International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:3, No:3, 2009

ILMI Approach for Robust Output Feedback Control of Induction Machine

Abdelwahed Echchatbi, Adil Rizki, Ali Haddi, Nabil Mrani, Noureddine Elalami

***Abstract*—**In this note, the robust static output feedback stabilisation of an induction machine is addressed. The machine is described by a non homogenous bilinear model with structural uncertainties, and the feedback gain is computed via an iterative LMI (ILMI) algorithm.

***Keywords*—**Induction machine, Static output feedback, robust stabilisation.

I. Introduction

INDUCTION machine has received particular interest through many researches in industry application systems, since it's intensively used in applications requiring high dynamic performances.

Controllers for such machines must have some characteristics as limiting both currents and flux in their respective nominal ranges, while driving the motor torque along a giving profile.

On other hand, the controller synthesis for drives using an induction machine is a rather difficult problem that must deal with nonlinear dynamics, multivariable inner structure of the system and no availability of flux sensors. Moreover, the most applications must be sensorless to insure required safety, high level availability of devices and low costs implementation, which increase problems difficulty. So, sensorless control for both synchronous and asynchronous machine was the one of the most attractive problems trough the last two decades.

In the last years, the most effective approaches to this problem were based on linearising and decoupling between stator parameters leading to the so called field oriented control (FOC). In such control problem, the knowledge of stator resistance and self induction coefficient is necessary. Those parameters are not constants due to the large heat domain of work and variable saturation level of the machine.

A.Echchatbi with the Centre d'excellence en Ingnierie et Management industriel, CIGMA, Faculté des Sciences et Techniques Settat, Morocco. (corresponding author to provide phone: +212-523-400-736; fax: +212-523-400-969; e-mail: echchatbi@gmail.com).

A.Rizki is with Faculté des Sciences et Techniques Settat, Morocco. (e-mail : arizki73@yahoo.fr).

A.Haddi, is with Laboratoire des Technologies Innovantes, Ecole Nationale des Sciences Appliquées Tanger Morocco.(e-mail: alihaddi00@yahoo.fr).

N. Elalami is with Ecole Mohamadia d'Ingénieurs, Rabat, Morocco

N. Mrani is with Faculté des Sciences et Techniques Errachidia, Morocco, (e-mail: nmrani@gmail.com).

In this paper, the robust static output feedback stabilisation of an induction machine described by a non homogenous bilinear model is focus on. The static output feedback problem is one of the most important open questions in control engineering [10]. Many analytical and numerical methods were proposed in Schumacher 1980 [8]; Barmish 1985[2]; Bernstein 1992[1] Iwasaki and Skelton 1995[5], Kucera and de Souza 1995[6], Oliveira and Geromel 1997[7]; Syrmos et al. 1997 [10] and the references therein.

In Cao et al. 1998 [3], an iterative linear matrix inequality (ILMI) approach is proposed to design a Static Output Feedback Stabilisation of a linear time invariant system. Zheng et al. 2002 [11] study the design of multivariable PID controllers via LMI approach that Lin et al. 2004 improve by transforming the problem to that of SOF controller design for a system in descriptor form.

II. Induction Machine Model

In this section, the classical structure of induction machine is considered. Such machines are driven by variable frequency voltage to provide suitable torque which is a combination of flux and current components.

Then, the considered equations are a two axes representation of induction machine under appropriates simplifying hypothesis. It's a set of a non linear differential equations with current and flux components as variables. The state vector is build with measured variables (currents) and estimated ones (flux).

$$x = \left[i_{ds}, i_{qs}, \varphi_{ds}, \varphi_{qs}\right]^T$$

Where:

i_{ds} and i_{qs} are projections of the components current stator on a (d,q) axes.

φ_{ds} and φ_{qs} are projections of the components flux stator on a (d,q) axes.

and the control $u = \left[v_{ds}, v_{qs}, \omega_s\right]^T$ applied by the inverter to achieve a variable speed control.

The dynamic equations describing the motor behaviour in the general form can be written as :

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= Dx \end{aligned} \tag{1}$$

With

$$f(x) = Ax = \begin{bmatrix} -\frac{R_s L_r + R_r L_s}{\sigma L_s L_r} & 0 & \frac{R_r}{\sigma L_r L_s} & \frac{\omega}{\sigma L_s} \\ 0 & -\frac{R_s L_r + R_r R_s}{\sigma L_s L_r} & -\frac{\omega}{\sigma L_s} & \frac{R_r}{\sigma L_r L_s} \\ -R_s & 0 & 0 & \omega \\ 0 & -R_s & -\omega & 0 \end{bmatrix} x$$

and

$$g(x) = \begin{bmatrix} \frac{\sigma}{R_s} & 0 & x_2 \\ 0 & \frac{1}{\sigma L_s} & -x_1 \\ 1 & 0 & x_4 \\ 0 & 1 & -x_3 \end{bmatrix}$$

where $\sigma = 1 - \left(M^2 / L_s L_r\right)$

and R_s, R_r, L_s, L_r and M stand for nominal machine electromagnetic parameters.

Such system can be written in the homogenous bilinear form:

$$\dot{x} = Ax + \sum_{i=1}^{3} u_i B_i x + Cu$$

With $B_1 = B_2 = 0$; $B_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$

and

$$C = \begin{bmatrix} \frac{\sigma}{R_s} & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} ; D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_i u_i(t) B_i x(t) + Cu(t) \\ y(t) = Dx(t) \\ u(t) = Ky(t) \end{cases} \tag{2}$$

In the following section, the uncertain system (2) is considered with $A=A_0 + \Delta A$ and $C=C_0 + \Delta C$, where A_0 and B_0 are nominal matrices build by nominal parameters.

ΔA and ΔC are bounded uncertainties with a known maximum values. Those uncertainties arise from variations in limited ranges of stator resistance and/or inductance. Thus the following inequalities hold:

$$\|\Delta A\| < a \quad \text{and} \quad \|\Delta C\| < c$$

The maximum values a and c can be easily determined from classical identifying tests.

Therefore the closed loop system is:

$$\begin{cases} \dot{x} = (A_0 + C_0 KD)x + \sum_i K_i D x B_i x + (\Delta A + \Delta C \cdot K \cdot D)x \\ y = Dx \end{cases} \tag{3}$$

III. Robust Output Feedback Stabilisation

Consider a linear time-invariant system described by:

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx \end{aligned} \tag{4}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^r$ is the control vector and $y \in \Re^m$ is the output vector.

Definition: system (4) is said to be stabilizable via static output feedback if there exists K such that the closed loop systems:

$$\dot{x} = (A + BKC)x$$

is stable. And it's said to be *α-stabilizable via static output feedback* if K places the closed-loop poles to the left of a vertical line Re(s) = - α (for some real α) in the complex plane.

Theorem:
Under the next hypothesis:

i. The system (1) linear part is α-stabilizable via static output feedback for some known real number α.
ii. The state initial conditions are in some ball of radius R.

the output feedback u(t) =Ky(t) exponentially stabilise the system (2).

Proof:
First, we will focus on the linear part of the system.
The nominal closed-loop matrix is defined as: $\tilde{A} = A_0 + C_0 KD$

Where a static gain feedback K is chosen so that allows the matrix $\tilde{A}$ to have all its eigenvalues with negative real part. Then the matrix $\tilde{A}$ is called stable. And the linear part of the system is output feedback stabilizable.

Cao et al [3] give a theorem on the necessary and sufficient condition for static output feedback stabilizability of a linear system as follows:

$$A^T P + PA - PBB^T P + [B^T P + KC]^T (B^T P + KC) < 0 \quad (5)$$

Which is a quadratic matrix inequality (QMI) derived from the Lyapunov's stability theorem.

To compute the gain K, one can apply the iterative LMI algorithm developed by Cao et al. [3].

In the following, the robust output feedback stabilisation of the considered system is investigated.

The solution of system (2) is:

$$x(t) = e^{\tilde{A}t} x_0(t) + \int_0^t e^{\tilde{A}(t-s)} (\Delta A + \Delta CKD) x(s) ds$$
$$+ \int_0^t e^{\tilde{A}(t-s)} \sum_i K_i D x(s) B_i x(s) ds$$

Hence :

$$\|x(t)\| = \left\|e^{\tilde{A}t}\right\| \|x_0(t)\| + \int_0^t \left\|e^{\tilde{A}(t-s)}\right\| (\|\Delta A\| + \|\Delta C\|\|KD\|) \|x(s)\| ds$$
$$+ \int_0^t \left\|e^{\tilde{A}(t-s)}\right\| \left\|\sum_i K_i B_i\right\| \|D\| \|x(s)\|^2 ds$$

From the Hill-Yosida theorem, There exist M>0 and ω<0 that verify:

$$\left\|e^{\tilde{A}t}\right\| \le M e^{\omega t}$$

$$\frac{\|x(t)\| e^{-\omega t}}{\|x_0(t)\|} \le M + \int_0^t e^{-\omega s} (a + c\|K\|\|D\|) \frac{\|x(s)\|}{\|x_0(t)\|} ds$$
$$+ \int_0^t M \left\|\sum_i K_i B_i\right\| \|x_0\| e^{\omega s} \frac{\|x(s)\|^2}{\|x_0\|^2} ds$$

We denote $m(t) = \dfrac{\|x(t)\| e^{-\omega t}}{\|x_0(t)\|}$ then:

$$m(t) \le M + \int_0^t M + (a + c\|K\|\|D\|) m(s) ds$$
$$+ \int_0^t M \left\|\sum_i K_i B_i\right\| \|x_0\| e^{\omega s} m^{2(s)} ds$$

This can be written in the compact form as:

$$m(t) \le M + \int_0^t f_1(s) m(s) ds + \int_0^t f_2(s) m^2(s) ds$$

with : $f_1(s) = M(a + c\|K\|\|D\|) = M\beta$ and

$$f_2 = M \left\|\sum_i K_i B_i\right\| \|x_0\| e^{\omega s}$$

The application of the Gronwall-Bellman lemma in its general form [4] leads to the second hypothesis, which arises from the following ones (see the theorem in the Appendix). The same result can be obtained from the classical form of Gronwall-Bellman lemma.

We define R as:

$$R = -\frac{\omega + M\beta}{M^2 \left\|\sum_i K_i B_i D\right\|}$$

the second hypothesis of the theorem is :

$$(H) \Rightarrow 1 - \int_{\grave{a}}^{\infty} M g_2(t) dt > 0$$

$$g_2(t) = f_2(t) \exp\left(\int_0^t f_1(s) ds\right)$$
$$= M \left\|\sum_i K_i B_i\right\| \|x_0\| e^{\omega t} \cdot \left(e^{M(a + c \cdot \|K\|\|D\|)t}\right)$$

$$(H) \Rightarrow 1 - M^2 \left\|\sum_i K_i B_i\right\| \cdot \|x_0\| \cdot \int_0^{\infty} e^{(\omega + M\beta)t} dt$$
$$= 1 - M^2 \left\|\sum_i K_i B_i\right\| \|x_0\| \left[\frac{e^{(\omega + \beta)t}}{\omega + \beta}\right]_0^{\infty}$$

The hypothesis H is written as:

$$1 - M^2 \left\|\sum_i K_i B_i\right\| \|x_0\| \left[-\frac{1}{\omega + M\beta}\right] > 0$$

$$\Rightarrow M^2 \left\|\sum_i K_i B_i\right\| \|x_0\| \left(\frac{1}{\omega + M\beta}\right) > -1$$

Hence the initial conditions must satisfy:

$$\|x_0\| \le \frac{-(\omega + \beta)}{M^2 \left\|\sum_i K_i B_i\right\|}$$

It follows:

$$m(t) \le \frac{Me^{M\beta t}}{1 + \frac{M^2 \left\| \sum_i K_i B_i \right\| \left\| x_0 \right\|}{\omega + M\beta}}$$

Hence :

$$\left\| x(t) \right\| \le \frac{Me^{(\omega + M\beta)t}}{1 + \frac{M^2 \left\| \sum_i K_i B_i \right\| \left\| x_0 \right\|}{\omega + M\beta}}$$

Then the system is exponentially stable. ■

IV. Iterative LMI

It can be noted that the hypothesis of α-stabilizability of the linear part system arises from the condition that the uncertainties and the closed-loop system must verify:
$\omega + M\beta < 0$.
Where ω stand for the maximum eigenvalues of the closed-loop matrix (A-CKD).

Corollary.
The linear system (2) is α/2-stabilizable via static output feedback if and only if there exist two matrices P > 0 and K satisfying the following matrix inequality:

$$A^T P + PA - PBB^T P + [B^T P + KC]^T (B^T P + KC) - 2\alpha P < 0$$

Cao et al. propose an iterative algorithm to solve the above QMI as an ILMI. In the next, we use the same algorithm where we verify the α-stabilizability according to the hypothesis (ii) of our theorem.

Then the algorithm can be written as :

Step1. Select Q > 0, and solve P from the following algebraic Riccati equation

$$A^T P + PA - PCC^T P + Q = 0$$

Set i = 1 and X_1 = P.

Step2. Solve the following optimization problem for P_i, K and α_i.

OP1 : Minimize α_i subject to the following LMI constraints

$$\begin{bmatrix} A^T P_i + P_i A - X_i CC^T P_i - P_i CC^T X_i + X_i CC^T X_i - \alpha_i P_i & \left(C^T P_i + KD\right)^T \\ \left(C^T P_i + KD\right) & -I \end{bmatrix} < 0 \quad (6)$$

$$P_i = P_i^T > 0 \quad (7)$$

Denote α_i^* as the minimized value of α_i.

Step3. If $\alpha_i^* < M\beta$, K is a stabilizing static output feedback gain. Stop.

Step4. Solve the following optimisation problem for P_i and K.

OP2 : Minimize trace(Pi) subject to the above LMI constraints (6) and (7) with $\alpha_i = \alpha_i^*$. Denote P_i^* as the α_i that minimized trace(P_i).

Step5. If $\left\| X_i - P_i^* \right\| < \delta$, a prescribed tolerance, go to Step6.

Step6. The system may not be stabilizable via static output feedback. Stop.

V. Conclusion

In this paper, the problem of the robust static output feedback stabilization of an induction machine is studied. We improve the results available on the SOF for the linear systems to the case of non homogenous bilinear model of the induction machine by using a generalization of the Bellman-Gronwall lemma. Then we proposed to follow the same iterative LMI algorithm developed in Cao et al. to compute the gain feedback.

Appendix

A generalization of Bellman-Gronwall lemma:

If a, b, n, k∈ℜ, where a<b, n>1 and k>0
And

$$f_i : [a,b] \to \Re^+, \quad i = 1..n$$

a set of integrable functions that verify :

$$\forall \alpha, \beta \in [a,b], \alpha < \beta :$$

$$g_i(t) = f_i(t) \exp \int_a^t (n-1) f_1(s) ds$$

$$\int_\alpha^\beta g_i(t) dt > 0$$

$x : [a,b] \to \Re^+$ a bounded function that satisfies $\forall t \in [a,b]$:

$$x(t) \le k + \int_a^t \sum_{i=1}^n f_i(s) (x(s))^i ds$$

Then under the following hypothesis:

$$1 - (n-1) \int_a^b \sum_{i=2}^n k^{i-1} g_i(t) dt > 0$$

for $t \in [a,b]$:

$$x(t) \le \frac{k \exp\left(\int_a^t f_1(s) ds \right)}{\left[1 - (n-1) \int_a^b \sum_{i=2}^n k^{i-1} g_i(t) dt \right]^{\frac{1}{n-1}}}$$

REFERENCES

[1] D. Bernstein, “Some open problems in matrix theory arising in linear systems and control,” *Linear Algebra Appl.*, Vol. 162 – 164, pp. 409 – 432.
[2] B. R. Barmish, “Necessary and Sufficient conditions for quadratic stabilizability of an uncertain system,” *J. Optimization Theory Appl.*, vol. 46, pp. 339 – 408.
[3] Y. Cao, J. Lam, Y. Sun, “Static Output Feedback Stabilization: An ILMI Approach,” *Automatica*, Vol. 34, No. 12, pp. 1641 – 1645, 1998.
[4] N. Elalami, A. Echchatbi, A. Bouaziz, "Observation et Stabilisation Robuste des Systèmes Bilinéaires incertains," *Conférence Internationale Francophone d’Automatique*, Nantes 2002.
[5] T. Iwasaki, and R. E. Skelton, “Parametrization of all stabilizing controllers via quadratic Lyapunov functions,” *J. Optimization Theory Appl.* No. 85, pp 291-307.
[6] V. Kucera and C. de Souza, “A necessary and sufficient condition for output feedback stabilizability,” *Automatica*, Vol. 31, N° 9, pp. 1357 – 1359, 1995.
[7] C. Lin, Q. Wang, T. H. Lee, “An improvement on multivariable PID controller design via iterative LMI Approach,” *Automatica*, Vol. 40, pp. 519 – 525, 2004.
[8] M. C. Oliveira and J.C. Geromel, “Numerical comparison of output feedback design methods,” *Proc. of America Control Conf.* Albuquerque, U.S.A. 1997.
[9] J. Schumacher, “Compensator synthesis using (C, A, B) pairs,” *IEEE Trans. Automat. Control.*, vol. 25, pp. 1133 – 1138.
[10] V.L. Syrmos, C. Abdallah, P. Dorato and K. Grigoriadis, “Static output feedback: a survey,” *Automatica*, Vol. 33, N° 2, pp. 125-137.
[11] F. Zheng, Q.-G., Wang and T.H. Lee, “On the design of multivariable PID controllers via LMI approach,” *Automatica*, Vol. 38, pp. 517-526, 2002.

N. Elalami was born in Rabat 1962, he obtained an Ing degree in Electrical Engineering at the Ecole Mohamadia d’Ingénieurs in Rabat 1983, and a Phd Thesis at the University of Perpignon France 1986.
He is o Professor of Control Engineering at the Ecole Mohamadia d’Ingénieurs in Rabat.
A. Echchatbi was born in Casablanca Morocco 1973, he obtained a Master degree in Electrical Engineering at the Hassan II University in 1998, and a National Doctorate at the Ecole Mohamadia d’Ingénieurs in Rabat 2003.
He was assistant professor of Electrical Engineering in the High School Of Technology Casablanca until 2005, and then he is a professor at the Faculté des Sciences et Techniques University Hassan 1er Settat.
Prof. Abdelwahed Echchatbi is a co-founder of “Centre d’excellence en Ingénierie et Management Industriel” CIGMA Settat.
A. Haddi was born in Rabat 1970, he obtained an qualified teacher degree in Electrical Engineering at the Normal Hign School of Technical Teaching, Rabat, 1996, and a National Doctorate at the Ecole Mohamadia d’Ingénieurs in Rabat 2005.
He was with the Electrical Engineering Department of the Normal Hign School of Technical Teaching, Rabat, and now he is o Professor of Control Engineering at the Ecole Nationale des Sciences Appliquées Tanger, Morocco.
N. Mrani was born in Rabat 1973, he obtained a Master degree in Electrical Engineering at the Faculté des Sciences University ElJadida Morocco, and a National Doctorate at the Ecole Mohamadia d’Ingénieurs in Rabat 2004.
He is a professor at the Faculté pluridisciplinaire d’Errachidia, University Mohammed Bnou Abdillah Morocco.
A. Rizki was born in Settat Morocco 1973, he obtained a Master degree in Computer Engineering at the Hassan 1er University in 2002.
He is assistant professor of Electrical Engineering in the Faculté des Sciences et Techniques University Hassan 1er Settat.