

Order Reduction of Linear Dynamic Systems using Stability Equation Method and GA

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Abstract—The authors present an algorithm for order reduction of linear dynamic systems using the combined advantages of stability equation method and the error minimization by Genetic algorithm. The denominator of the reduced order model is obtained by the stability equation method and the numerator terms of the lower order transfer function are determined by minimizing the integral square error between the transient responses of original and reduced order models using Genetic algorithm. The reduction procedure is simple and computer oriented. It is shown that the algorithm has several advantages, e.g. the reduced order models retain the steady-state value and stability of the original system. The proposed algorithm has also been extended for the order reduction of linear multivariable systems. Two numerical examples are solved to illustrate the superiority of the algorithm over some existing ones including one example of multivariable system.

Keywords—Genetic algorithm, Integral square error, Order reduction, Stability equation method.

I. INTRODUCTION

THE approximation of high order systems by low order models is one of the important problems in system theory. The use of a reduced order model makes it easier to implement analysis, simulations and control system designs. Numerous methods are available in the literature for order-reduction of linear continuous systems in time domain as well as in frequency domain [1]-[7]. Further, the extension of single-input single-output (SISO) methods to reduce multi-input multi-output (MIMO) systems has also been carried out in [8]-[11]. Each of these methods has both advantages and disadvantages when tried on a particular system. In spite of several methods available, no approach always gives the best results for all systems.

The stability equation method [12] is one of the most popular techniques among the various model order reduction methods available in the literature. This method preserves stability in the reduced model, if the original high-order system is stable, and retains the first two time-moments of the

system, thus ensuring steady-state response matching for impulse, step and ramp inputs between the original high-order system and the reduced order model. Some interesting variants on the basic technique have also appeared in the literature [13]-[14], which shows that the stability equation approach may be applied to non-minimum phase [13] and oscillatory systems [14]. This method was also combined with Pade approximation method by Chen et al. [15] and in order to overcome the drawback of approximating the non-dominant poles of the original system, an improved method for Pade approximants using the stability equation method was proposed by Pal [16]. In [17], Parthasarathy and Jayasimha combined this method with modified Caue continued fraction technique in order to retain the rank of the high-order system in the reduced order model.

Further, numerous methods of order reduction are also available in the literature [18]-[23], which are based on the minimization of the integral square error (ISE) criterion. However, a common feature in these methods [18]-[22] is that the values of the denominator coefficients of the low order system (LOS) are chosen arbitrarily by some stability preserving methods such as dominant pole, Routh approximation methods, etc. and then the numerator coefficients of the LOS are determined by minimization of the ISE. In [23], Howitt and Luss suggested a technique, in which both the numerator and denominator coefficients are considered to be free parameters and are chosen to minimize the ISE in impulse or step responses.

Nowadays, Genetic algorithm (GA) is becoming popular to solve the optimization problems in different fields of application mainly because of their robustness in finding an optimal solution and ability to provide a near optimal solution close to a global minimum. Unlike strict mathematical methods, the GA does not require the condition that the variables in the optimization problem be continuous and different; it only requires that the problem to be solved can be computed. GA employs search procedures based on the mechanics of natural selection and survival of the fittest. The GAs, which use a multiple point instead of a single point search and work with the coded structure of variables instead of the actual variables, require only the objective function thereby making searching for a global optimum simpler [24]-[25].

The present attempt is towards evolving a new algorithm for order reduction, which combines the advantages of the stability equation method and the error minimization by GA.

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The proposed algorithm consists of pole synthesis of the low-order system (LOS) by stability equation method while the zeros are determined by minimizing the integral square error between the transient responses of original and LOS using GA. The algorithm has also been extended for the order reduction of linear multivariable systems. In the following sections, the algorithm is described in detail with the help of two numerical examples.

II. DESCRIPTION OF THE ALGORITHM

Let the transfer function of the original high-order system (HOS) of order 'n' be :

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_o + a_1 s + a_2 s^2 + \dots + a_{n-1} s^{n-1}}{b_o + b_1 s + b_2 s^2 + \dots + b_{n-1} s^{n-1} + s^n} \quad (1)$$

and let the same of low-order system (LOS) of order 'r' to be synthesized is :

$$G_r(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)} = \frac{\alpha_o + \alpha_1 s + \dots + \alpha_{r-1} s^{r-1}}{d_o + d_1 s + d_2 s^2 + \dots + d_{r-1} s^{r-1} + s^r} \quad (2)$$

Further, the method consists of following steps :

Step-1: Determination of the denominator coefficients of LOS:

For stable original system $G_n(s)$, the denominator $D(s)$ of the HOS is bifurcated in the even and odd parts in the form of stability equations as [13] :

$$\left. \begin{aligned} D_e(s) &= \sum_{i=0,2,4}^n b_i s^i = b_o \prod_{i=1}^{m_1} \left(1 + \frac{s^2}{z_i^2} \right) \\ D_o(s) &= \sum_{i=1,3,5}^n b_i s^i = b_1 s \prod_{i=1}^{m_2} \left(1 + \frac{s^2}{p_i^2} \right) \end{aligned} \right\} \quad (3)$$

where m_1 and m_2 are the integer parts of $n/2$ and $(n-1)/2$, respectively and $z_1^2 < p_1^2 < z_2^2 < p_2^2 \dots$

Now by discarding the factors with large magnitudes of z_i^2 and p_i^2 in (3), the stability equations for r^{th} order LOS are obtained as :

$$\left. \begin{aligned} D_e^r(s) &= b_o \prod_{i=1}^{m_3} \left(1 + \frac{s^2}{z_i^2} \right) \\ D_o^r(s) &= b_1 s \prod_{i=1}^{m_4} \left(1 + \frac{s^2}{p_i^2} \right) \end{aligned} \right\} \quad (4)$$

where m_3 and m_4 are the integer parts of $r/2$ and $(r-1)/2$, respectively.

Combining these reduced stability equations and therefore proper normalizing it, the r^{th} order denominator $\tilde{D}(s)$ of LOS is obtained as :

$$\tilde{D}(s) = D_e^r(s) + D_o^r(s) = \sum_{i=0}^{r-1} d_i s^i + s^r \quad (5)$$

Therefore, the denominator polynomial in (2) is now known, which is given by

$$\tilde{D}(s) = d_o + d_1 s + d_2 s^2 + \dots + d_{r-1} s^{r-1} + s^r \quad (6)$$

Step-2: Determination of the numerator coefficients of the LOS by Genetic Algorithm :

In the present study, GA is employed to minimize the objective function 'J', which is the integral square error in between the transient responses of HOS and LOS and is given by :

$$J = \int_0^\infty [y(t) - y_r(t)]^2 dt \quad (7)$$

where, $y(t)$ and $y_r(t)$ are the unit step responses of original ($G_n(s)$) and reduced ($G_r(s)$) order systems, and the parameters to be determined are the numerator coefficients of LOS α_i ($i = 0, 1, \dots, (r-1)$).

For different problems, it is possible that the same parameters for GA do not give the best solution and so these can be changed according to the situation. In Table I, the typical parameters for GA optimization routines, used in the present study are given. One more important point that affects the optimal solution more or less is the range for unknowns. For the very first execution of the program, wider solution space can be given and after getting the solution one can shorten the solution space nearer to the values obtained in the previous iteration. The normalized geometric ranking, which is one of the ranking methods, is used as selection function to select individuals in the population for the next generations. Also arithmetic crossover as the crossover function and non-uniform mutation as mutation operator is adopted. The computational flowchart of the GA optimization is shown in Fig. 1.

TABLE I
TYPICAL PARAMETERS USED BY THE GENETIC ALGORITHM

Name	Value (type)
Number of Generations	200
Population Size	50
Type of Selection	Normal Geometric [0.08]
Type of Crossover	Arithmetic [2]
Type of Mutation	Nonuniform [2 200 3]
Termination method	Maximum Generation

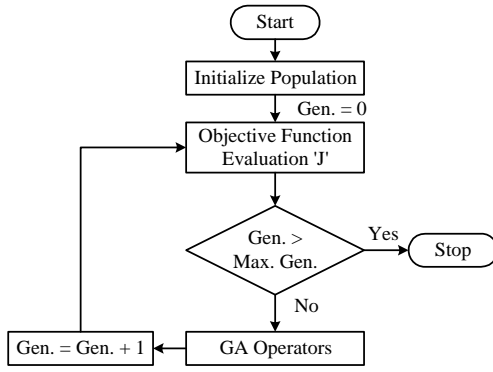


Fig. 1. Flowchart of Genetic Algorithm.

III. EXTENSION TO MULTIVARIABLE SYSTEMS

Let, the transfer function matrix of the HOS of order ' n ' having ' p ' inputs and ' m ' outputs be :

$$[G(s)] = \frac{1}{D(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) & a_{13}(s) & \dots & a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & a_{23}(s) & \dots & a_{2p}(s) \\ \vdots & \vdots & \vdots & \dots & \vdots \\ a_{m1}(s) & a_{m2}(s) & a_{m3}(s) & \dots & a_{mp}(s) \end{bmatrix} \quad (8)$$

$$\text{or, } [G(s)] = [g_{ij}(s)], \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, p \quad (9)$$

is a $m \times p$ transfer matrix.

The general form of $g_{ij}(s)$ of $[G(s)]$ in (9) is taken as :

$$g_{ij}(s) = \frac{a_{ij}(s)}{D(s)} = \frac{a_o + a_1 s + a_2 s^2 + \dots + a_{n-1} s^{n-1}}{b_o + b_1 s + b_2 s^2 + \dots + b_{n-1} s^{n-1} + s^n} \quad (10)$$

Let, the transfer function matrix of the LOS of order ' r ' having ' p ' inputs and ' m ' outputs to be synthesized is :

$$[R(s)] = \frac{1}{\tilde{D}(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) & b_{13}(s) & \dots & b_{1p}(s) \\ b_{21}(s) & b_{22}(s) & b_{23}(s) & \dots & b_{2p}(s) \\ \vdots & \vdots & \vdots & \dots & \vdots \\ b_{m1}(s) & b_{m2}(s) & b_{m3}(s) & \dots & b_{mp}(s) \end{bmatrix} \quad (11)$$

$$\text{or, } [R(s)] = [r_{ij}(s)], \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, p \quad (12)$$

is a $m \times p$ transfer matrix.

The general form of $r_{ij}(s)$ of $[R(s)]$ in (12) is taken as :

$$r_{ij}(s) = \frac{b_{ij}(s)}{\tilde{D}(s)} = \frac{\alpha_0 + \alpha_1 s + \dots + \alpha_{r-1} s^{r-1}}{d_o + d_1 s + d_2 s^2 + \dots + d_{r-1} s^{r-1} + s^r} \quad (13)$$

The proposed algorithm consists of pole synthesis of the low order system (LOS) by stability equation method while the zeros are determined by minimizing the integral square error 'J' as given in (7), between the transient responses of original and LOS using GA. Basically; the method starts with fixation of the denominator of the LOS by the stability equation method followed by the determination of coefficients of the numerator polynomials of each element ($r_{ij}(s)$) of the LOS transfer matrix $[R(s)]$ by minimizing the error 'J', between the transient responses of original ($g_{ij}(s)$) and reduced ($r_{ij}(s)$) order models using Genetic algorithm.

IV. NUMERICAL EXAMPLES

Two numerical examples are chosen from the literature for the comparison of the low order system (LOS) with the original high order system (HOS). The proposed algorithm is described in detail for one example while only the result of the other example is given.

An error index ISE [5] known as integral square error in between the transient parts of original and reduced order systems is calculated to measure the goodness of the LOS (i.e. the smaller the ISE, the closer is $G_r(s)$ to $G_n(s)$), which is given by :

$$\text{ISE} = \int_0^\infty [y(t) - y_r(t)]^2 dt \quad (14)$$

where, $y(t)$ and $y_r(t)$ are the unit step responses of original ($G_n(s)$) and reduced ($G_r(s)$) order systems.

Example-1. Consider a fourth-order system [19], [22] described by the transfer function as :

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \quad (15)$$

If a second-order model $G_2(s)$ is desired, then the steps to be followed are as under:

Step-1: Bifurcating the denominator of the above HOS in even and odd parts, we get the stability equations as :

$$\begin{aligned} D_e(s) &= 24 + 35s^2 + s^4 \\ &= 24 \left(1 + \frac{s^2}{0.6997} \right) \left(1 + \frac{s^2}{34.3004} \right) \end{aligned}$$

$$\begin{aligned} D_o(s) &= 50s + 10s^3 \\ &= 50s \left(1 + \frac{s^2}{5} \right) \end{aligned}$$

Now by discarding the factors with large magnitudes of z_i^2 and p_i^2 in $D_e(s)$ and $D_o(s)$ respectively, the stability

equations for the second-order reduced model are given by:

$$D_e^2(s) = 24 \left(1 + \frac{s^2}{0.6997} \right) \text{ and } D_o^2(s) = 50s.$$

Combining these reduced stability equations and thereafter normalizing it, the denominator of the second-order reduced model is given by :

$$\tilde{D}(s) = s^2 + 1.45771s + 0.6997 \quad (16)$$

Step-2: By using the GA to minimize the objective function 'J', as described earlier, we have:

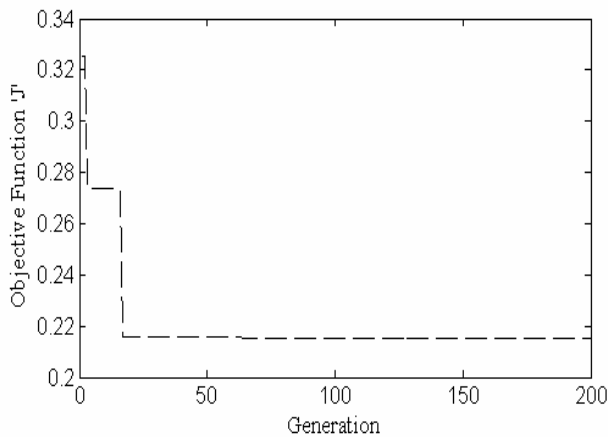
$$\tilde{N}(s) = 0.7442575s + 0.6991576$$

Therefore, finally $G_2(s)$ is given as :

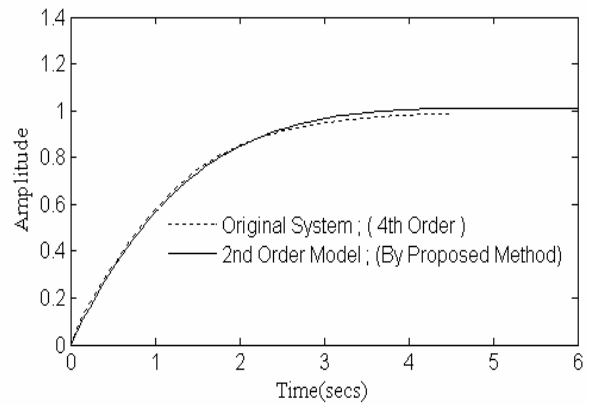
$$G_2(s) = \frac{0.7442575s + 0.6991576}{s^2 + 1.45771s + 0.6997} \quad (17)$$

with an I.S.E. of 1.644548×10^{-3} .

Fig. 2(a)-(b) presents diagrams of convergence of the objective function 'J', step responses of original ($G_4(s)$) and reduced order ($G_2(s)$) models, respectively. A comparison of the proposed algorithm with the other existing methods for a second-order reduced model is given in Table II. It can also be seen in Table II that, the proposed algorithm gives low value of ISE in comparison to the previously developed stability equation methods [13], [15]-[17] and [31].



(a)



(b)

Fig. 2 (a) Convergence of objective function 'J'. (b) Step responses of $G_4(s)$ and $G_2(s)$.

TABLE II
COMPARISON OF REDUCED ORDER MODELS

Method of order reduction	Reduced Models	ISE
Proposed Algorithm	$\frac{0.7442575s + 0.6991576}{s^2 + 1.45771s + 0.6997}$	1.644548×10^{-3}
Chen <i>et al.</i> [13]	$\frac{0.6997(s+1)}{s^2 + 1.45771s + 0.6997}$	2.665530×10^{-3}
Chen <i>et al.</i> [15]	$\frac{0.6997(s+1)}{s^2 + 1.45771s + 0.6997}$	2.665530×10^{-3}
Pal [16]	$\frac{s + 34.2465}{s^2 + 239.8082s + 34.2465}$	1.534272
Parthasarathy and Jayasimha [17]	$\frac{s + 0.6997}{s^2 + 1.45771s + 0.6997}$	34.014055×10^{-3}
Davison [26]	$\frac{-s^2 + 2}{s^2 + 3s + 2}$	$220.237945 \times 10^{-3}$
Shieh and Wei [27]	$\frac{s + 2.3014}{s^3 + 5.7946s + 2.3014}$	142.56072×10^{-3}
Krishnamurthy and Seshadri [28]	$\frac{20.5714s + 24}{30s^2 + 42s + 24}$	9.589125×10^{-3}
Pal [29]	$\frac{16.0008s + 24}{30s^2 + 42s + 24}$	11.688145×10^{-3}
Gutman <i>et al.</i> [30]	$\frac{2[48s + 144]}{70s^2 + 300s + 288}$	45.592871×10^{-3}
Prasad and Pal [31]	$\frac{s + 34.2465}{s^2 + 239.8082s + 34.2465}$	1.534272
Safonov and Chiang [32]	$\frac{s + 5.403}{s^2 + 8.431s + 4.513}$	4.51613×10^{-2}

Example-2. Consider a sixth-order two input two output system [33] described by the transfer function matrix :

$$[G(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix}$$

$$= \frac{1}{D(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix} \quad (18)$$

where, the common denominator $D(s)$ is given by :

$$D(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$

$$= 6000 + 13100s + 10060s^2 + 3491s^3 + 571s^4 + 41s^5 + s^6$$

and

$$a_{11}(s) = 6000 + 7700s + 3610s^2 + 762s^3 + 70s^4 + 2s^5$$

$$a_{12}(s) = 2400 + 4160s + 2182s^2 + 459s^3 + 38s^4 + s^5$$

$$a_{21}(s) = 3000 + 3700s + 1650s^2 + 331s^3 + 30s^4 + s^5$$

$$a_{22}(s) = 6000 + 9100s + 3660s^2 + 601s^3 + 42s^4 + s^5$$

The proposed algorithm is successively applied to each element of the transfer function matrix of above multivariable system and the reduced order models $r_{ij}(s)$ of the LOS $[R(s)]$ are obtained. The general form of second-order reduced transfer function matrix is taken as:

$$[R(s)] = \frac{1}{\tilde{D}(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) \\ b_{21}(s) & b_{22}(s) \end{bmatrix} \quad (19)$$

where, $\tilde{D}(s) = s^2 + 1.34952s + 0.6181$.

and

$$b_{11}(s) = 0.8503087s + 0.6171331,$$

$$b_{12}(s) = 0.4617562s + 0.2466069,$$

$$b_{21}(s) = 0.4093304s + 0.3086095,$$

$$b_{22}(s) = 0.9976611s + 0.6171125.$$

The step responses of original and reduced order models are compared in Fig. 3(a-d). Further, a comparison of the proposed algorithm with some existing order reduction techniques (for second-order reduced models), is also shown as given in Table III, by comparing the ISE 'E', in between the transient parts of original ($g_{ij}(s)$) and reduced ($r_{ij}(s)$) order models. This ISE is calculated for each element ($r_{ij}(s)$) of the transfer function matrix of the LOS $[R(s)]$, and it is given by :

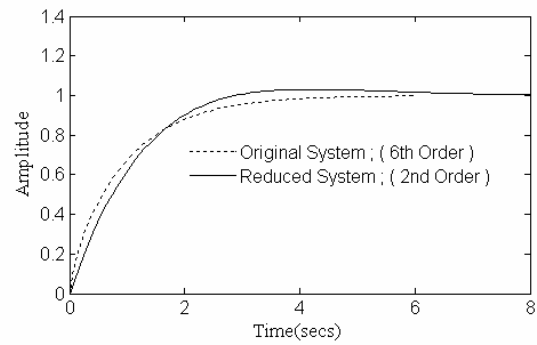
$$E = \int_0^\infty [g_{ij}(t) - r_{ij}(t)]^2 dt \quad (20)$$

where, $i = 1, 2$; $j = 1, 2$ and $g_{ij}(t)$, $r_{ij}(t)$ are the unit step responses of original and reduced order models, respectively.

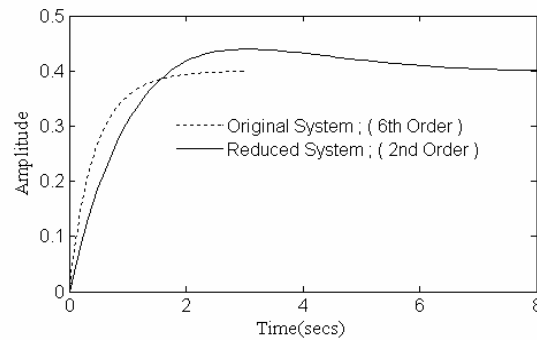
TABLE III
COMPARISON OF REDUCED MODELS

I.S.E. 'E' for r_{ij} ($i = 1, 2$; $j = 1, 2$)				
$E = \int_0^\infty [g_{ij}(t) - r_{ij}(t)]^2 dt$				
Method of Order Reduction	r_{11}	r_{12}	r_{21}	r_{22}
Proposed Algorithm	0.014498	0.008744	0.002538	0.015741
Prasad and Pal [10]	0.136484	0.002446	0.040291	0.067902
Safonov and Chiang [32]	0.590617	0.037129	0.007328	1.066123
Prasad <i>et al.</i> [34]	0.030689	0.000256	0.261963	0.021683

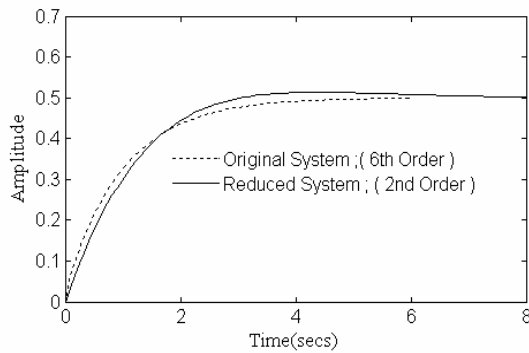
It can be seen in Table III that, the proposed algorithm gives low value of 'E' for all r_{ij} ($i = 1, 2$; $j = 1, 2$) in comparison to the other existing techniques.



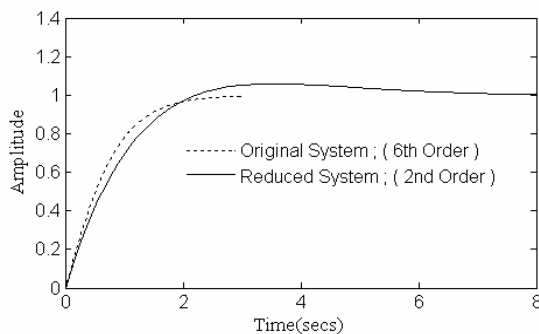
(a)



(b)



(c)



(d)

Fig. 3 (a) Comparison of step responses; $u_1 = 1, u_2 = 0$. (b) Comparison of step responses; $u_1 = 0, u_2 = 1$. (c) Comparison of step responses; $u_1 = 1, u_2 = 0$. (d) Comparison of step responses; $u_1 = 0, u_2 = 1$.

V. CONCLUSIONS

An algorithm which combines the advantages of the stability equation method and the error minimization by Genetic algorithm has been presented, to derive stable reduced order models for linear time invariant dynamic systems.

In this algorithm, the poles are determined by the stability equation method and the zeros are synthesized by minimizing the integral square error between the transient responses of original and low order systems using Genetic algorithm. The algorithm has also been extended for the order reduction of linear multivariable systems. The algorithm is simple, rugged and computer oriented. The algorithm has been implemented in Matlab 7.0 on a Pentium-IV processor and the computation time is negligible being less than 1 minute. The matching of the step response is assured reasonably well in the method. The ISE in between the transient parts of original and reduced order systems is calculated and compared in the tabular form as given in Tables II and III, from which it is clear that the proposed algorithm compares well with the other existing techniques of model order reduction. The algorithm preserves model stability and avoids any error in between the initial or

final values of the responses of original and reduced order models. To be more precise, the salient features of the proposed method can be summarized as follows:

- (i) The stability and steady state values of the original system are preserved.
- (ii) The method is computationally very attractive since only linear algebraic equations are required to be solved to arrive at reduced order models.
- (iii) The method can be easily applied to the systems where there are no dominant poles or where the dominant poles are difficult to identify (e.g. a system with poles at $-1, -1 \pm \sqrt{27}, -1.5$).

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