E-37	9,2554E-11	1,0333E-10	2,4290E-10	3,4148E-10	4,4572E-10	5,0128E-10	5,8029E-10
	24						1,2122E-36	4,1927E-11	1,2367E-10	2,1386E-10	2,5559E-10	3,6898E-10	4,0607E-10
	28							1,0898E-36	6,8994E-11	1,4458E-10	1,5016E-10	2,2340E-10	2,8681E-10
	32								1,2047E-36	6,7435E-11	1,1063E-10	1,1536E-10	1,9714E-10
	36									1,5666E-36	3,9795E-11	6,5899E-11	1,6074E-10
	40										2,5985E-36	3,7875E-11	1,0347E-10
	44											1,4130E-36	6,7821E-11
	48												1,8765E-36

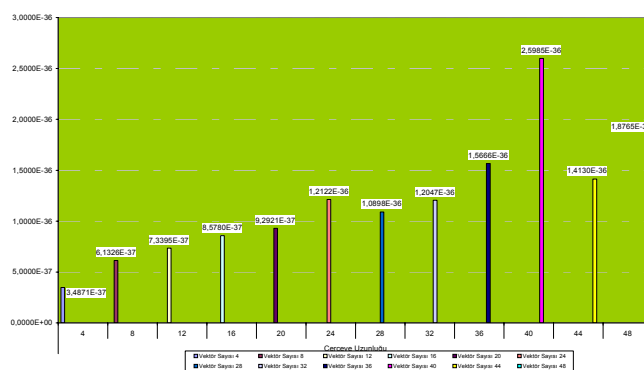


Fig. 1 Mean square errors to the number of vectors / the length of frames = 1 / 1

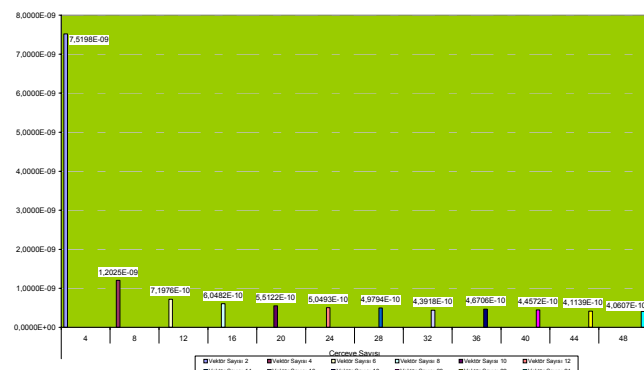


Fig. 2 Mean square errors to the number of vectors / the length of frames = 1 / 2

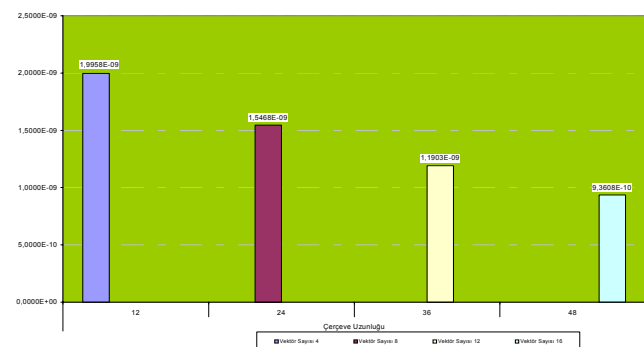


Fig. 3 Mean square errors to the number of vectors / the length of frames = 1 / 3

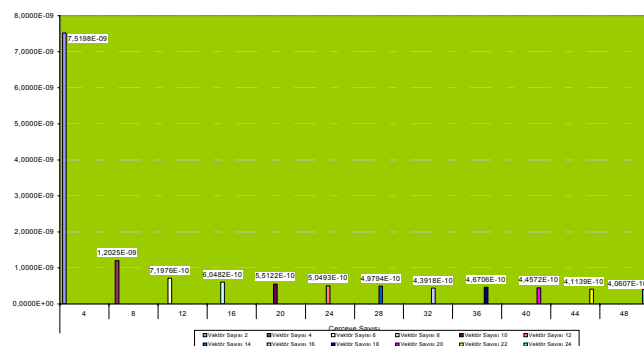


Fig. 4 Mean square errors to the number of vectors / the length of frames = 1 / 4

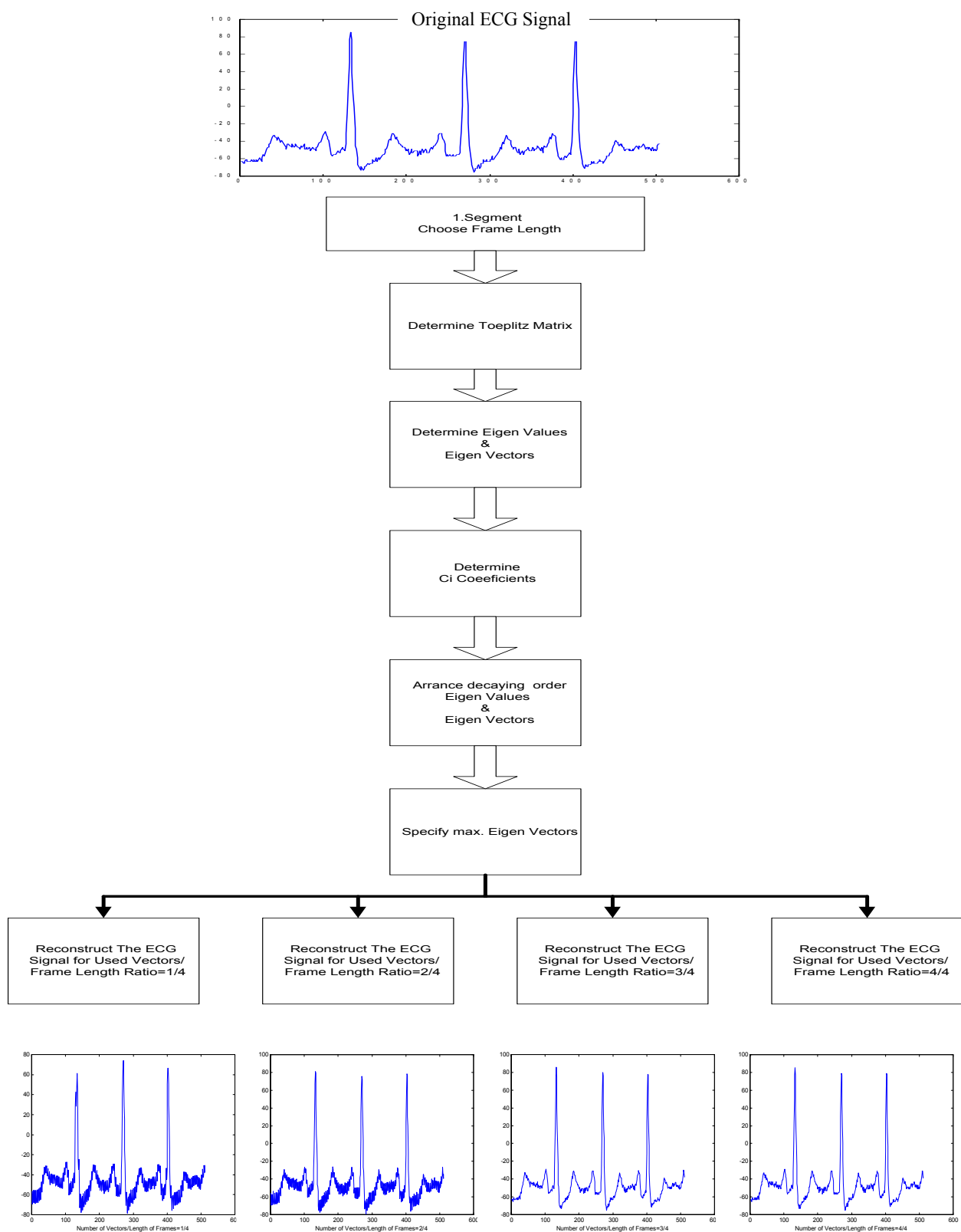


Fig. 5 Flow chart of KLT for ECG signal