

# Further Investigation of Elastic Scattering of $^{16}\text{O}$ on $^{12}\text{C}$ at Different Energies

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**Abstract**—The aim of this work is to study the elastic transfer phenomenon which takes place in the elastic scattering of  $^{16}\text{O}$  on  $^{12}\text{C}$  at energies near the Coulomb barrier. Where, the angular distribution decrease steadily with increasing the scattering angle, then the cross section will increase at backward angles due to the  $\alpha$ -transfer process. This reaction was also studied at different energies for tracking the nuclear rainbow phenomenon. The experimental data of the angular distribution at these energies were compared to the calculation predictions. The optical potential codes such as SPIVAL and Distorted Wave Born Approximation (DWUCK5) were used in analysis.

**Keywords**—Transfer reaction, DWBA, Elastic Scattering, Optical Potential Codes.

## I. INTRODUCTION

A N experiment was performed with a 24 and 28 MeV  $^{16}\text{O}$  beam on a  $^{12}\text{C}$  target in the cyclotron DC-60 located in Astana, Kazakhstan, to study the elastic scattering of  $^{16}\text{O}$  on  $^{12}\text{C}$ ; the reaction also was analyzed at different energies for tracking the phenomenon nuclear rainbow. Its aims were to extend the measurements to very large angles, and attempt to uniquely identify the elastic scattering potential. Nuclear rainbow scattering is well known in  $\alpha$ -particle Scattering [1] and in some light heavy-ion scattering such as  $^{16}\text{O}+^{16}\text{O}$ ,  $^{16}\text{O}+^{12}\text{C}$ , and  $^{12}\text{C}+^{12}\text{C}$  [2], for which absorption is weak. The rainbow scattering and the associated Airy structure can be well described by a deep folding type potential. The angular distributions of rainbow scattering are sensitive to the potential up to very internal region, which made it possible to determine the interaction potential uniquely and precisely.

The study of refractive nuclear rainbow scattering, where massive composite particles interact and penetrate each other, is a unique source of information on this interaction of nucleons inside nuclear matter.

The scattering of massive particles is described by their wave properties, which are determined by the de Broglie wavelength. This is given by the energy,  $E$ , and the reduced mass,  $\mu$ , of the particle as,  $\lambda = \hbar / (2\mu E)^{1/2}$ .

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The waves associated with nuclear particles are not only refracted (changing their wavelength and direction) but usually also absorbed, giving rise to diffraction. Diffraction occurs if a geometrical object removes flux from incoming waves. The corresponding intensity pattern as function of scattering angle, observed at far distances known as Fraunhofer diffractive scattering. The pattern shows intensity maxima which are separated by very sharp minima; the distance in angle between these minima is determined by the radius of the object  $R$  and wavelength  $\lambda$ . We remark that, with increasing the energy the Airy minima is shifted toward small angles. In elastic scattering process, the rainbow angle  $\theta_N$  can be given analytically if Woods-Saxon form factor is assumed for the real part of the optical potential

$$\theta_N \approx \left| V_c - 0.56 V \left( \frac{R}{a} \right)^{1/2} \right| / E_{c.m.} \quad (1)$$

The nucleus-nucleus interaction potential is a key ingredient in the analysis of nuclear reactions. By using the potential between nuclei we can evaluate the cross sections of different nuclear reactions [3]. The interaction potential between nuclei consists of nuclear, Coulomb and centrifugal parts. The Coulomb and centrifugal interactions of two nuclei are well-known. In contrast to this the nuclear part of nucleus-nucleus interaction is known worse.

$$U(R) \approx V_c(R) - V \left[ 1 + \exp\left(\frac{R - R_v}{a_v}\right) \right]^{-1} - iW \left[ 1 + \exp\left(\frac{R - R_w}{a_w}\right) \right]^{-1} \quad (2)$$

## II. GENERAL DESCRIPTION OF EXPERIMENT

The experiment was carried out with a 24 and 28 MeV  $^{16}\text{O}$  beam from the DC-60 cyclotron at Astana, Kazakhstan. A self supported carbon foil  $20\mu\text{g}/\text{cm}^2$  thick was used as the target. Scattered particles were detected by silicon surface barrier detector ORTEC company sensitive layer with a thickness of 100 microns. The energy resolution of the registration system was 250-300 keV, which is mainly determined by the energy spread of the primary beam. Only one detector was used in our measurements which detect  $^{16}\text{O}$  fragment with the ability also to identify the  $^{12}\text{C}$  peaks.

### III. RESULTS AND DISCUSSION

In addition to our experimental data obtained from the cyclotron DC-60 at energies 24 and 28 MeV, we also analyzed the elastic scattering of  $^{16}\text{O}$  on  $^{12}\text{C}$  at different energies (260, 230, 200, 170, 132, 80, 65, 42, 35) MeV from literature survey [4, 5], in order to make analysis for this reaction in a wide range of energies. Good agreement between the experimental data and the theoretical predictions has been obtained with optimal optical potential parameters using different optical potential codes such as SPIVAL and DWUCK5. The optical model code SPIVAL was used successfully for fitting the experimental data at energies (260, 230, 200, 170 and 132) MeV as shown in figure 1, the optimal optical parameters are listed in table 1. Fairly good results could be obtained by using SPIVAL with  $l$ -dependent imaginary potential at energies (80, 65, and 42) MeV as shown in figure 2, with optimal parameters listed in table 2. The phenomenon of nuclear rainbow is clearly shown at energies 132, 200, 170, and 230 as shown in figure 1. The most interesting feature for this reaction is shown at small energies but somewhat greater than the Coulomb barrier energy as (24 and 28 MeV), where the phenomenon of  $\alpha$ -cluster transfer is clearly shown, the differential cross section at forward angles decrease steadily with a sharp increase at the backward angles. In this case, the optical potential model code SPIVAL could be used for fitting the experimental data in the forward hemisphere, and at the backward angles the distorted wave Born approximation (DWUCK5 Code) could be used successfully for fitting the experimental data. The angular distributions for the elastic transfer reaction between  $^{16}\text{O}$  on  $^{12}\text{C}$  at energies 24 and 28 MeV are represented in figures 3 and 4. The potential parameters are listed in table 1, the coulomb radius  $r_c$  was fixed at 0.95 fm during the search. The  $J_V$  and  $J_W$  values obtained with the WS1 also agree closely with the global systematic found for light heavy ions elastic scattering.

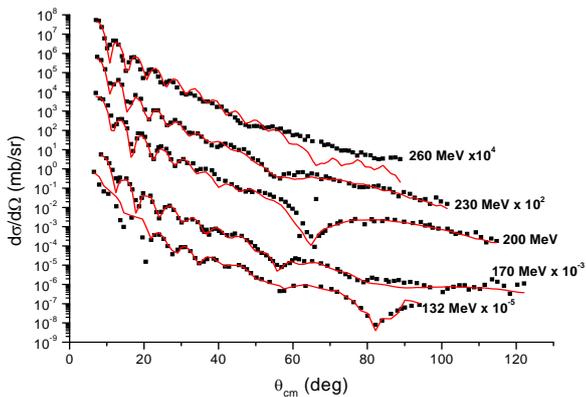


Fig. 1. The angular distributions for elastic scattering of  $^{16}\text{O}$  on  $^{12}\text{C}$  at energies 260, 230, 200, 170, and 132 MeV using optical potential code SPIVAL. Points are experimental data and solid curves are calculated cross sections.

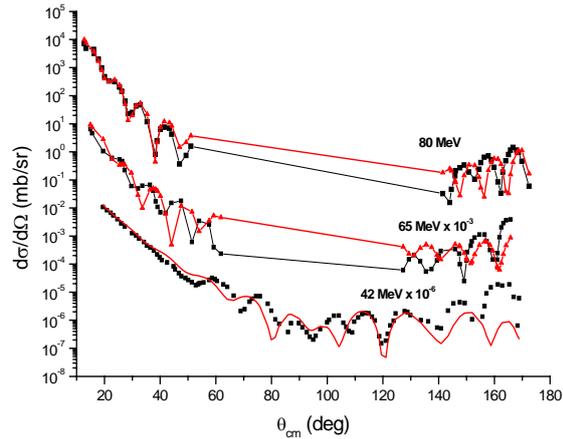


Fig. 2. The angular distribution of  $^{16}\text{O}$  on  $^{12}\text{C}$  at energies 80, 65, and 42 MeV respectively using SPIVAL code with  $l$ -dependent imaginary potential. Points are experimental data and solid curves are calculations.

It was suggested [6] that especially in heavy ion elastic scattering-in a certain energy region-the use of a  $l$ -dependent imaginary potential is necessary. This should take into account the fact that high angular momentum waves are weakly absorbed relative to low angular momentum waves. In the optical model the imaginary potential  $W$  is replaced by

$$W(l) \approx W \frac{l}{1 + \exp(l - L_c)\Delta L} \quad (3)$$

The quality of the experimental data description on the basis of some theoretical function (functional of several variables) can be estimated using the  $\chi^2$ -method, which is represented as

$$\chi^2 = \frac{1}{N} \sum_{i=1}^N \left[ \frac{\sigma_i^t(\theta) - \sigma_i^e(\theta)}{\Delta \sigma_i^e(\theta)} \right]^2 = \frac{1}{N} \sum_{i=1}^N \chi_i^2 \quad (4)$$

where  $\sigma^e$  and  $\sigma^t$  are the experimental and theoretical (i.e. calculated by using some given values of the scattering phase shifts  $\delta_{s,l}^j$ ) differential cross sections of the elastic scattering of nuclear particles for  $i$ -th scattering angle,  $\Delta \sigma^e$  is the error of the experimental differential cross sections for these angles,  $N$  - the number of measurements. The less the value  $\chi^2$  is, the better is the description of the experimental data in terms of the selected theoretical representation. Usually the results of calculations can be considered as wholly satisfactory if  $\chi^2$  is about 1, i.e. the deviation of the calculated values from the experimental ones is in average equal to the value of the experimental errors. The description can be considered as good if every partial  $\chi^2$  for each scattering angle is less than 1, and thus, the average  $\chi^2$  is always less than 1.

TABLE I  
THE OPTIMAL OPTICAL POTENTIAL PARAMETERS OBTAINED USING THE SPIVAL CODE FOR  $^{16}\text{O}$  ELASTICALLY SCATTERING ON  $^{12}\text{C}$  AT DIFFERENT ENERGIES,  $R=r_0(A_p^{1/3}+A_t^{1/3})$

E (MeV)	$V_0$ (MeV)	$r_r$ (fm)	$a_r$ (fm)	$W_0$ (MeV)	$r_i$ (fm)	$a_i$ (fm)	$J_v$ (MeV/F <sup>3</sup> )	$J_w$ (MeV/F <sup>3</sup> )	$r_c$ (fm)
260	168.29	0.769	0.801	24.86	1.163	0.454	271.8	101.1	0.95
230	180.58	0.763	0.825	22.32	1.162	0.622	291.9	95.40	0.95
200	213.22	0.683	0.924	17.84	1.219	0.561	293.6	85.50	0.95
170	255.90	0.629	0.970	16.41	1.245	0.521	311.6	82.59	0.95
132	288.19	0.586	0.986	13.62	1.224	0.561	310.8	66.02	0.95
80	319.44	0.546	0.906	8.25	1.294	0.512	274.7	46.27	0.95
65	334.21	0.518	0.955	7.263	1.304	0.582	276.5	42.40	0.95
42	365.07	0.495	0.924	6.80	1.320	0.407	267.2	39.49	0.95
28	496.01	0.621	0.571	5.40	1.22	0.707	288.8	41.23	0.95
24	500.39	0.603	0.383	5.20	1.35	1.307	312.4	43.46	0.95

The relationship between energy and real potential depth (V), imaginary potential depth (W) were calculated. The strength parameters in table 1 can be presented be:  $V_0 = 242.03 - 0.271E$ , and  $W_0 = 4.854 + 0.14E - 1.9 \cdot 10^{-4} E^2$

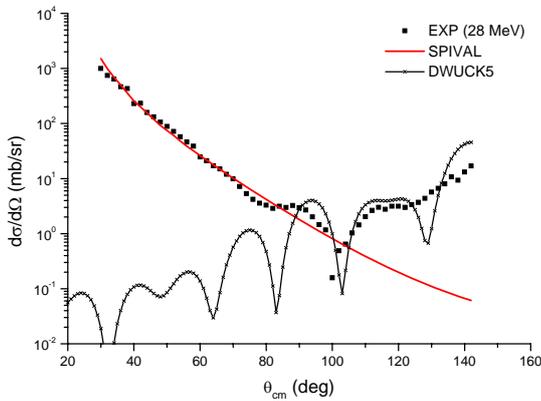


Fig. 3. The angular distribution for elastic scattering of  $^{16}\text{O}$  on  $^{12}\text{C}$  at energy  $E=28$  MeV. Points are measured cross-sections and solid lines are calculations using optical potential code SPIVAL and DWBA.

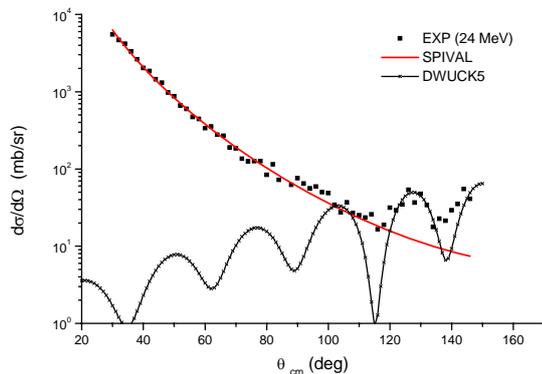


Fig. 4. The angular distribution for elastic scattering of  $^{16}\text{O}$  on  $^{12}\text{C}$  at energy  $E=24$  MeV. Points are measured cross-sections and solid lines are calculations using optical potential code SPIVAL and DWBA.

TABLE II  
THE OPTIMAL OPTICAL POTENTIAL PARAMETERS OBTAINED USING SPIVAL WITH  $l$ -DEPENDENT IMAGINARY POTENTIAL FOR  $^{16}\text{O}$  ELASTICALLY SCATTERING ON  $^{12}\text{C}$ , COULOMB RADIUS WAS FIXED AT 0.95FM, AND  $R=r_0(A_p^{1/3}+A_t^{1/3})$

E (MeV)	$V_0$ (MeV)	$r_r$ (fm)	$a_r$ (fm)	$W_0$ (MeV)	$r_i$ (fm)	$a_i$ (fm)	$L_c$	$\Delta L$
42	25.85	1.231	0.439	46.56	1.195	0.246	9.2	1
65	30.63	1.125	0.657	38.37	1.05	0.718	15	1
80	100.8	0.999	0.606	23.29	1.156	0.458	18	1

#### IV. CONCLUSION

The elastic scattering of  $^{16}\text{O}$  on  $^{12}\text{C}$  has been studied in a wide energy range. The rainbow phenomenon has been observed at energies 230, 200, 170, and 132 MeV, in this range of energies the optical model code SPIVAL could be used effectively for fitting the experimental data. While, at lower energies such as 80, 65 and 42 MeV good results could be obtained using SPIVAL with  $l$ -dependent imaginary potential. Optical model calculations with  $l$ -dependent imaginary potentials were also applied to the data and relatively good agreement was found. However, the parameter  $L_c$  could not be fixed by any physical argument. It can be chosen arbitrarily by changing the depth of the imaginary potential adequately. Thus, as long as no physical concept exists to determine  $L_c$  or the depth of the imaginary potential, the inclusion of the  $l$ -dependence consists mainly in an increase in the number of parameters to fit the data.

The most interesting feature of this reaction is observed at low energies near the coulomb barrier as 24 and 28 MeV, the  $\alpha$ -transfer process results in the increase of the differential cross-section at backward angles. In this case SPIVAL code could be used for fitting the experimental data at forward hemisphere and Distorted Wave Born Approximation (DWUCK5 Code) could be used for fitting the experimental data at backward angles using potential parameters already obtained from SPIVAL.

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