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A Soft Set based Group Decision Making Method with Criteria Weight

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***Abstract*—**Molodstov's soft sets theory was originally proposed as general mathematical tool for dealing with uncertainty problems. The matrix form has been introduced in soft set and some of its properties have been discussed. However, the formulation of soft matrix in group decision making problem only with equal importance weights of criteria, which does not show the true opinion of decision maker on each criteria. The aim of this paper is to propose a method for solving group decision making problem incorporating the importance of criteria by using soft matrices in a more objective manner. The weight of each criterion is calculated by using the Analytic Hierarchy Process (AHP) method. An example of house selection process is given to illustrate the effectiveness of the proposed method.

***Keywords*—**Soft set, Soft Matrix, Soft max-min decision making (*SMmDM*), Analytic hierarchy process (AHP)

I. Introduction

EVEN though many approaches have been proposed in solving decision making problem, each method is limited to certain types of problems. Soft set theory was first proposed by Molodstov in 1999, as a mathematical tool for dealing with problems in many fields involving data that contain uncertainties.

Application of soft set theory has enormous potential in many areas and several directions, some of which are reported by Molodstov [1]. Later Maji and Roy [2] presented some new operations on soft set theory such as equality of two soft set, subsets and super sets of soft sets, complement of soft sets and so on based on Molodstov definition [1]. Aktas and Cagman [3] initiated the notion of soft group and also compared soft sets to fuzzy sets and rough sets.

Cagman and Enginoglu [4] redefined Molodstov [1] operation on soft sets and defined four products of soft sets and four decision functions in decision making problem, namely uni-int decision function, int-int decision function, uni-uni decision function and int-uni decision function. In real life application Herawan & Deris [5] have been applied soft sets in decision making for patient suspected influenza. Maji and Roy [6] applied soft set theory in decision making problems that is based on the concept of knowledge reduction in rough set theory. Cagman and Enginoglu defined soft matrices, where soft sets representation in matrix form [7].

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By far, parameters are considered to have equal importance or with equal weighted and this does not portray the true opinion of decision maker. Maji and Roy [6], applied criteria weight in their decision making problem, however the criteria weight are determined based on decision maker observations only without any proper procedure.

This paper improves Maji and Roy [6] method to find weights of criteria by using AHP method and apply soft matrices concept [7] to solve group decision making problems. A house selection process is presented to illustrate how to use our method in practical applications.

II. Soft Matrices

Molodstov [1] defined the soft set theory in following manner:

Definition 1: Let U be an initial universe set and E be a set of all parameters in relation to object U. Parameter are often attributes, characteristics or properties of object. Let $P(U)$ denotes the power set of U and $L \subseteq E$. A pair (F, L) is called a soft set over U, where F is a mapping given by $F : L \to P(U)$.

In other words, a soft set over U is parameterized family of subsets of the universe U. For $\varepsilon \in L, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, L).

A. Soft Matrices

Cagman and Enginoglu [7] developed soft decision making method, by the following definitions.

Definition 2: Let U be an initial universe set, $P(U)$ denotes the power set of U and E be a set of all parameters and $L \subset E$. A soft set (f_L, E) on the universe is defined by the set of ordered pairs $(f_L, E) = \{(e, f_L, (e)) : e \in E, f_L(e) \in P(U)\}$, where $f_L, E \to P(U)$ such that $f_L(e) = \phi$ if $e \neq L$.

Here, f_L is called an approximate function of the soft set (f_L, E). The set $f_L(e)$ is called $e-$*approximate value set* or *e-approximate set* which consist of related objects of the parameter $e \in E$.

Definition 3: Let (f_L, E) be a soft set over U. Then subset of $U \times E$ is uniquely defined by $T_L : \{(u, e) : e \in L, u \in f_L(e)\}$

which is called a relation form of (f_L, E). The characteristics function of T_L is written by:

$$X_{T_A} : U \times E \to \{0,1\}, \qquad X_{R_A}(u,e) = \begin{cases} 1, & (u,e) \in T_A \\ 0, & (u,e) \notin T_A \end{cases}.$$

If $U = \{u_1, u_2, \ldots, u_m\}$, $E = \{e_1, e_2, \ldots, e_n\}$ and $L \subseteq E$ then the T_L can be presented by a table as in the following form:

R_L	e_1	e_2	$\ldots$	e_n
u_1	$X_{T_L}(u_1,e_1)$	$X_{T_L}(u_1,e_2)$	$\ldots$	$X_{T_L}(u_1,e_n)$
u_2	$X_{T_L}(u_2,e_1)$	$X_{T_L}(u_2,e_2)$	$\ldots$	$X_{T_L}(u_2,e_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$X_{T_L}(u_m,e_n)$	$X_{T_L}(u_m,e_2)$	$\ldots$	$X_{T_L}(u_m,e_n)$

If $A_{ij} = X_{T_L}(u_i, e_j)$, we can define matrix

$$[A_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

which is called a $m \times n$ soft matrix of the soft set (f_L, E) over U.

B. Product of Soft Matrices

Definition 4: Let $[r_{ij}], [s_{ik}] \in SM_{m\times n}$. The *And–product* between $[r_{ij}]$ and $[s_{ik}]$ is defined by $\wedge : SM_{m\times n} \times SM_{m\times n} \to SM_{m\times n^2}$, $[r_{ij}] \wedge [s_{ik}] = [t_{ip}]$ where $[t_{ip}] = \min\{r_{ij}, s_{ik}\}$ such that $p = (n-1) + k$.

III. Methodology

Two procedures are involved in the proposed method. The first procedure is to determine the weight of criteria. Here we utilize AHP method introduced by Saaty [8]. The second procedure is to solve the decision making problem. The method of Cagman and Enginoglu is used. The details of both procedures are given below:

A. Criteria Weight Determination

Analytical hierarchy process (AHP) was first proposed by Saaty in 1971[8, 9, 10]. It is a structure for dealing with complex decisions and has been extensively studied and refined since then. The AHP is also be used in determining criteria weights of decision maker under group decision environment. The procedure of the AHP involves four steps as follows:

Step 1: Confirm the evaluation criteria and alternatives of decision making problems.

Step 2: Decompose the complex problem into a hierarchical structure with decision element. Each of these decision element defined by decision maker based on the Saaty's 1-9 scale.

Step 3: Employs pair wise comparison among decision elements and form comparison matrices. Each decision-maker $(DM_1, DM_2, \ldots DM_k)$, individually carries out pair-wise comparison and represented as:

$$\tilde{A}^k = [\tilde{a}_{ij}]^k \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{2m} & \cdots & \tilde{a}_{mn} \end{bmatrix},$$

where k is the number of decision makers, n is the number of the related elements at this level and $[\tilde{a}_{ij}^{-1}] = 1/\tilde{a}_{ij}$.

Step 4: Use the eigenvalue method to estimate the relative weights of the elements for each decision makers $W^k = (w_1, w_2, \ldots, w_i)$, where $\Sigma W_i^k = 1$.

B. Method of Soft Matrix Theory

Cagman and Enginoglu [7] defined soft matrices to solve problems by using soft max-min decision making method $(SMmDM)$ and *And–Product*. They defined soft max-min decision function as follows:

Definition 5: Let $[d_{ip}] \in SM_{m\times n^2}$, $I_k = \{p : \exists i, d_{ip} \neq 0, (k-1)n, < p \leq kn\}$ for all $k \in I = \{1,2,\ldots,n\}$. Then max-min decision function, denoted *Mm*, is defined as follows:

$$Mm : SM_{m\times n^2} \to SM_{m\times 1}, \qquad Mm[d_{ip}] = \left[\max_{k\in I}\{r_k\}\right],$$

$$\text{where, } r_k = \begin{cases} \min\limits_{p\in I_k}\{d_{ip}\}, & \text{if } I_k \neq \phi \\ 0, & \text{if } I_k = \phi. \end{cases}$$

The one column soft matrix $Mm[d_{ip}]$ is called max-min decision soft matrix.

Definition 6: Let $U = \{u_1, u_2, u_3, \ldots, u_m\}$ be an initial universe and $Mm[d_{ip}] = [e_{i1}]$. Then a subset of U can be obtained by using $[e_{i1}]$ as the following expression, $Opt[e_{il}](U) = \{u_i : u_i \in U, e_{il} = \max\{r_k\}\}$, which is called an optimum set of U.

Now, by using definition 5 and 6, *SMmDM* method is constructed. The algorithm for calculating the *SMmDM method is given as follows:*

Step 1: From the given parameters, choose the feasible subsets of the set of parameters,

Step 2: Use matrix form to construct the soft matrix for each set of parameters,

Step 3: Find the convenient product for the soft matrices,

Step 4: Find a max-min decision soft matrix,

Step 5: Find an optimum set of U,

$$Opt_{Mm}(U) = [u_1 \quad u_2 \quad \cdots \quad u_n]^T.$$

IV. The Generalization of Soft Matrices with Weight of Criteria

Razak and Mohamad [11] generalized the method of Cagman and Enginoglu [7] to n[th] decision maker using soft max-min decision making approach in particular taking into consideration the associative law of soft matrices [11].

In this paper, the method of Razak and Mohamad is employed together with the weight of each criteria calculated using the AHP technique. The new algorithm is given as follows:

Step 1 : From the given parameters, choose the feasible subsets of the set of parameter, $E=\{e_1, e_2,\ldots e_n\}$

Step 2 : Use the matrix form to construct the soft matrix for each set of parameters,

$$[r_{ij}]_{m\times n}=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

Step 3 : Calculate weight of criteria choose by every decision makers using AHP procedure in IV.

Step 4 : Input the criteria weight w_k and compute the values for each alternative and then construct the soft matrices, $W_n=(w_1,w_2,\ldots,w_k)$ as a criteria weight for every decision maker. $[r_{ij}]\times w_k$, where w_k is the importance weight of parameter n.

$$M_{ij}=[r_{ij}]_{m\times n}\begin{bmatrix} r_{11}\otimes w_1 & r_{12}\otimes w_2 & \cdots & r_{1n}\otimes w_k \\ r_{21}\otimes w_1 & r_{22}\otimes w_2 & \cdots & r_{2n}\otimes w_k \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}\otimes w_1 & r_{m2}\otimes w_2 & \cdots & r_{mn}\otimes w_k \end{bmatrix}$$

Step 5 : Find the *And- product* of the combination soft matrices $(\text{e.g.}(DM_{n-1}\wedge DM_n=A))$.
$[M_{ij}]$ and $[N_{ik}]$

$$[t_{io}]=[M_{ij}]\wedge[N_{ik}]=\begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1n} \\ M_{21} & M_{22} & \cdots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1} & M_{m2} & \cdots & M_{mn} \end{bmatrix}\wedge\begin{bmatrix} N_{11} & N_{12} & \cdots & N_{1n} \\ N_{21} & N_{22} & \cdots & N_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ N_{m1} & N_{m2} & \cdots & N_{mn} \end{bmatrix}$$

Step 6 : Find the minimum of *And-product* between $[M_{ij}]$ and $[N_{ik}]$, i.e.

$$t_{io}=\begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1} & t_{m2} & \cdots & t_{mn} \end{bmatrix}$$

where $t_{io}=\min\limits_{O=i,2,\ldots,n}[M_{ij}\wedge N_{ik}]$.

Step 7 : Find the *And-product* between $[t_{ir}]$ and $[P_{il}]$

$$[v_{ip}]=[t_{ir}]\wedge[P_{il}]=\begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1} & t_{m2} & \cdots & t_{mn} \end{bmatrix}\wedge\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mn} \end{bmatrix},$$

$$[v_{ip}]=\begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix},$$

where $v_{ip}=\min\limits_{p=1,2,\ldots,n}[t_{ir}\wedge u_{il}]$.

Step 8 : Find the max-min decision soft matrix,

$$Mm(([M_{ij}]\wedge[N_{ik}])\wedge[P_{il}])=[u_1 \quad u_2 \quad \cdots \quad u_n]^T.$$

Step 9 : Find an optimum set of U .

$$opt_{Mm}(U)=\{u_1,u_2,\ldots,u_n\}.$$

Note: The significant change in procedure is at step 3 as compared to Razak and Mohamad [11].

V. Case Study: Selection of House with Weight of Criterion

As an illustration, we revisit house selection problem in the case study [11]. Consider U to be a set of house under consideration and E be a set of sub criteria in house selection. We wish to solve house selection problems that consider weight for each criterion for every decision makers involved in this problem. Assume that, in this problem we have an expert group A, B and C (Mr. X family) as a decision maker to evaluate house in U.

As stated in [11], there are three main criteria and follow by nine sub criteria in this problem. The first main criterion is neighborhood, with two sub criteria aesthetics and safety. Exterior, interior and systems are sub criteria for criterion property. Community is the third criteria with four sub criteria; school, government, social, and entertainment.

Based on these criteria the decision maker will choose their own criteria. Weight of criteria will be calculated based on AHP approach. Soft max-min decision making will be used as a method to solve this group decision making problems with weight of criteria for each decision makers. The solution is obtained by using Microsoft Excel 2007.

A. House Selection Problem

A soft set (f_A,E) describes the "attractiveness of houses" that decision maker will select.

Assume that $U=\{h_1,h_2,h_3,h,h_4,h_5,h_6\}$ is a universe consisting of six houses as possible alternatives, and $E=\{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9\}$ is a set of parameters considered by decision maker (criteria set of house selection), where e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, and e_9 represent the parameters "aesthetic", "safety", "exterior", "interior", "system", "school", "government", "social" and "entertainment" respectively.

B. Constructing the comparison matrices in AHP

The evaluation matrix for each criterion according to decision makers A, B and C are constructed via pair wise comparison using nine point scale developed by Saaty and are given as follows:

$$A=\begin{bmatrix} 1 & 1/8 & 1/5 & 1/4 & 1/7 & 1/5 & 1/3 & 1/2 & 1/3 \\ 8 & 1 & 1/4 & 1/4 & 1/5 & 1/3 & 1/4 & 1/3 & 1/2 \\ 5 & 4 & 1 & 1/5 & 1/6 & 1/4 & 1/4 & 1/3 & 1/3 \\ 4 & 4 & 5 & 1 & 1/6 & 1/4 & 1/4 & 1/2 & 1/2 \\ 7 & 5 & 6 & 6 & 1 & 1/3 & 1/4 & 1/3 & 6 \\ 5 & 3 & 4 & 4 & 3 & 1 & 1/5 & 1/4 & 1/2 \\ 3 & 4 & 4 & 4 & 4 & 5 & 1 & 1/5 & 1/3 \\ 2 & 3 & 3 & 2 & 3 & 4 & 5 & 1 & 1/2 \\ 3 & 2 & 3 & 2 & 1/6 & 2 & 3 & 5 & 1 \end{bmatrix}$$

$$B=\begin{bmatrix} 1 & 1/7 & 1/6 & 5 & 1/8 & 1/4 & 1/3 & 1/2 & 1/2 \\ 7 & 1 & 8 & 1/3 & 1/5 & 7 & 1/3 & 1/2 & 6 \\ 6 & 1/8 & 1 & 1/5 & 1/7 & 1/4 & 5 & 1/3 & 1/3 \\ 1/5 & 3 & 5 & 1 & 1/7 & 6 & 1/4 & 6 & 1/3 \\ 8 & 5 & 7 & 7 & 1 & 7 & 1/4 & 1/3 & 1/2 \\ 4 & 1/7 & 4 & 1/6 & 1/7 & 1 & 1/5 & 1/3 & 1/2 \\ 3 & 3 & 1/5 & 4 & 4 & 5 & 1 & 1/4 & 1/3 \\ 2 & 2 & 3 & 1/6 & 3 & 3 & 4 & 1 & 1/5 \\ 2 & 1/6 & 3 & 3 & 2 & 2 & 3 & 5 & 1 \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 1/8 & 5 & 1/5 & 1/7 & 6 & 4 & 1/3 & 3 \\ 8 & 1 & 8 & 7 & 1 & 1/3 & 6 & 1/3 & 6 \\ 1/5 & 1/8 & 1 & 1 & 1/8 & 6 & 1/4 & 6 & 4 \\ 5 & 1/7 & 1 & 1 & 1/6 & 1/4 & 1/3 & 6 & 1/3 \\ 7 & 1 & 8 & 6 & 1 & 7 & 1/4 & 7 & 1/3 \\ 1/6 & 3 & 1/6 & 4 & 1/7 & 1 & 1 & 1/4 & 6 \\ 1/4 & 1/6 & 4 & 3 & 4 & 1 & 1 & 1/3 & 1 \\ 3 & 3 & 1/6 & 1/6 & 1/7 & 4 & 3 & 1 & 1 \\ 1/3 & 1/6 & 1/4 & 3 & 3 & 1/6 & 1 & 1 & 1 \end{bmatrix}.$$

C. Calculation of Soft max-min decision making method (SMmDM)

Step 1 : Decision makers choose their own set of parameters given as:

$A=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$,

$B=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$,

$C=\{e_1, e_2, e_4, e_5, e_6, e_7, e_8\}$.

Step 2 : The soft matrices representing the evaluation of each decision makers to the chosen parameters are as follows:

$$[A_{ij}]=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \quad [B_{ik}]=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_{il}]=\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Step 3 : Weight of each criterion is calculated using AHP and are obtained as:

$[w_A]$ = (e_1=0.02, e_2=0.04, e_3=0.05, e_4=0.08, e_5=0.17, e_6=0.13, e_7=0.17, e_8=0.18, e_9=0.16)

$[w_B]$ = (e_1=0.04, e_2=0.14, e_3=0.05, e_4=0.10, e_5=0.19, e_6=0.05, e_7=0.13, e_8=0.13, e_9=0.17)

$[w_C]$ = (e_1=0.09, e_2=0.23, e_3=0.08, e_4=0.06, e_5=0.22, e_6=0.07, e_7=0.09, e_8=0.09, e_9=0.07)

Step 4 : Multiply each parameter with weight of criteria for each decision makers. We obtain:

$$[A_{ij} \times w_A]=[P_{ij}]=\begin{bmatrix} 0.02 & 0.04 & 0.05 & 0.08 & 0.17 & 0.13 & 0.17 & 0.18 & 0 \\ 0 & 0.04 & 0.05 & 0 & 0.17 & 0 & 0 & 0.18 & 0 \\ 0 & 0.04 & 0.05 & 0 & 0.17 & 0.13 & 0.17 & 0 & 0 \\ 0.02 & 0.04 & 0 & 0.08 & 0.17 & 0 & 0 & 0.18 & 0 \\ 0.02 & 0.04 & 0.05 & 0 & 0.17 & 0.13 & 0.17 & 0 & 0 \\ 0.02 & 0.04 & 0.05 & 0.08 & 0.17 & 0.13 & 0.17 & 0 & 0 \end{bmatrix}$$

$$[B_{ik} \times w_B]=[Q_{ik}]=\begin{bmatrix} 0.04 & 0.14 & 0.05 & 0.10 & 0.19 & 0.05 & 0.13 & 0.13 & 0 \\ 0 & 0.14 & 0.05 & 0.10 & 0.19 & 0.05 & 0.13 & 0 & 0 \\ 0.04 & 0.14 & 0.05 & 0.10 & 0.19 & 0 & 0.13 & 0 & 0 \\ 0 & 0.14 & 0.05 & 0 & 0.19 & 0.05 & 0.13 & 0.13 & 0 \\ 0.04 & 0.14 & 0.05 & 0.10 & 0 & 0.05 & 0 & 0.13 & 0 \\ 0.04 & 0.14 & 0.05 & 0 & 0.19 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_{il} \times w_C]=[R_{il}]=\begin{bmatrix} 0.09 & 0.23 & 0 & 0.06 & 0.22 & 0.07 & 0.09 & 0.09 & 0 \\ 0 & 0.23 & 0 & 0.06 & 0 & 0.07 & 0 & 0.09 & 0 \\ 0.09 & 0.23 & 0 & 0.06 & 0.22 & 0.07 & 0.09 & 0.09 & 0 \\ 0 & 0.23 & 0 & 0 & 0 & 0.07 & 0 & 0 & 0 \\ 0.09 & 0.23 & 0 & 0 & 0.22 & 0.07 & 0.09 & 0 & 0 \\ 0.09 & 0.23 & 0 & 0.06 & 0.22 & 0.07 & 0.094 & 0.09 & 0 \end{bmatrix}$$

Step 5 : Using the *And–Product*, the product of soft matrices between $[P_{ij}]$ and $[Q_{ik}]$ is obtained as follows:

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\left[\begin{array}{ccccccccccccccccccccccccccc}
0.02 & 0.02 & 0.02 & 0.02 & 0.02 & 0.02 & 0.02 & 0.02 & 0 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0 & 0.04 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0 & 0 & 0 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0 & 0.04 & 0 & 0 & 0.04 & 0.05 & 0.05 & 0.05 & 0.05 & 0 & 0.05 & 0 & 0 \\
0 & 0.02 & 0.02 & 0 & 0.02 & 0.02 & 0.02 & 0.02 & 0 & 0 & 0.04 & 0.04 & 0 & 0.04 & 0.04 & 0.04 & 0.04 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0.02 & 0.02 & 0.02 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0.04 & 0.04 & 0.04 & 0 & 0.04 & 0 & 0 & 0 & 0 & 0.04 & 0.05 & 0.05 & 0 & 0.05 & 0 & 0 & 0 & 0
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0.04 & 0.08 & 0.05 & 0.08 & 0.08 & 0.05 & 0.08 & 0.08 & 0 & 0.04 & 0.14 & 0.05 & 0.10 & 0.17 & 0.05 & 0.13 & 0.13 & 0 & 0.04 & 0.13 & 0.05 & 0.10 & 0.13 & 0.05 & 0.13 & 0.13 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.14 & 0.05 & 0.10 & 0.17 & 0.05 & 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.04 & 0.14 & 0.05 & 0.10 & 0.17 & 0 & 0.13 & 0 & 0 & 0.04 & 0.13 & 0.05 & 0.10 & 0.13 & 0 & 0.13 & 0 & 0 \\
0 & 0.08 & 0.05 & 0 & 0.08 & 0.05 & 0.08 & 0.08 & 0 & 0 & 0.14 & 0.05 & 0 & 0.17 & 0.05 & 0.13 & 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.04 & 0.14 & 0.05 & 0.10 & 0 & 0.05 & 0 & 0.13 & 0 & 0.04 & 0.13 & 0.05 & 0.10 & 0 & 0.05 & 0 & 0.13 & 0 \\
0.04 & 0.08 & 0.05 & 0 & 0.08 & 0 & 0 & 0 & 0 & 0.04 & 0.14 & 0.05 & 0 & 0.17 & 0 & 0 & 0 & 0 & 0.04 & 0.13 & 0.05 & 0 & 0.13 & 0 & 0 & 0 & 0
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\left.\begin{array}{ccccccccccccccccccccccccccc}
0.04 & 0.14 & 0.05 & 0.10 & 0.17 & 0.05 & 0.13 & 0.13 & 0 & 0.04 & 0.14 & 0.05 & 0.10 & 0.18 & 0.05 & 0.13 & 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.14 & 0.05 & 0.10 & 0.18 & 0.05 & 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.04 & 0.14 & 0.05 & 0.10 & 0.17 & 0 & 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.14 & 0.05 & 0 & 0.18 & 0.05 & 0.13 & 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.04 & 0.14 & 0.05 & 0.10 & 0 & 0.05 & 0 & 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.04 & 0.14 & 0.05 & 0 & 0.17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

Which is a 6×81 *matrix*

Step 6 : Observe that there are 9 blocks 6×9 elements in the above matrix. For each block, we choose the minimum value for each row. We then obtain:

$$
[d_{ir}] = \begin{bmatrix}
0.02 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Step 7 : Using the *And-product* $[d_{ir} \wedge R_{il}]$ is obtained as:

$[d_{ir} \wedge R_{il}] =$

$R_1 = [$ 0.02 0.02 0 0.02 0.02 0.02 0.02 0.02 0 0.04 0.04 0 0.04 0.04 0.04 0.04 0.04 0 0.04 0.04 0 0.04 0.04 0.04 0.04 0.04 0
0.04 0.04 0 0.04 0.04 0.04 0.04 0.04 0 0.04 0.04 0 0.04 0.04 0.04 0.04 0.04 0 0.04 0.04 0 0.04 0.04 0.04 0.04 0.04 0
0.04 0.04 0 0.04 0.04 0.04 0.04 0.04 0 0.04 0.04 0 0.04 0.04 0.04 0.04 0.04 0 0.04 0.04 0 0.04 0.04 0.04 0.04 0.04 0 $]$

$R_2 = R_3 = R_4 = R_5 = R_6 = \{0,0,\ldots,0\}$ (81 zeros) where R_i indicates the elements in row *i*.

Hence, the min for the *And-product* $[d_{ir}] \wedge [R_{il}] = [t_{ip}]$ is:

$$
[t_{ip}] = \begin{bmatrix}
0.02 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0.04 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Step 8 : From step 7, we obtain the max value in each row as:

$$
Mm\left(\left([P_{ij}] \wedge [Q_{ik}]\right) \wedge [R_{il}]\right) = Mm[t_{ip}] = \begin{bmatrix} 0.04 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Step 9 : Finally, the optimum set of U according to $Mm\left(\left([P_{ij}] \wedge [Q_{ik}]\right) \wedge [R_{il}]\right)$ is calculated

$$
Opt_{Mm(([P_{ij}] \wedge [Q_{ik}]) \wedge [R_{il}])}(U) = \{u_1\},
$$

It is clear that the optimum set of universal set (U) is u_1, hence h_1 is the selected house that Mr. X family want to buy.

VI. Conclusion

In this paper, we have presented a new method in soft matrices incorporating determination weight of criteria. AHP approach is used to determine the weight of each criterion for every decision makers and we solved group decision making problem by using soft max-min decision making method (*SMmDM*). We provide a numerical example of house selection problem that demonstrated the feasibility of this method.

This method can be applied to many fields in other group decision making problems that contained uncertainty data and would be beneficial to extend the proposed method to subsequent study.

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