

# Unipolar Anamorphosis and its use in Accessibility Analyses

T. Hudecek, Z. Zakova

**Abstract**—The paper deals with cartographic visualisation of results of transport accessibility monitoring with the use of a semi-automated method of unipolar anamorphosis, developed by the authors in the GIS environment. The method is based on transformation of distance in the map to values of a geographical phenomenon. In the case of time accessibility it is based on transformation of isochrones converted into the form of concentric circles, taking into account selected topographic and thematic elements in the map. The method is most suitable for analyses of accessibility to or from a centre and for modelling its long-term context.

The paper provides a detailed analysis of the procedures and functionality of the method, discussing the issues of coordinates, transformation, scale and visualisation. It also offers a discussion of possible problems and inaccuracies. A practical application of the method is illustrated by previous research results by the authors in the field of accessibility in Czechia.

**Keywords**—accessibility, GIS, transformation, unipolar anamorphosis

## I. INTRODUCTION, OBJECTIVES

TRANSPORT accessibility has been studied and analysed at the Faculty of Science of the Charles University in Prague in the long-term. There has arisen the need to visualise the results of the research for the geographic community and others. The most common method used is the method of isochronic maps. A great degree of illustrativeness and a relatively simple procedure to create these maps made them popular already over a hundred years ago [1].

The cartographic method of unipolar anamorphosis can be used. However, this method of Euclidean transformation used in common maps requires either considerable cartographic erudition or, in the case of GIS automation, resolution of a number of methodological and topological issues. It also places high demand on input digital data and their processing.

In the following chapters we outline the issue of accessibility research and the possibilities of its visualisation. Main emphasis is put on the introduction of a semi-automated method of unipolar anamorphosis performed in GIS, i.e.

Euclidean transformation from space to time, modelling accessibility to one central point (centre, city).

## II. ACCESSIBILITY AND ITS VISUALISATION

Accessibility is currently a topical issue in European [2], [3] and Czech [4], [5] transport geography research. In the time of advanced integration of the European Union (e.g. the international ESPON project) accessibility analyses in the form of temporal distance between centres and their hinterland are an important part of regional development and cross-border cooperation.

First geographical concepts of accessibility dealt merely with monitoring so called connectivity [6], i.e. the number of connections between two centres. Gradually, impedance (representation of distance) was taken into account. This does not necessarily have to be classical Euclidean distance but it can also be time or price of the journey [7]. Over time, this Euclidean perception of space has been substituted thanks to the development of IT technology. The previously most important aspect of accessibility – distance – lost its importance and has been substituted by temporal or travelcost surface accessibility [8].

Even though there are various other approaches to the study of accessibility, e.g. the potential of opportunities for interaction [9], accessibility of places [10], citizen accessibility [11], the ability of people to pursue activities in their neighbourhood [12] it is temporal accessibility that is the central and most frequent topic of all research.

Due to improvement in time accessibility there is reduction (also contraction, shrinking) of space. This is achieved by continuous advancement in the quality and speed of means of transport and by improvement of the quality of infrastructure, which leads to so called time-space convergence [13]. Even though in some spheres of human activity complete time-space convergence has been reached, i.e. a balance in accessibility on the whole planet by the means of e.g. Internet, telephone and partially postal and banking services [14], in the case of transport of material goods the process is still ongoing.

This time-space convergence can lead to a graphical output in the form of so called "shrinking map" or "time-space map". Shrinking maps represent a visually attractive but methodologically problematic method because when we conserve the limited number of three dimensions, we cannot transform space so that it preserves "all" distances [15]. This means that we can only work with a single central point (city, settlement centre) and only consider relationships towards it. Therefore, visualisation with the use of unipolar anamorphosis is more suitable than visualisation with the use of shrinking maps.

Despite simplification of the problem by a transformation in relation to one central point the method of unipolar anamorphosis is not easy and requires high cartographic

T. Hudecek is with the Charles University in Prague, Faculty of Science, Czech Republic, Albertov 6, 128 43, Prague 2, CZ (phone: 0042-22195-1410; e-mail: hudecek@dr.com).

Z. Zakova is with the Charles University in Prague, Faculty of Science, Czech Republic, Albertov 6, 128 43, Prague 2, CZ (e-mail: zuzka-zuzu@seznam.cz).

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erudition. A good solution to make this method available for both professional and laic audience is the use of automated or, at least, semi-automated creation in the GIS environment. Unipolar anamorphosis offers a suitable method when analysing two or more phenomena represented by isolines – e.g. time accessibility of a centre and the share of commuters from neighbouring municipalities.

### III. UNIPOLAR ANAMORPHOSIS AS A CARTOGRAPHIC METHOD

Unipolar anamorphosis is a type of real anamorphosis. Another type is non-unipolar anamorphosis which includes axial and general distortion [16]. It consists in deformation of space in relation to a central point. It is constructed along concentric curves, mainly circles, that represent a constant value of a real variable. If the real variable expresses the value of a geographical phenomenon, we define it as geographical unipolar anamorphosis.

Distorted maps were first mentioned in the 19<sup>th</sup> century [17]. The 1960s and 1970s saw a development of cartographic anamorphosis; this issue was dealt with by many authors, e.g. [18]-[19]. From mid-80s, when cartography started using information technology, research focused on the created of algorithms for different types of anamorphosis.

Nevertheless, all these studies focused primarily on so called cartograms, when distortion of the original cannot be identified in relation to one specific point and only various shape and polygon area distortion is made (so called value-by-area map). In contrast, unipolar anamorphosis represents a deformation of space in relation to a central point. A variation of the method was used, for example, for *town plans by the German publisher Falk* where space (topographic surface) in the central part of the map was magnified due to a large number of phenomena in this part of the map.

However, geography also works with spaces where Euclidean metric fails. A good example is the economic accessibility of a place, i.e. the price for which it is possible to reach the place (cost distance) or, in the case of space-time, time distance. A distorted map then represents the temporal, or possibly economic, distance of a place from a central point, which means that the map is transformed into geographic space. This then leads to a visual change in the position of points (approximation to or distance from the centre) depending on the value of the represented variable.

An important factor that allows the use of unipolar anamorphosis is the concentricity of input geographic data. Even though time accessibility can be in its nature considered a concentric geographic phenomenon, in some situations isochrones can be of such a nature that rules out the use of unipolar anamorphosis. For example, the map by Max Eckert [20] represents isochrones of time accessibility from Berlin in 1909 (see Fig 1). In the case of Africa there are situations when the inland is less accessible than the shore. If we wanted to use unipolar anamorphosis, Africa would have to be represented inversely, which could create an impression of it "caving in" onto itself and its resulting shape would not be identifiable anymore.

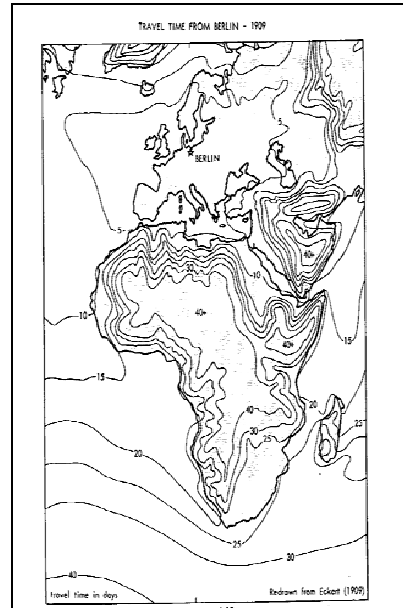


Fig. 1 Impossible task for unipolar anamorphosis – map by Max Eckert [20]

This problem can be solved by introducing a requirement for the value of the geographic element to have the form of cumulative sums from the centre towards the edge (even if it was different in individual directions). However, in reality there are few geographic phenomena that comply with the above mentioned condition. Usually, the fall in the values is local or there are "islands" of a significantly higher value than their surroundings. In such cases the cartographer's experience plays an important role in deciding whether changes are or are not suitable for the given purpose.

The number of distorted elements in the map also has to be considered. Each geographic element has a spatial relation to the other ones. The more elements are represented, the more interactions and spatial relations are taken into account. In these terms, anamorphosis is a radical method of thematic cartography, when spatial relations between elements can be distorted. The more relations there are in the map, the higher the probability of them being distorted by anamorphosis, which would make orientation in the map more difficult for the users. Therefore, the main construction problem is the respect of topology and spatial relations between the geographical elements in the distorted map. However, the selection of geographic elements is also influenced by the level of detail of the distorted map, i.e. the scale.

### IV. UNIPOLAR ANAMORPHOSIS IN GIS

When creating geographical unipolar anamorphosis, the cartographer has to consider the following: the suitability of the geographical phenomenon, the purpose of the resulting distorted map, the number of distorted elements, and the scale of the distorted space. Even though there are numerous advantages related to the degree of illustrativeness of the representation, this method should be used with care. Distortion of space can often be so extensive that there is a

real danger of the user finding it difficult to decipher the map. All these steps can be used with GIS and their automation is not only possible but highly advantageous.

The essence of unipolar anamorphosis is transformation of space along concentric curves. Because of the character of the transformation it is advantageous to use polar coordinates ( $\rho$  and  $\varepsilon$ ) to localise points in the map, where  $\rho$  indicates the distance of the point towards the distortion centre  $S$  and  $\varepsilon$  indicates the angle of the connector of the point and the beginning of the selected axis lying in the plane. Unipolar anamorphosis maintains the angle  $\varepsilon$  and transforms the distance  $\rho$  to  $\rho'$ .

In the case of geographical unipolar anamorphosis it is not possible to describe the function  $f$  that describes the transformation of the distance  $\rho$  to  $\rho'$  using an elementary mathematical equation (as in the case of mathematical unipolar anamorphosis). Because the value of the function  $f$  changes unevenly in the represented area depending on the characteristics of the represented geographical phenomenon we can simplify the description of the function  $f$  and note that it ensures that the points  $P$  with the same value  $G$  of the geographical phenomenon are of the same, or similar, distance from the centre  $S$ . Therefore,  $\rho' = G \cdot m_a$  where  $m_a$  indicates the distortion scale (e.g. 1 centimetre corresponds to 10 minutes).

The scale of the distorted map is expressed as the ratio of the value of the represented phenomenon and the distance from the distortion centre and it determines how many centimetres from the central point correspond to which value of the represented phenomenon  $G$ . Therefore, the scale logically does not apply to any other distance in the distorted map than the distance calculated from the central point.

In digital spatial data, position is usually expressed by the rectangular coordinates  $x, y$ . Conversion to polar coordinates encompasses the calculation of the arctangent function which, however, has the same value for two different angles, thus complicating an unequivocal localisation of the point. This can be solved by the use of a different expression of polar coordinates with the direction vector  $u_s \rightarrow$  (from the distortion centre  $S$  to the localised point  $P$ ) and its magnitude  $|u_s \rightarrow|$ . Then

$$P = S + u_s \rightarrow \quad (1)$$

apply for individual coordinates. Transformation of the point  $P$  from the original to the distorted position  $P^{ana}$  can be described by the equation

$$P^{ana} = S + u_s \rightarrow \cdot (\rho' / \rho) \quad (2)$$

where the ratio of the distorted distance  $\rho'$  and the original distance  $\rho$  expresses the relative displacement of the point in the direction  $u_s \rightarrow$  from the point  $S$ . If we substitute the influence of scales for the transition into an analogue format, we get

$$P^{ana} = S + u_s \rightarrow \cdot (G/\rho) \cdot m_a \quad (3)$$

where  $G$  expresses the value of the geographical phenomenon,  $\rho$  expresses the original distance, and  $m_a$  expresses the scale of the distortion.

Automation of distortion was performed in ArcGIS by ESRI with the use of the Python 2.6 programming language. The issue of distortion automation is different for thematic content (where the accurate value of the represented phenomenon is used) and for topographic content. However, the basic problem remains the same in both cases; it is necessary to resolve the issue of decomposition of geometrical structures (polygons and lines) into individual vertices, recalculate their position and then reconstruct them while preserving the input topology. In topographic elements there is one more problem: to derive the value of the geographical phenomenon.

As an example of thematic elements in the case of accessibility we can state the layer of points (municipalities) with a value of time accessibility in the network, the layer of lines (isochrones) and the layer of polygons (accessibility zones). The algorithm of distortion is based on three steps. First, the access to individual points, or vertices, is ensured and their spatial coordinates, ID and the corresponding accessibility values are saved in a Python list. Each element (point, line, polygon) is saved into one object, thus ensuring the same vertex topology. Second, spatial coordinates in the list are recalculated according to accessibility values and all is again saved in a new list, or possibly an object. Third, an empty file (*shapefile*) is created according to geometry specified by the user and the element from the new distorted coordinates is repeatedly drawn (see Fig. 2).

There is one partial problem in the algorithm - a suitable selection of the scale - because the calculation has to take into account both the scale of the distortion and the scale of the map. These two parameters were substituted by the use of a fixed point. This is a point that has the same position both in the original and the distorted map. The scale of distortion is then expressed as the ratio of accessibility of the fixed point and its aerial Euclidean distance to the centre  $S$ .

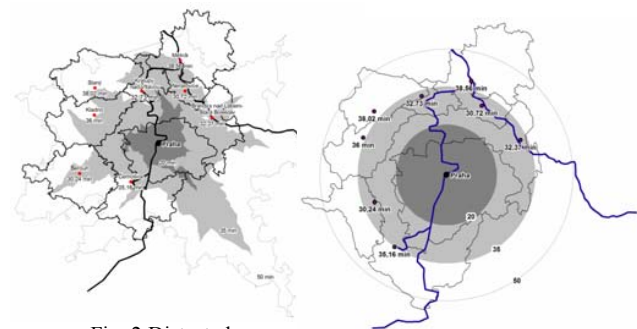


Fig. 2 Distorted accessibility in the area of Prague

Topographic elements serve as reference orientation points. Adding further geographical elements in the map makes the user's orientation easier, especially if the shape of the element is specific enough and well known. Thus, the aim is not to accurately express the value of the geographical phenomenon in the supporting elements (which are often hard to determine) but to place supporting elements in the distorted space in a meaningful manner, so that orientation in the distorted map is

easier for the user. Therefore, it is desirable for the points to have a sufficiently significant shape, conserving a sufficient degree of generalisation at the same time. The requirement of a generalised shape of input topographical elements is related to the main deficiency of unipolar anamorphosis, namely the violation of spatial relations.

Topographical input can be distorted with the use of interpolation of the values of accessibility from point thematic elements or with the use of analysis of the degree of distortion in the area surrounding the point.

The difference of these two approaches lies in the different value of the interpolated phenomenon (accessibility, degree of distortion). In the first case, there is interpolation of known values of accessibility from the layer of thematic points of the map, e.g. the layer of municipalities. The second case is based on the estimate of the degree of distortion in the area surrounding the point. The mathematical foundations of unipolar anamorphosis show that deformation of space happens on the half line *centre – analysed point*. First, the extent of displacement on the given half line is determined for the thematic points and this value is then interpolated to the remaining area.

Due to other stages of distortion, it is essential for the resulting raster of interpolated values to have the nature of cumulative totals from the centre towards the edges. Emphasis is put on global values; local minimums and maximums are eliminated. Because of wide variability of input data it is impossible to unequivocally state the most suitable interpolation method. Good results can be obtained by *Universal Kriging* (the Kriging model depends on the nature of input data), and possibly *Local Polynomial Interpolation* with emphasis on global values.

For distortion of topographic elements the original algorithm of distortion is extended with two parts - interpolation of values of the geographical phenomenon (resulting in a raster) and preprocessing of the input topographic layer. Interpolation of values has been described above. Preprocessing consists in a transformation of the input layer to a layer of points, so that a raster value can be matched to it. The transformation is performed in the following sequence: polygon to lines to points. This sequence is necessary in order to preserve the vertex topology. The next step consists in matching each vertex with an interpolated raster value. Distorted spatial coordinates are then calculated for each point and, in the end, the points are again connected into input elements. Lastly, the topologic correctness of the distorted layer is checked and topologic errors are corrected.

The above described algorithms can be used to distort thematic elements, i.e. points (municipalities), lines (isochrones) and polygons (accessibility zones). Topographic elements can also be distorted, i.e. points (e.g. touristic localities), lines (e.g. rivers) and polygons (e.g. administrative territories). These improve orientation in the distorted map. The algorithm is used to define the distorted layer, distortion centre, attributes of the value of the represented phenomenon, fixed point, raster with interpolated values in the case of distortion of topographic elements and geometry of the resulting layer.

## V. CONCLUSION

Unipolar anamorphosis is the most suitable cartographic method used to represent mutual relation of accessibility and other geographical phenomena in the territory. Transformation of topographical input, represented by distance, to time enables clear illustration and subsequent analysis of the relationships in the given territory. As an example, we can state the comparison of accessibility represented by isochrones with another, not necessarily a planar, phenomenon. By creating circles out of isochrones of irregular shape we obtain not only the above mentioned transformation but we also ensure transparency and a significant degree of illustrativeness of the given phenomenon in relation to accessibility.

Automation, or rather semi-automation, of the method of unipolar anamorphosis opens the research of accessibility to a wider cartographic and geographic audience, as well as to a wider public.

Transformation of cartographical expression of space to another selected phenomenon gives a wide range of possibilities of further research and use. The evolutionary point of view in relation to accessibility in the given territory (state) provides the possibility of immediate comparison of an improvement or deterioration in accessibility at specific places and in specific directions. This implies the possibility of use even outside research and investigation, e.g. in the field of transport planning policy.

## REFERENCES

- [1] V. Nový, *Isochronická mapa Čech – s úvodem o izochronách vubec*. Prague, CZ, Zemepisná knihovna, 1904.
- [2] K. Spiekermann, M. Wegener „Trans-European Networks and Unequal Accessibility in Europe,” *EUREG*, vol. 4, pp. 35-42, 1996.
- [3] J. Brimberg, J.H. Walker, R.F. Love „Estimation of travel distances with weighted  $l_p$  norm: Some empirical results,” *Journal of Transport Geography*, vol. 1, pp. 62-72., 2007.
- [4] M. Marada, T. Hudeček „Accessibility Of Peripheral Regions: A Case of Czechia”, *Europa XXI*, vol. 15., pp. 43-51, 2006.
- [5] T. Hudeček, *Dostupnost v Česku v období 1991-2001*. Prague, CZ, Charles University, 2010.
- [6] N. Spence, B. Linneker „Evolution of the motorway network and changing levels of accessibility in Great Britain. *Journal of Transport Geography*”, vol. 2, pp. 247-264, 1994.
- [7] S. Moryadas, J. Lowe, *The Geography of Movement*. Boston, US, Houghton Mifflin Company, 1975.
- [8] J.S. Brainard, A.A. Lovett, I.J. Batemann „Using isochrone surfaces in travelcost models,” *Journal of Transport Geography*, vol. 2, pp. 117-126, 1997.
- [9] S. Hanson, *The Geography of Urban Transportation – second edition*. London, UK, The Guilford Press, 1995.
- [10] I.B. Thomson „High-speed transport hubs and Eurocity status: the case of Lyon,” *Journal of Transport Geography*, vol. 1., pp. 29-37, 1995.
- [11] J. Gutierrez, P. Urbano „Accessibility in the European Union: the impact of the trans-European road network.” *Journal of Transport Geography*, vol. 1, pp. 15-25, 1996.
- [12] M. Taylor, P. Bonsall, W. Young, *Understanding Traffic Systems: Data, Analysis and Presentation – second edition*. US, Ashgate Publishing Company, 2000.
- [13] D. Janelle „Metropolitan Expansion, Telecommuting and Transportation,” In: Hanson, S. (ed.): *The Geography of Urban Transportation – second edition*. pp. 407-434, 1995.
- [14] D. Adler, D. Janelle, A. Philbrick, J. Sommer, *Human Geography in a shrinking world*. Mass., US, Duxbury press, 1975.
- [15] N. Ahmed, H.J. Miller „Time-space transformations of geographic space for exploring, analyzing and visualizing transportation systems. *Journal of Transport Geography*, vol. 1. pp. 2-17, 2007.

- [16] Z. Murdych, *Metody anamorfozy mapy pro geograficke ucely*. Prague, CZ, Charles University, 1993.
- [17] W. Tobler „Thirty Five Years of Computer Cartograms.” *Annals of the Association of American Geographers*, vol. 1, pp. 58-73. 2004.
- [18] W. Bunge, *Theoretical Geography*. First Edition. Lund, Sweden, C.W.K. Gleerup, 1962.
- [19] J. Olson „Noncontiguous area cartograms.” *The Professional Geographer*, vol. 4, pp. 371 – 380, 1976.
- [20] W. Tobler, *Map transformation of geographic space*. US, Washington, 1961.