

# Experimental investigation of the maximum axial force in the folding process of aluminum square columns

A. Niknejad, G. H. Liaghat, A. H. Behraves, and H. Moslemi Naeini

**Abstract**—In this paper, a semi empirical formula is presented based on the experimental results to predict the first pick (maximum force) value in the instantaneous folding force- axial distance diagram of a square column. To achieve this purpose, the maximum value of the folding force was assumed to be a function of the average folding force. Using the experimental results, the maximum value of the force necessary to initiate the first fold in a square column was obtained with respect to the geometrical quantities and material properties. Finally, the results obtained from the semi empirical relation in this paper, were compared to the experimental results which showed a good correlation.

**Keywords**—Honeycomb, Folding force, Square column, Aluminum, Axial loading.

## I. INTRODUCTION

Honeycombs are composed of a network of joined parallel columns. The mechanical properties of a honeycomb depend on the geometry of the cross section of the elementary column, its base material and the joining technique employed [1]. Sandwich panel with honeycomb core is used in transportation and aerospace industries, because of its high strength and stiffness to the weight ratio. Among honeycomb properties, its folding behavior under axial loading is the most important, since the highest portion of the absorbed energy occurs during this mechanism [2].

In recent decades, many researchers have investigated the honeycomb behavior under the various loading. Wierzbicki and Abramowicz introduced “Basic Folding Mechanism” and calculated the average folding force of a square column based on this theoretical model of deformation [3]. The extensionally folding modes in an angular element were investigated by Wierzbicki and Heyduk [4]. Abramiwicz calculated the effective folding distance in thin-walled columns with considering the strain hardening [5]. Dynamic folding of square column was experimentally analyzed by Abramowicz and Jones [6]. Wierzbicki and Abramowicz carried out an

experimental and theoretical investigation on the crushing process in polyurethane foam-filled square columns [7]. Then, they theoretically calculated the mean folding force in square and hexagonal columns by introducing Corner Element with selectable angle [8]. Liaghat *et al.* compared the theoretical results with those of experimental and evaluated the analytical relations [9], and then, theoretically and numerically investigated honeycomb behavior under the impact loading [10-15]. Effect of metal fillers such as aluminum foam in a square column and its behavior under bending were studied by Santosa and Wierzbicki through experimental and numerical methods [16, 17]. Chen *et al.* analytically calculated mean folding force in a multi-cell square column [18]. Zhang *et al.* calculated the mean crushing force in multi-cell columns, based on Super Folding Element [19].

Review of the published works on the folding behavior of columns and honeycombs, reveals that only theoretical calculation of the mean folding force in single-cell and multi-cell columns has been introduced. In this paper, the maximum requested value of the instantaneous folding force of a square column is calculated. On the other hand, in our previous research work [2], it was shown that the theoretical relationship of the instantaneous folding force in square columns presents a singularity in the start of folding process. Thus, at time of zero, the theoretical relationship can not predict a finite value for the folding force. In this paper, maximum value of the axial force requested for the first fold creation in a square column is thus calculated.

## II. THEORY

Please As mentioned above, Wierzbicki and Abramowicz introduced a “Basic Folding Mechanism” as shown in Fig. 1 and calculated the dissipated energy rate of the folding deformation in the Basic Folding Mechanism [3]. They assumed the dissipated energy rate of the folding deformation to be equal to the work of the external force. By this analytical investigation, they could calculate an average value for the folding force in a square column.

In folding process, the dissipated energy rate is resulted from the continuous and discontinuous velocity fields and is calculated as the following [3]:

$$\dot{E}_{\text{int}} = \int_S (M^{\alpha\beta} \dot{\kappa}_{\alpha\beta} + N^{\alpha\beta} \dot{\lambda}_{\alpha\beta}) \cdot dS + \int_L M_0 \dot{\theta} \cdot d\ell \quad (1)$$

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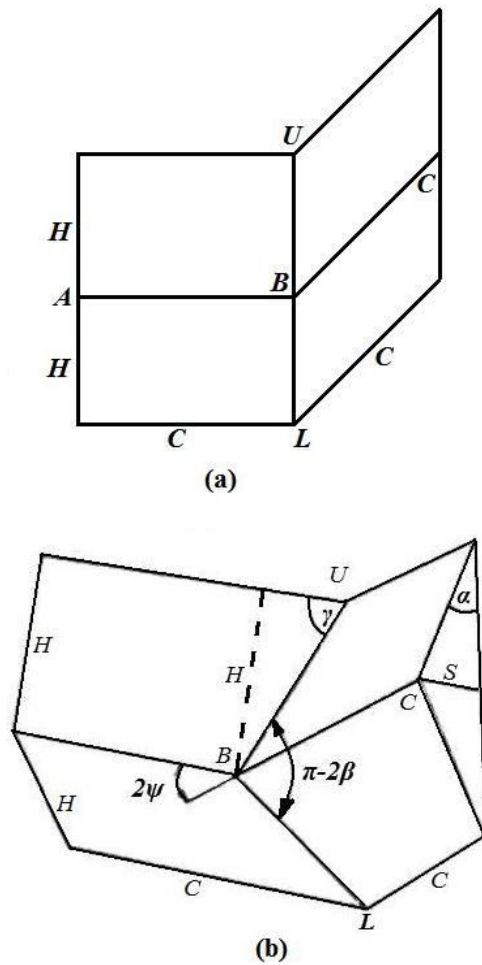


Fig. 1 Basic Folding Mechanism, (a) before folding, (b) after folding [1]

which  $N^{\alpha\beta}$ ,  $M^{\alpha\beta}$ ,  $\dot{\kappa}_{\alpha\beta}$ , and  $\dot{\lambda}_{\alpha\beta}$  are stress resultants, stress couples, the rate of curvature, and the rate of extension, in the continuous deformation field, respectively. Note that both the extent of continuous plastic deformations  $S$  and the length of hinge lines  $\ell$  increase during the deformation progresses [3].  $\theta$  is a finite rotation around every hinge line. In the above formula, the first integral yields the dissipated energy rate of the extensional deformation on a small area that is called toroidal surface and the second integral calculates the dissipated energy rate of the non-extensional deformation, or in other word, dissipated energy rate of bending around the hinge lines. The rate of external work, required for compressing a Basic Folding Mechanism, should be equated with the internal dissipated energy rate. So the following relation is yielded:

$$\dot{E}_{ext} = \dot{E}_{int} \quad (2)$$

Wirzbicki and Abramowicz predicted the average folding force of a square column by introducing the Super Folding Element and a theoretical analysis [3]:

$$P_m = 9.56\sigma_o h^{5/3} C^{1/3} \quad (3)$$

where  $P_m$  is the average folding force,  $\sigma_o$  is the material flow stress,  $h$  is the wall thickness of the column, and  $2C$  is the length of every edge in the square cross section.

Equation (3) was theoretically obtained using the concept that the work of the axial external force is equal to the amount of the internal dissipated energy in the column.

Then, Wierzbicki and Abramowicz rewrote the above relation as the following, using Corner Element [8]:

$$P_m = 13.06\sigma_o h^{5/3} C^{1/3} \quad (4)$$

The above equation results in the mean crushing force of a column with the square cross section which showed a good correlation with the experimental results [8].

It shows that the mean folding force of a square column depends on the five-third power of the wall thickness and one-third power of the half length of the square column edge. According to the experimental results, the actual crushing force versus the folding distance (the axial length variation of the column) diagram shows that the value of the folding force initially increases and then decreases continuously. In other word, when each fold is created, an increasing trend and then a decreasing trend occur in the force-displacement curve. In Equations (3) and (4), the material flow stress,  $\sigma_o$  is calculated as below [17]:

$$\sigma_o = \sqrt{\frac{\sigma_y \sigma_u}{1+n}} \quad (5)$$

where  $\sigma_y$ ,  $\sigma_u$ , and  $n$  are yield stress, ultimate stress, and exponent of strain-hardening, respectively. Fig. 2 shows a sample diagram of the folding force versus the axial displacement (folding distance) for a column with a square cross section. The figure shows that the folding force of each fold varies around an average value (shown by the horizontal dash line). This diagram has a primary peak that is the maximum value of the force along the curve, and it occurs when the first fold creates in the column. The purpose of this research work is to present a formula to predict this maximum value which is the first peak value of the axial force in the crushing force diagram versus axial displacement for a square column.

According to Fig. 2 and similar diagrams this result obtains that the average value of the folding force increases, when the first peak of the instantaneous folding force increases. Investigation of experimental results shows that there is a relationship between the maximum folding force and its average value. In this paper, it is assumed that this relationship

is linear. So, according to the experimental results, a semi empirical formula is assumed such as the following:

$$P_{\max} = K \cdot P_m \quad (6)$$

In which,  $P_{\max}$  is the maximum value of the folding force of a square column and  $K$  is a material type dependent coefficient that is calculated as follows. According to Equation (4) theoretically introduced by Wierzbicki and Abramowicz, and substituting in Equation (6), the following relationship is then resulted:

$$P_{\max} = 13.06 K \sigma_o h^{5/3} C^{1/3} \quad (7)$$

This equation shows that the maximum folding force of a square column depends on the five-third power of the wall thickness and one-third power of the half length of the square column edge. The unknown coefficient  $K$ , is obtained through the experimental method. In this research work, several axial compressing tests were then carried out on aluminum square column with various dimensions and cross sections to determine the value of coefficient  $K$ .

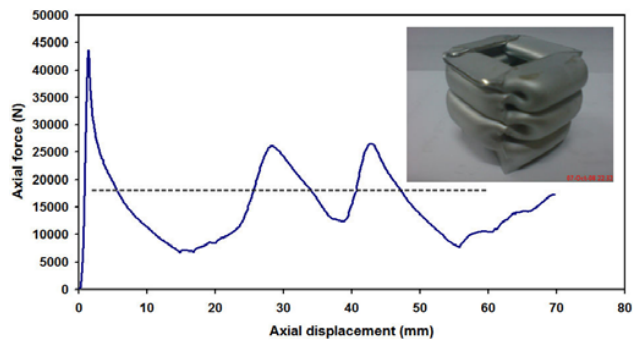


Fig. 2 The experimental folding force versus axial displacement in a square column

### III. EXPERIMENTAL

Rectangular and square columns were used in axial static compression test, and folds were created in the columns. All columns were made of aluminum alloy. A dumbbell shape specimen of aluminum columns was prepared and used in the static tension test. An Instron Machine, model 5500R, was used in the tests. The resultant stress-strain diagram is shown in Fig. 3.

The columns were made of an aluminum with mechanical properties: Young's modulus  $E = 71 \text{ GPa}$ , Poisson's ratio  $\nu = 0.3$ , yield stress  $\sigma_y = 173 \text{ MPa}$ , ultimate Stress  $\sigma_u = 236.9 \text{ MPa}$  and the power of strain hardening  $n = 0.121$ . The flow stress of the aluminum alloy is calculated by Equation (5) as following:

$$\sigma_o = \sqrt{\frac{(173) \cdot (236.9)}{(1 + 0.121)}} = 191.2 \text{ MPa} \quad (8)$$

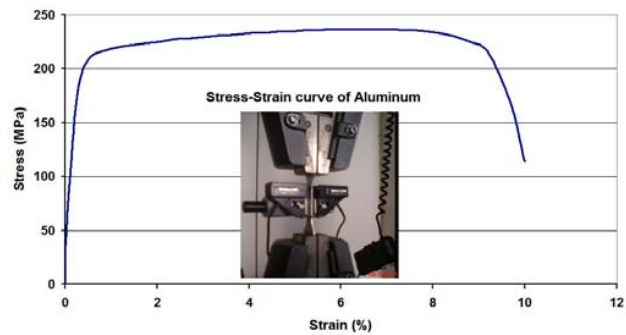


Fig. 3 Stress-strain diagram of aluminum alloy

A specimen of aluminum column with a square cross section of  $35 \text{ mm} \times 35 \text{ mm}$  in dimensions, the wall thickness of  $1.5 \text{ mm}$  and length of  $100 \text{ mm}$  was used in the static folding test. Fig. 4 shows the experimental diagram of the instantaneous folding force versus the axial displacement. This graph is the result of the first fold creation in a square column, the folding force initially increases from zero value until it reaches to a maximum value and then it follows the decreasing trend, so a fold is formed completely.

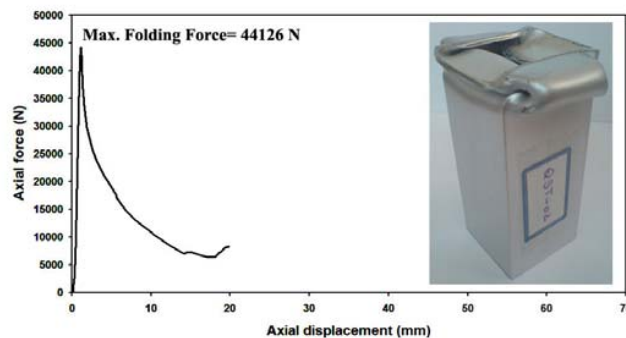


Fig. 4 Instantaneous folding force versus axial displacement in a square column ( $35 \text{ mm} \times 35 \text{ mm}$ )

For the folding process to start in a column, an initial value of axial force which is the maximum value in Fig. 4 is needed. In other word, the results of the axial compression test show the starting point of plastic deformation and folding creation in the square column occur around the maximum force zone in Fig. 4. The purpose of this research is the prediction of the maximum force that is necessary for starting and creating the folding process in a column. Maximum value of the instantaneous folding force in the square column with the mentioned characteristics was obtained from Fig. 4 which is equal to  $44126 \text{ N}$ .

According to the experimental results of the square column with cross section  $35 \text{ mm} \times 35 \text{ mm}$ , wall thickness  $1.5 \text{ mm}$ , length  $100 \text{ mm}$ , material flow stress  $\sigma_o = 191.2 \text{ MPa}$ , and the value obtained for the maximum force,  $44126 \text{ N}$ , the value of coefficient  $K$  is obtained as 3.47. Substituting this value in

Equation (7), the final relationship for prediction of the maximum value of folding force in a column with rectangular and square cross sections under axial loading is obtained as following:

$$P_{\max} = 45.32\sigma_o h^{5/3} C^{1/3} \quad (9)$$

According to above relation, the maximum value of the folding force in a square column depends on the five-third power of the wall thickness and the one-third power of half length of the every square edge, directly.

#### IV. RESULTS AND DISCUSSION

Further axial static compression tests were applied on rectangular and square columns to investigate the validation of the proposed relationship for prediction of the maximum folding force in rectangular and square columns.

An aluminum column with rectangular cross section with dimensions of  $54.1\text{mm} \times 34.5\text{mm}$ , wall thickness  $1.4\text{mm}$ , length  $100\text{mm}$ , and material flow stress  $\sigma_o = 191.2\text{MPa}$ , was used in a statically folding test. Fig. 5 shows the experimental diagram of the instantaneous folding force versus the axial displacement. This graph is the result of the first fold creation in a square column. From Fig. 5, the maximum folding force of this aluminum rectangular column was obtained equal to  $43434\text{N}$ . Corresponding value from the theoretical Equation (9) is calculated as  $42638\text{N}$ , that shows 1.8 percent error.

To further investigate, the predicted value of maximum folding force by Equation (9) is compared with that of experimental results obtained by other researchers.

For this purpose, Fig. 6 shows the folding force diagram versus axial displacement in a square column of AA6060 aluminum alloy with half of the edge length  $C = 40\text{mm}$ , wall thickness  $h = 2.5\text{mm}$ , yield stress  $\sigma_y = 195\text{MPa}$  and ultimate stress  $\sigma_u = 220\text{MPa}$  [20]. The maximum value of the folding force from the proposed relationship in this paper has been obtained equal to  $142764\text{N}$ . The corresponding value from the diagram of the experimental test is equal to  $142\text{KN}$  revealing an error of 0.5%.

The folding force diagram versus axial displacement in a square column of AA6060 T4 aluminum alloy with half of the edge length  $C = 40\text{mm}$ , wall thickness  $h = 1.88\text{mm}$ , yield stress  $\sigma_y = 80\text{MPa}$ , ultimate stress  $\sigma_u = 170\text{MPa}$  and the exponent of strain hardening  $n = 0.23$  is shown in Fig. 7 [17]. The maximum value of the folding force of this column according to the given data has been calculated to be  $46670\text{N}$ . From equation (9) the corresponding experimental value is obtained as  $48.5\text{KN}$ . The difference is 3.8 %.

Comparing the maximum values of the folding force in square and rectangular columns obtained from Equation (9) and the results of the experimental tests applied by the authors of this article and experimental results of the other researchers show that the proposed theoretical relation in this paper could

fairly predict the value of maximum force with some minor error percentage. A combination of the theoretical Equation (4) which was introduced by Wierzbicki and Abramowicz [8] and the experimental results are the base of the proposed Equation (9).

Reviewing the results from other sources indicates the proposed Equation (9) in this paper, can be considered as the first semi empirical relationship for prediction of the maximum value of the folding force in square columns.

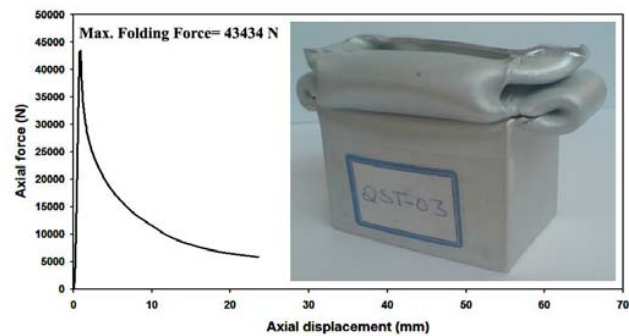


Fig. 5 Instantaneous folding force versus axial displacement in a rectangular column ( $54.1\text{mm} \times 34.5\text{mm}$ )

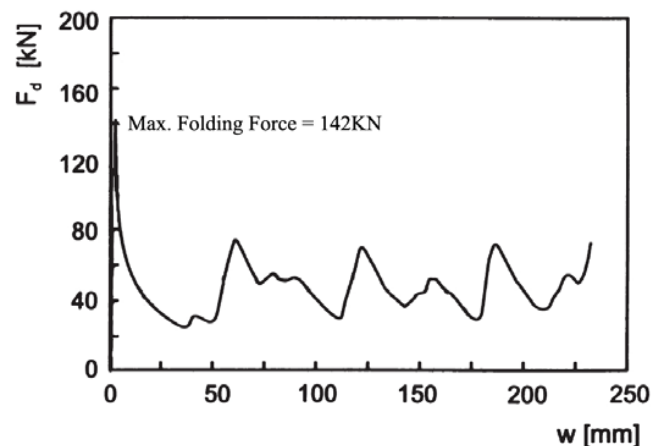


Fig. 6 Experimental folding force versus axial displacement in a square column ( $80\text{mm} \times 80\text{mm}$ ) [20]

#### V. CONCLUSION

In this paper, a semi empirical relationship is proposed to predict the maximum value of the axial force in the folding process of a square column. Semi empirical Equation (9) predicts the maximum value of the force that is necessary to initiate the first fold in a column with the square cross section. According to the relationship, the maximum axial folding force in a square column is dependent on a constant coefficient, geometrical dimensions of the column such as the wall thickness and half of the square length. Comparing the predicted values of this semi empirical relation and results of the experimental tests prepared by the authors of the present article as well as those extracted from the other cited

researchers, shows the result of Equation (9) has a good correlation with the experimental results.

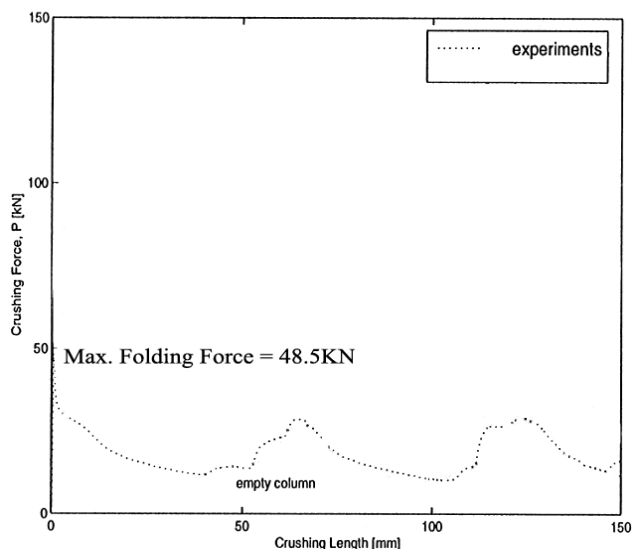


Fig. 7 Experimental folding force versus axial displacement in a square column ( $80\text{mm} \times 80\text{mm}$ ) [17]

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