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A Study of Indentation Energy in Three Points Bending of Sandwich beams with Composite Laminated Faces and Foam Core

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***Abstract*—**This paper deals with analysis of flexural stiffness, indentation and their energies in three point loading of sandwich beams with composite faces from Eglass/epoxy and cores from Polyurethane or PVC. Energy is consumed in three stages of indentation in laminated beam, indentation of sandwich beam and bending of sandwich beam. Theory of elasticity is chosen to present equations for indentation of laminated beam, then these equations have been corrected to offer better results. An analytical model has been used assuming an elastic-perfectly plastic compressive behavior of the foam core. Classical theory of beam is used to describe three point bending. Finite element (FE) analysis of static indentation sandwich beams is performed using the FE code ABAQUS. The foam core is modeled using the crushable foam material model and response of the foam core is experimentally characterized in uniaxial compression.

Three point bending and indentation have been done experimentally in two cases of low velocity and higher velocity (quasi-impact) of loading. Results can describe response of beam in terms of core and faces thicknesses, core material, indentor diameter, energy absorbed, and length of plastic area in the testing. The experimental results are in good agreement with the analytical and FE analyses. These results can be used as an introduction for impact loading and energy absorbing of sandwich structures.

***Keywords*—**Three point Bending, Indentation, Foams, Composite laminated beam, Sandwich beams, Finite element

I. Introduction

SANDWICH structures are composed of composite laminates as skins and low density foam as core that present suitable properties for flexural stiffness and absorbing energy without weight penalty. These characteristics exhibit important role in three point and impact loading. Energy in these kinds of loadings are absorbed in three stages of indentation in laminated beam, indentation of sandwich beam and bending of sandwich beam.

The contact law between two isotropic bodies was first developed by Hertz [1]. Willis [2] investigated the contact behavior between a transversely isotropic half-space and a rigid sphere. Yang and Sun [3] have conducted several static indentation tests on glass/epoxy. Enboa and Shyu [4] have presented experimental results for contact and Low velocity impact response of composite laminates by rigid spheres. Christoforou [5] has introduced a perfect theory for indentation in composites too. In this research, last theory selected and has been corrected to offer better results to compare with experiments.

In loading of sandwich structures, foam has a weak resistance such that indentation is further related to composite laminate skins. Much research effort has been given to this problem in order to model a response of sandwich structures to local load. An excellent review article by Arate[6] provides a through overview of research work on this subject. Soden[7] also presented an analytical model for indentation of sandwich beam assuming plastic behavior for core. Shuaeib and Soden [8] extended experimentally this work. Zenkert, et al [9] have recently studied indentation of sandwich beams. They presented an elastic-perfectly plastic compressive behaviour of foam core that elastic part of indentation is described by Winkler foundation model. Yang and Qiao [10] presented impact fully backed composite sandwich structures. Hazizzian and Cantwell [11] have investigated low velocity impact of sandwich structures and attended to absorbed energy in structures.

In the present paper, three point bending and indentations on composite laminates and sandwich beams have been done experimentally in two cases of low velocity and higher velocity (quasi-impact) of loading. Also they have been simulated with FE codes and have been modelled analytically with modified equations for indentation of composites and elastic-perfect plastic model of foam for indentation of sandwich beams and Allen's classical theory [12] for three point bending of sandwich beam .Also Results have been presented in terms of core and faces thicknesses, core material, indentor diameter, energy absorbed, and length of plastic area in the testing.

II. Properties of Beam Components

A. FOAMS

Energy absorption of polymeric foam has caused their extensive application in mechanical structures. In this research, PVC and Polyurethane foam were used in sandwich beams. These foams show a special behavior in uniaxial compression in according of Fig. 1. The foam properties were obtained from uniaxial compression tests according to that given in ASTM D5308 standard [13]. Loading velocities were selected in two cases of $2_{mm/min}$ and $100_{mm/min.}$ Finite element

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analysis is performed using the FE code ABAQUS. For validation of FE results, uniaxial compression is modeled by Abaqus that result is presented in Fig. 2 and show good agreement to each other. Also foam properties are displayed in Table I.

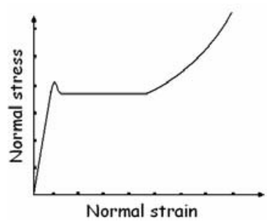


Fig. 1 Foam behavior in uniaxial compression

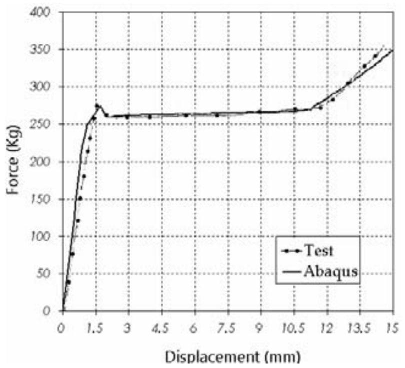


Fig. 2 PVC loading in uniaxial with velocity 2mm/min

TABLE I
MECHANICAL PROPERTIES OF FOAMS

Foam	υ	2 mm / min		100 mm / min	
		E Mpa	σ_y Mpa	E Mpa	σ_y Mpa
PU	0.42	6.15	0.274	6.9	0.315
PVC	0.32	20.4	1.11	25.1	1.25

B. Composite Laminate Properties

In the present study, composite laminates were fabricated from Woven Eglass / epoxy in hand lay up style. They were prepared in two forms of 2layers and 4layers and their properties were obtained from tests according to that given in ASTM D3039 standard. In lateral direction, according to Fig 3, behavior of laminated is divided to two regions. In initial compression state, resin represents low resistance until reach to point *a*. Then in second region laminate acts stronger. Composite laminate properties are presented in Table II

III. INDENTATION OF COMPOSITE LAMINATED

A. Indentation Theory

Well known contact law has been derived by Hertz for isotropic materials, that special case of it is the contact of an elastic sphere on an elastic half space.

$$F = K\alpha^{1.5} \tag{1}$$

$$K = (4/3)(R_s)^{0.5}[(1-\upsilon_s^2)/E_s + (1-\upsilon_t^2)/E_t]^{-1} \tag{2}$$

s: isotropic indentor *t:* isotropic half space

A modified contact law was employed by Willis [2] for a study on contact of transversely isotropic half-space. Where :

$$K = (4/3)(R_s)^{0.5}[(1-\upsilon_s^2)/E_s + 1/E_2]^{-1} \tag{3}$$

E_2 : Transverse modulus in the half space

Another theory was presented by Christoforou [5] that it's results are closer to experiment. According to Fig 4 and theory of elasticity, we have

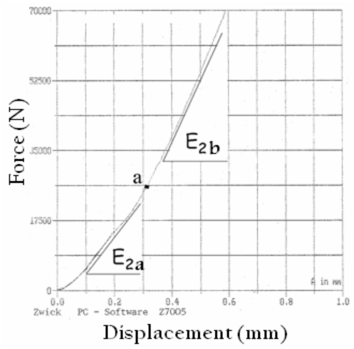


Fig. 3 Lateral compression in 4layer composite laminate

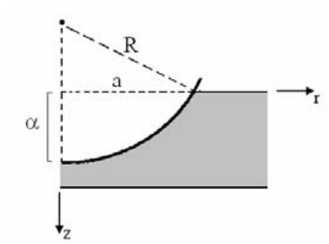


Fig. 4 Indentation of composite laminated

for $\alpha << 1$ $\quad a^2 = 2R\alpha$

And the stresses are [5]:

$$\sigma_r = \frac{\upsilon_{rz} E_r a^2}{4(1-\upsilon_{rz}\upsilon_{zr})Rh}(1-\frac{r^2}{2a^2}) \tag{4}$$

$$\sigma_z = \frac{E_z a^2}{4(1-\upsilon_{rz}\upsilon_{zr})Rh}(1-\frac{r^2}{2a^2}) \tag{5}$$

$$\sigma_\theta = \frac{\upsilon_{rz} E_r a^2}{4(1-\upsilon_{rz}\upsilon_{zr})Rh}(1-\frac{3r^2}{2a^2}) \tag{6}$$

was only the component considered σ_z is small and υ_{rz}

$$F(a) = 2 * \int_0^a \sigma_z \cdot L \cdot \delta r \tag{7}$$

Or:

$$F(\alpha) = \frac{2^{2.5}}{3} . \frac{E_2 . L}{(1-\upsilon_{rz}\upsilon_{zr})h}\sqrt{R}\ \alpha^{1.5} \tag{8}$$

Where:

R : laminate wide L : indentor diameter
E_2 : transverse modulus of laminates
h : laminate thikness α : indentation

TABLE II
Mechanical Properties of Composite Laminate

Layers	Thickness mm	E_1 Mpa	Mpaσ_y	G_{12} Mpa	G_{32} Mpa	G_{13} Mpa	$E_{2.a}$ Mpa	$E_{2.b}$ Mpa	α_{cr} mm	σ_{cr} Mpa
2	0.92	11e3	62	8	8	15	19	27	0.15	6.4
4	0.52	14e3	130	12	12	20	30	50	0.31	10.5

B. Modification of Indentation Theory

In this part, equations of last section have been modified. Assuming to Fig 3 and according to Fig 5, after point a, behavior of laminate should be presented with two elasticity modulus.

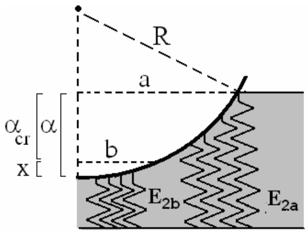


Fig. 5 Two modulus consideration for laminates indentation

Thus:

$$x = \alpha - \alpha_{cr} \qquad \alpha > \alpha_{cr}$$

$$b^2 = 2Rx \qquad a^2 = 2R\alpha$$

Therefore equation leads to:

$$F(a) = 2 * \left(\int_0^b \sigma_{z_2} L\,\delta r + \int_b^a \sigma_{z_1} L\,\delta r\right) \tag{9}$$

$$F(a) = bL\sigma_{cr} + \frac{2}{3}.\frac{E_{2a}a^2L}{(1-\upsilon^2).h.R}.(a-b) + \frac{2}{3}.\frac{E_{2b}b^2}{(1-\upsilon^2).h.R}b \tag{10}$$

C. Indentation Test in Composite Laminate

Laminate indentations have done experimentally with two different diameter indentores, 10mm and 20mm (Fig 7), loading velocity was chosen 0.2mm/min and laminate has been supported by a rigid plate. Also indentation was modeled in Abaqus (Fig. 6).

Four series of results: elasticity theory, modified theory, Abaqus and test were displayed in Fig's 8,9,10.

Results in initial steps of indentation show a suitable convergence and after 20% indentation of thickness, results difference increases rapidly. Modified theory presents a more satisfactory behaviour than elasticity theory after 40% indentation and FE results have better prediction than both of theories.

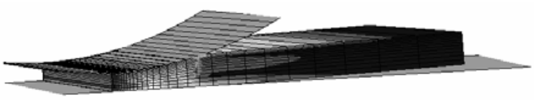


Fig 6 Modeling of loading in Abaqus

IV. Indentation of Sandwich Beam

In this section, Abrate's suggestion [6] and Zenkert's approach [9] were chosen to introduce indentation of sandwich beams.

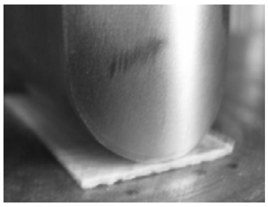


Fig 7 Laminated composite indentation

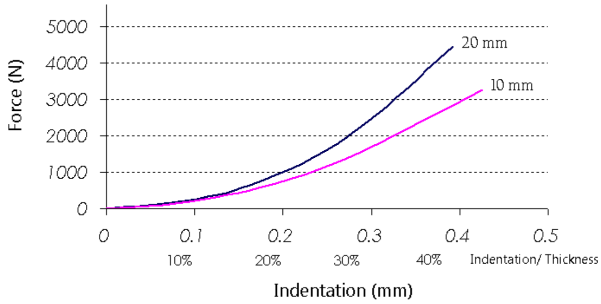


Fig. 8 Indentation in 4layer laminate with 10_{mm} ,20_{mm} indentor

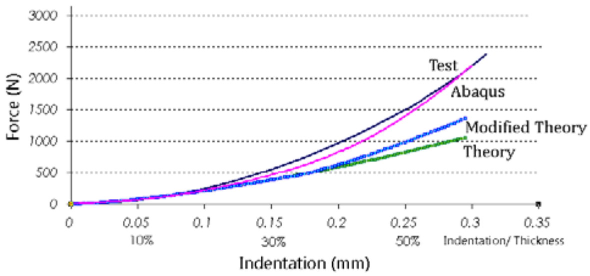


Fig. 9 Indentation in 2layer laminate with 10_{mm} indentor

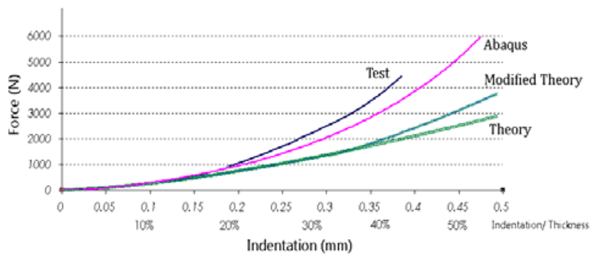


Fig. 10 Indentation in 4layer laminates with 20_{mm} indentor

A. Indentation Theory

The model assumes an elastic Winkler foundation for the elastic core, and a perfectly plastic foundation for part of core that undergoes crushing, as schematically shown in Fig 11,[9].

1) Elastic Solution

When the load P is small, the entire foundation is elastic and governing equation is:

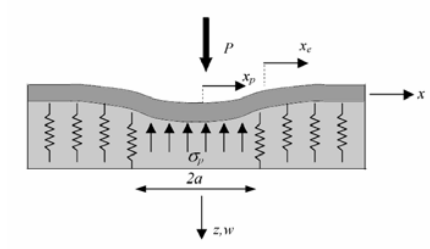


Fig. 11 Indentation model of sandwich beams [9]

$$D_f \frac{d^4 w_{el}}{dx_e^{\,4}} + Kw_{el} = 0 \tag{11}$$

Where:

w_{el} : face sheet deflection $\quad$ k : foundation modulus

$$k = \frac{E_c}{t_c} \tag{12}$$

t_c : core thickness $\quad$ E_c : core elastic modulus

and boundary conditions are

$$w'_{el}(0) = 0 \qquad w_{el}(0) = \alpha \qquad w'_{el}(\infty) = 0$$

$$w_{el}(\infty) = 0$$

A.2 Perfectly Plastic Solution

As the load increases, a part of core with length of 2a undergoes plastic deformation and core shows a uniform constant reaction σ_p on the face sheet.

Thus, governing equation is

$$D_f \frac{d^4 w_{pl}}{dx_p^{\,4}} + \sigma_p = 0 \tag{13}$$

Where:

σ_p : plateau compressive yield stress

and boundary conditions are

$$w'_{pl}(0) = 0 \qquad w_{pl}(0) = \alpha$$

$$\left(\frac{d^3 w_{pl}}{dx^3_{\,p}}\right)_{x_p=0} = \frac{P}{2D_f}$$

General equation for face deflection is obtained by combining the elastic and plastic of Eqs. (11) and (13)

Soden [7] has also presented an analytical model for perfectly plastic behavior of core.

$$P = \frac{4}{\sqrt{3}} \left(\frac{2}{3}\right)^{0.25} bt_f^{\;0.75} \sigma_{pcore}^{\;0.75} E_f^{\;0.25} \alpha^{0.25} \tag{14}$$

B. Indentation Test in Sandwich Beams

The properties of manufactured sandwich beams have been presented in Table III.

Foam materials, skins thicknesses and foam thickness are varied between samples.

Indentation have done experimentally with two different diameter indentores, 10mm and 20mm. Loading velocities were chosen 2mm/min and 80mm/min. Higher velocity observes strain rate effect in indentation. Sandwich beams have been supported by a rigid plate (Fig. 12). Also indentation was modeled in Abaqus.(Fig. 13).

V. Results of Sandwich Indentation

Effect of indentor diameter is hot considerable and according to Fig. 14 there isn't any change in elastic area for two indentors but a little shift are seen in plastic area. Abaqus have also presented similar results for effect of indentor diameter.

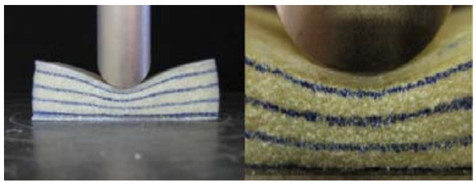


Fig 12 Indentation in sandwich beam with PU core

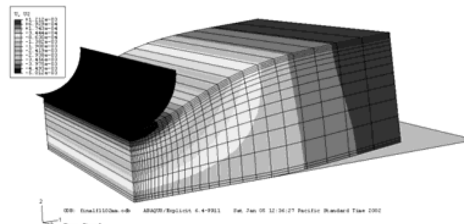


Fig 13 Indentation modeling of sandwich beam in Abaqus

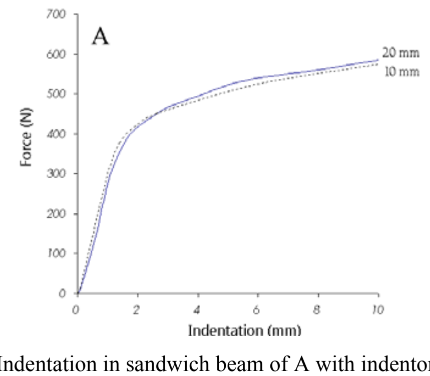


Fig 14 Indentation in sandwich beam of A with indentores 10_{mm}, 20mm

Four series of results: Zenkert's theory (IV.A), Soden's theory (IV.A.1), Abaqus results and indentation test (IV.B) were displayed in Fig 15 for several samples of sandwich beams introducing in Table III.

Indentation results show that increasing of core and skins stiffness enhances sandwich resistance in indentation loading. Also thickness increasing of sandwich components has a suitable effect in rising of resistance. In plastic area, Soden's theory shows a good approximation comparing to other results. Sandwich beams mentioned in Table III have been modeled with Zenkert's theory and a comparison between their indentations was presented in Fig. 16.

TABLE III Properties of used sandwich beams

A	B	C	D	E	F	Sandwich beams
40	40	30.5	30.5	30.05	30.5	Wide mm
60	60	60	60	60	60	Length mm
21.6	20.5	10.2	9.7	11.9	11.6	Thickness mm
PU	PU	PVC	PVC	PU	PU	Core
40	40	70	70	40	40	Core density (Kg/m^3)
19.76	19.46	8.36	8.66	10.06	10.56	Core thickness mm
4layer	2layer	4layer	2layer	4layer	2layer	Composite skins
0.92	0.52	0.92	0.52	0.92	0.52	Skins thickness mm

Finally loadings were performed in a low velocity of 2mm/min like quasi static loading. But according to Table I, increasing of loading velocity has an important effect on resistance of foams. So mentioned sandwich beams again have been loaded with a new velocity of 80mm/min. Experimental results of both velocities loading and Zenkert's theory using higher velocity results of Table I have been shown in Fig 17.

Deformation of upper skin in indentation of sandwich beam was shown and compared with presented equations (IV.A) in Fig 18. This deformation has been shown for Abaqus and theory results. In plastic area, results are very close to each other but many differences are seen in beginning of elastic area as indentation increases.

VI. Three Point Bending of Sandwich Beams

Manufactured beams mentioned in Table III have been loaded in three point bending. Under ASTM C393 standard and according to Fig 19, indentor and support diameters are 20mm and 30mm, also distance between supports is 200mm, loading velocities are 2mm/min and 100mm/min.

Allen's classic theory [12] was chosen to present bending stiffness for deriving equation (15), assumptions of theory are:

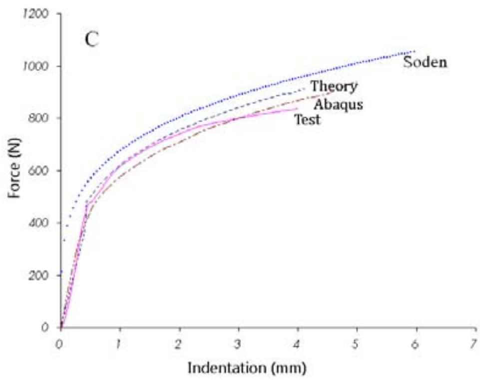
1. Elimination of core stiffness comparing to skins stiffness in longitudinal direction
2. Uniform shear stress distribution in lateral direction of core

$$\Delta = F.(\frac{l^3}{48.D} + \frac{l}{4.A.G}) \tag{15}$$

Three point bending was modeled in Abaqus and results of Abaqus, classic theory and experiments have been shown in Fig 20. Results of Abaqus for all Obtained results of Three point bending show that beams in Table III also were presented in Fig 21.

Theory and Abaqus predictions are converged in loading linear part and they are close to experimental results.

Loading velocity doesn't have a noticeable effect in sandwich behavior. Increasing of core and skins stiffness improves sandwich resistance in indentation loading but core thickness increasing causes local indentation and structure failure in lower displacement.



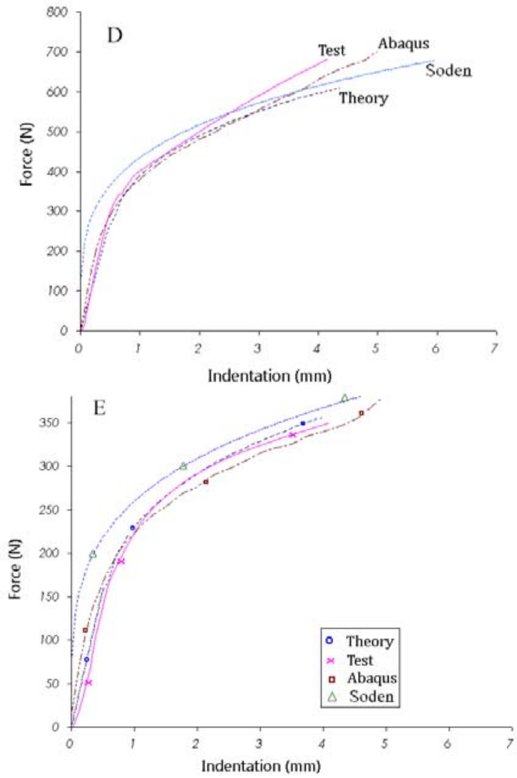


Fig 15 Indentation in sandwich beams of E, C, D

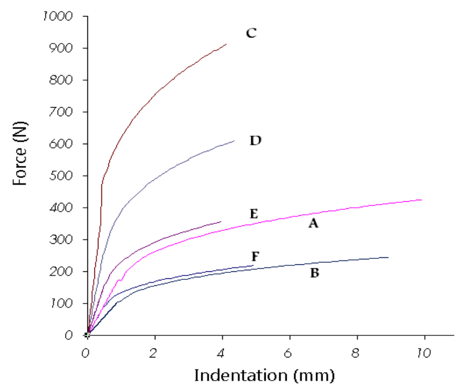


Fig. 16 Results of theory (4.1) for sandwich beams from Table III

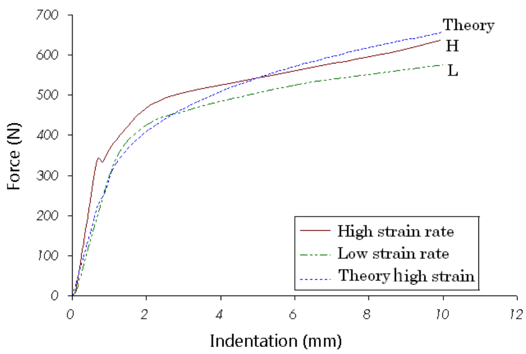


Fig 17 Higher velocity indentation in sandwich beam of A

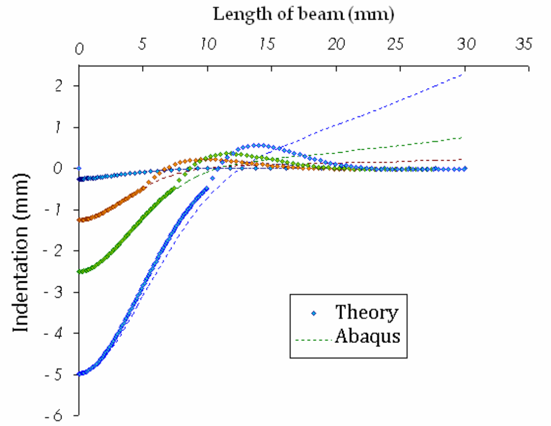


Fig 18 Deformation of upper skin in indentation of sandwich D

Fig 19 Three point bending in sandwich beam of A

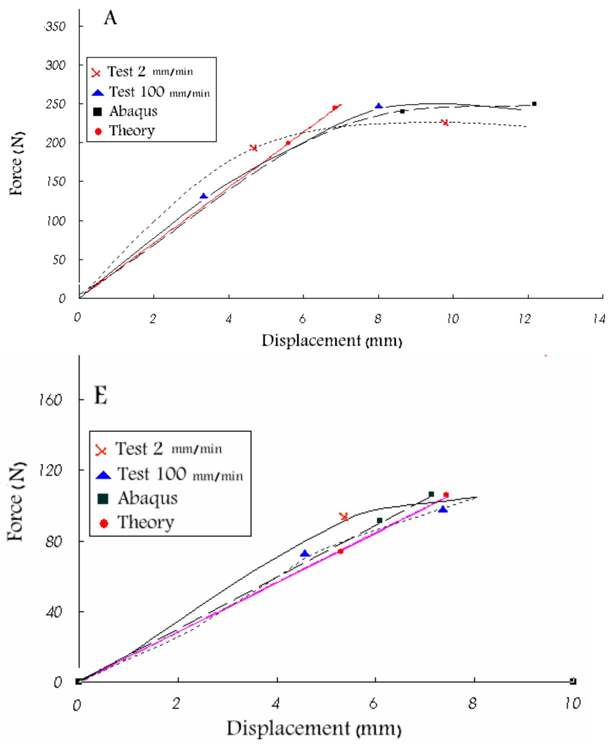


Fig. 20 Three point bending in sandwich beams A, E

Three point bending displacement is sum of skin laminate indentation, sandwich beam indentation and beam bending. Table IV shows mentioned displacements in sandwich beams and Table V presents consumed energies in indentations and displacements.

TABLE IV
COMPARISON OF INDENTATION AND BENDING DISPLACEMENT IN THREE POINT BENDING OF SANDWICH BEAMS .MM

A	B	E	F	C	D	Sandwich beams
0.082	0.05	0.048	0.03	0.090	0.06	Skin indentation
0.55	0.42	0.24	0.33	0.26	0.25	Total beam indentation
6	4	7	7	7	7	Total beam displacement

TABLE V
CONSUMED ENERGIES IN INDENTATION COMPARING TO TOTAL THREE POINT BENDING ENERGY (%) N.MM

9.1	10	3.4	4.7	3.7	3.5	Total indentation energy
1.36	1.25	0. 68	0.42	1.28	0.08	Skin indentation energy

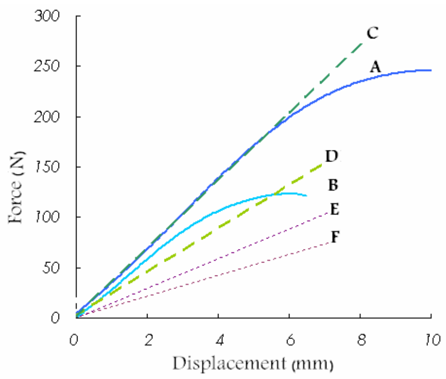


Fig. 21 Abaqus results for sandwich beams three point bending

VII. CONCLUSION

Foams have an important task in behavior of sandwich structures, so knowing their behaviour and it's suitable modeling helps to better prediction of behavior of sandwich structures loading. In indentation of composite laminate, presented theories offer suitable prediction comparing with test results in initial loading. Modified theory has a more satisfactory behavior than existence theory in 40% indentation and upper. Also FE results predict closer values to of tests in loading duration.

Theory of sandwich beam indentation has a good convergence with test and FE results. Also Soden's theory behaves closely to other theoretical results in plastic area.

In indentation loading, change of sandwich beam components shows that, increasing of core and skins stiffness, skins thicknesses and decreasing of foam thickness improve sandwich resistance. Loading velocity increasing shows more stiffness of beams and more energy absorption in sandwich beams. Diameter indentor has a clear effect in response of composite laminate indentation but has not obvious influence in sandwich indentation. Deformation of upper skin in sandwich indentation loading is predicted favorably by theory but differences in plastic area are increased by load rising.

In three point bending, theory and FE present acceptable results comparing with tests, results show that sandwich beams have a more flexural stiffness comparing with core and skin stiffness as thicknesses increase.

In three point bending duration, energy of laminate indentation is very little comparing with total loading energy but sandwich indentation consumes further energy that is notable portion of total energy.

Effect of consumed energies is very important on impact loading, so these results can be used in and may be as an introduction for absorption energy in impact loading.

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