FAT based Adaptive Impedance Control for Unknown Environment Position

N. Z. Azlan, H. Yamaura

***Abstract*—**This paper presents the Function Approximation Technique (FAT) based adaptive impedance control for a robotic finger. The force based impedance control is developed so that the robotic finger tracks the desired force while following the reference position trajectory, under unknown environment position and uncertainties in finger parameters. The control strategy is divided into two phases, which are the free and contact phases. Force error feedback is utilized in updating the uncertain environment position during contact phase. Computer simulations results are presented to demonstrate the effectiveness of the proposed technique.

***Keywords*—**Adaptive impedance control, force based impedance control, force control, Function Approximation Technique (FAT), unknown environment position.

I. Introduction

Force control plays an important role when a robot end-effector is in contact with the environment. In force control, is it desired that the end-effector maintains a desired force while tracking a reference position trajectory. Some applications of force control include deburring, grinding, massaging and object manipulation.

Impedance control is one of the main force control methods. Impedance function is realized by regulating the relationship between force and position/ velocity error [1]. The advantage of impedance control is it provides a uniform framework for controlling the robot both in free and contact spaces [2].

In impedance control, a reference position to produce the desired contact force can be determined if the location and stiffness of the environment are known exactly [2]. However, in some practical cases, the environment parameters are not known precisely.

Huang and Chien [3] proposed FAT based adaptive impedance controller for a flexible-joint electrically driven robots. The method is able to estimate time-varying uncertainties present in the system dynamics.

Therefore, this study proposes FAT based impedance control for controlling a robotic finger while operating under imprecise knowledge of the environment position, including the shape of the object that is in contact with the finger. The force error feedback is utilized in updating the uncertain environment position in the adaptive control law. The method in [4] is applied to cater for the uncertainties in the plant parameters.

This paper is organized as follows; review on the finger mechanism and its dynamic modeling are presented in Section II. The FAT based adaptive impedance control for unknown environment with plant parameter uncertainties are described in Section III. Simulation results are discussed in Section IV and finally conclusions are drawn in Section V.

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II. Finger Mechanism and Dynamic Model

The robotic finger is made of seven bar linkages with two degree of freedom (DOF) as shown in Fig. 1. The proximal phalanx is fixed, while the middle and distal phalanges are able to rotate. The mechanism is actuated by two motors which are Motor 2 and Motor s as shown in Fig. 1. Motor 2 is responsible for the angular displacement of the whole finger and Motor s actuates the slider mechanism in the middle phalanx to turn the distal phalanx. In this study, it is assumed that the angular displacement between link 4 and link 5 are constants or in other words, link 4 and link 5 are coupled.

Since the input torques from the Motor 2 and Motor s are provided at link 2 and the lead screw of the slider mechanism respectively, the angular displacement of link 2, θ_2 and the angular displacement of the lead screw in the slider mechanism, θ_s are chosen as the generalized coordinates. Using Lagrange equation, the mathematical model describing the robotic finger motion can be represented as [5]

$$A\ddot{\theta}_2(t)+\frac{1}{2}\frac{dA}{d\theta_2}\dot{\theta}_2^2(t)=\tau_2-\tau_{e2} \tag{1}$$

$$\left(B+m_r\left(-\frac{P}{2\pi}\right)^2+J_r\right)\ddot{\theta}_s(t) + \frac{1}{2}\frac{dB}{d\theta_s}\dot{\theta}_s^2(t)=\tau_s-\tau_{es} \tag{2}$$

where

$$A=\sum_{i=2}^{7}\left(m_i\left(\left(\frac{\partial X_{ci}}{\partial\theta_2}\right)^2+\left(\frac{\partial Y_{ci}}{\partial\theta_2}\right)^2\right)+J_i\left(\frac{\partial\theta_i}{\partial\theta_2}\right)^2\right), \tag{3}$$

$$B=\sum_{i=2}^{7}\left(m_i\left(\left(\frac{\partial X_{ci}}{\partial\theta_s}\right)^2+\left(\frac{\partial Y_{ci}}{\partial\theta_s}\right)^2\right)+J_i\left(\frac{\partial\theta_i}{\partial\theta_s}\right)^2\right), \tag{4}$$

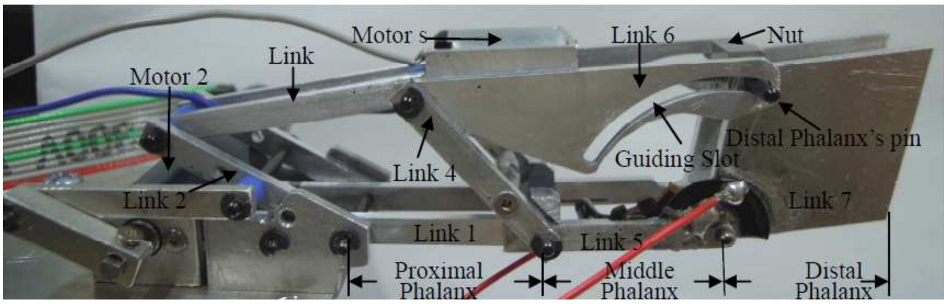

Fig. 1 Robotic finger mechanism

θ_i is the angular displacement of ith link, $\dot{\theta}_i$ is the angular velocity of ith link, $\ddot{\theta}_i$ is the angular acceleration of ith link, m_i is the mass of ith link, m_r is the mass of slider nut, J_i is the moment of inertia of ith link, J_r is the moment of inertia of slider nut, P is the lead screw pitch, X_{ci}, Y_{ci} are the x and y displacement of ith link's centroid from its origin respectively, τ_2 and τ_s are the driving torques from Motor 2 and Motor s respectively, τ_{e2} and τ_{es} are the external disturbance torques at link 2 and lead screw of the slider mechanism respectively. Fig. 2 describes the linkages' parameters used in the modeling.

The dynamic model of the finger can be rewritten in the Cartesian space as

$$M_x(X)\ddot{X}(t)+C_x(X,\dot{X})\dot{X}(t)+G_x(\dot{X})=F-F_e \qquad (5)$$

where $X, \dot{X}$ and $\ddot{X}$ are the 2×1 vector of position, velocity and acceleration respectively, $M_x(X)$ is the 2×2 symmetric positive definite inertia matrix, $C_x(X,\dot{X})$ is the 2×1 vector of Coriolis/ Centrifugal forces, $G_x(X)$ is the 2×1 vector of gravitational force, F is the 2×1 vector of control input from the actuators and F_e is the 2×1 vector of force exerted by the robotic finger on the environment.

τ_{e2} and τ_{es} are related to F_e by the finger's 2×2 Jacobian matrix,

$$[\tau_{e2} \quad \tau_{es}]^T = J^T F_e \qquad (6)$$

where

$$J=\begin{bmatrix} -l_5\sin\theta_5\dfrac{\partial\theta_5}{\partial\theta_2}-l_8\sin\theta_8\dfrac{\partial\theta_8}{\partial\theta_2} & -l_8\sin\theta_8\dfrac{\partial\theta_8}{\partial\theta_s} \\ l_5\cos\theta_5\dfrac{\partial\theta_5}{\partial\theta_2}-l_8\cos\theta_8\dfrac{\partial\theta_8}{\partial\theta_2} & l_8\cos\theta_8\dfrac{\partial\theta_8}{\partial\theta_s} \end{bmatrix}. \qquad (7)$$

J is assumed to be nonsingular in the finger's workspace.

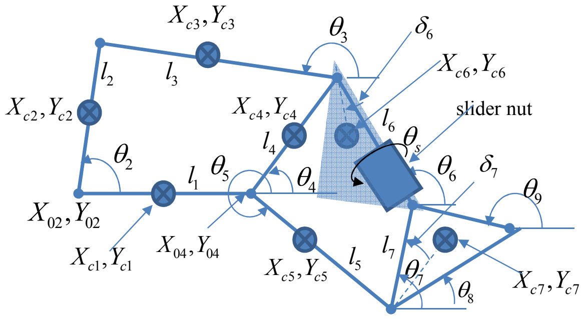

Fig. 2 Robotic finger parameters

III. FAT BASED ADAPTIVE IMPEDANCE CONTROL

A. Review on Function Approximation Technique (FAT)

When a time varying uncertainty exits in a system, traditional adaptive scheme can not be applied in controlling the system [3]. As an alternative, the FAT based strategy can be adopted to solve this problem.

The basic idea in FAT method is to represent the time varying uncertainties as orthogonal functions [6], consisting of summation of the multiplication of constant weighting matrices and time varying of basis function matrices. Since the weighting matrices are constant, the update laws can be easily found by proper selection of Lyapunov or Lyapunov-like functions.

Some of the orthogonal functions that can be utilized in FAT include Taylor polynomials, Chebyshev polynomials, Legendre polynomial, Hermite polynomials, Laguerre polynomials, Bessel polynomials and Fourier series [6].

For example, an unknown bounded period function, $\delta_f(x)$ with period $2T$ can be expanded using the Fourier series [6]

$$\delta_f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{T}+b_n\sin\frac{n\pi x}{T}\right] \qquad (8)$$

The function, $\delta_f(x)$ can be approximated and rewritten as

$$\delta_f(x)=W_f Z_f \qquad (9)$$

where

$$W_f = \begin{bmatrix} a_0 & a_1 & b_1 & \dots & a_M & b_M \end{bmatrix} \tag{10}$$

$$Z_f = \begin{bmatrix} \frac{1}{2} & \cos\frac{\pi x}{T} & \sin\frac{\pi x}{T} \dots & \cos\frac{M\pi x}{T} & \sin\frac{Mx}{T} \end{bmatrix}^T \tag{11}$$

and M is the maximum number of the summation terms, n chosen in (8). Similarly, the estimation of the function, $\hat{\delta}_f(x)$ can be represented as

$$\hat{\delta}_f(x) = \hat{W}_f Z_f + \varepsilon_f \tag{12}$$

where $\hat{W}_f$ is the estimation of W_f and ε_f is the approximation error matrices.

Since $\hat{W}_f$ is constant, the update laws for the parameter can be obtained easily by adaptive update law with a proper selection of the Lyapunov or Lyapunov-like function. However, it is very important that the valid range of the orthogonal functions to be orthonormal is ensured, to guarantee the effectiveness of the FAT strategy [6].

B. FAT based Adaptive Impedance Control for Unknown Environment Position

The main objective of the control strategy is to drive the robotic finger to exert the desired force on the environment or object under uncertain environment position. The uncertainty may arise due to the lack of information of the environment position, [1] or the distance that it needs to move into the object to exert the desired force [2].

This study proposes two phases control law for the force controllable direction, which are the free space phase and contact phase.

In non-contact or free space, the robotic finger is not in contact and is moving towards the environment. The target impedance in this phase governed by the standard force based impedance control for N DOF robot,

$$M_d\ddot{E} + B_d\dot{E} + K_d E = -E_f, \tag{13}$$

E and E_f are the position and force error respectively, described by

$$E = X - X_d \tag{14}$$

$$E_f = F_e - F_d \tag{15}$$

where X_d is the $N \times 1$ vector of the reference position for the robot end-effector and F_d is the $N \times 1$ vector of desired force respectively. M_d , B_d and K_d are the $N \times N$ diagonal symmetric positive definite desired inertia, damping and stiffness matrices which can be specified by the designer. The same target impedance is applied for position controllable direction for both free and contact phases.

In contact space, the robotic finger interacts with the environment. Since the environment position or object shape may be unknown in practical [1], a new adaptive control with compensators is proposed as

$$B_d\left(\dot{E}'+\Omega_1\right) + K_d\left(E'+\Omega_2\right) + \Omega_3 = -K_f\left(F_e - F_d\right) \tag{16}$$

where the compensator terms can be described as

$$\Omega_1 = \dot{\hat{\delta}}_{xe} \tag{17}$$

$$\Omega_2 = \hat{\delta}_{xe} \tag{18}$$

$$\Omega_3 = \frac{-B_d\dot{F}_d - K_d F_d}{k_e} \tag{19}$$

Ω_1, Ω_2 and Ω_3 are the compensators related to the velocity error, position error and force respectively, k_e is the environment stiffness, K_f is $N \times N$ diagonal symmetric positive definite force error factor matrix.

In target impedance (16), the tracking error is replace by the inaccurate estimated environment position, E' which can be represented mathematically as

$$X_e' = X_e + \delta_{xe} \tag{20}$$

where X_e is the unknown actual environment position and δ_{xe} is inaccuracy in the estimation.

Although the precise knowledge of object stiffness may also be unknown in practice, it is assumed to be known in advance at this stage of study.

For simplicity, consider that force is applied in one direction only and let $b_d, k_d, k_f, \dot{e}', e', x_e', x, f_d$ and f_e be the elements of $B_d, K_d, K_f, \dot{E}', E', X_e', X_e, F_d$ and F_e . Therefore, (16) can be rewritten as

$$b_d\left(\dot{e}'+\Omega_1\right) + k_d\left(e'+\Omega_2\right) + \Omega_3 = -k_f\left(f_e - f_d\right) \tag{21}$$

FAT can be utilized to approximate not only a flat environment, but also a varying function or non-flat environment. This can be done by representing the inaccuracy in the environment estimation and its true value as function of time, as

$$\dot{\hat{\delta}}_{xe} = \hat{W}_{xe}\dot{Z}_{xe} + \varepsilon_{\dot{\delta}xe}, \quad \dot{\delta}_{xe} = W_{xe}\dot{Z}_{xe} \tag{22}$$

$$\hat{\delta}_{xe} = \hat{W}_{xe}Z_{xe} + \varepsilon_{\delta xe}, \quad \delta_{xe} = W_{xe}Z_{xe}, \tag{23}$$

Where W_{xe} is the true value of weighting matrices, $\hat{W}_{xe}$ is the value to be estimated by adaptive law, Z_{xe} and $\dot{Z}_{xe}$ are matrices the basis functions, $\varepsilon_{\dot{\delta}xe}$ and $\varepsilon_{\delta xe}$ are the approximation error matrices which are assumed to be zero.

Substituting (22) and (23) into (21),

$$\begin{aligned} & b_d\left(\dot{x}-\dot{x}_e\right)-b_d\dot{Z}_{xe}\tilde{W}_{xe}+k_d\left(x-x_e\right)-k_dZ_{xe}\tilde{W}_{xe} \\ & +\frac{-b_d\dot{f}_d-k_df_d}{k_e}=-k_f\left(f_e-f_d\right) \end{aligned} \tag{24}$$

where

$$\tilde{W}_{xe}=W_{xe}-\hat{W}_{xe} \tag{25}$$

From the environment model, the force, f_e can be described as

$$f_e=k_e\left(x-x_e\right) \tag{26}$$

Rearranging the equation,

$$x=\frac{F_e}{k_e}+x_e\,,\ \dot{x}=\frac{\dot{F}_e}{k_e}+\dot{x}_e \tag{27}$$

Substituting (27) into (24) yields

$$\begin{aligned} & b_d\left(\dot{f}_e-\dot{f}_d\right)+K_d\left(f_e-f_d\right)-b_dk_e\dot{Z}_{xe}\tilde{W}_{xe} \\ & -k_dk_eZ_{xe}\tilde{W}_{xe}-b_d\dot{f}_d-k_df_d=-k_ek_f\left(f_e-f_d\right) \end{aligned} \tag{28}$$

Replacing the force error in (28) as e_f,

$$\begin{aligned} & b_d\dot{e}_f+\left(k_d+k_ek_f\right)e_f \\ & -b_dk_e\dot{Z}_{xe}\tilde{W}_{xe}-k_dk_eZ_{xe}\tilde{W}_{xe}=0 \end{aligned} \tag{29}$$

The update law for $\hat{W}_{xe}$ can be chosen by defining the Lyapunov-like function,

$$V=\frac{1}{2}e_f^Tb_de_f+\frac{1}{2}\left(\tilde{W}_{xe}^TQ_{xe}\tilde{W}_{xe}\right) \tag{30}$$

where Q_{xe} is a diagonal symmetric positive definite constant matrix.

Differentiating (30),

$$\dot{V}=e_f^Tb_d\dot{e}_f-Tr\left(\tilde{W}_{xe}^TQ_{xe}\dot{\hat{W}}_{xe}\right) \tag{31}$$

Substitute $b_d\dot{e}_f$ from (29),

$$\begin{aligned} \dot{V}= & -e_f^T\left(k_d+k_ek_f\right)e_f \\ & +Tr\left(\tilde{W}_{xe}\left[\begin{array}{l} B_dk_e\dot{Z}_{xe}e_f \\ +K_dk_eZ_{xe}e_f-Q_{xe}\dot{\hat{W}}_{xe} \end{array}\right]\right) \end{aligned} \tag{32}$$

From (32), the update law can be set as

$$\dot{\hat{W}}_{xe}=Q_{xe}^{-1}\left(B_dk_e\dot{Z}_{xe}+K_dk_eZ_{xe}\right)e_f \tag{33}$$

Substituting (33) into (32),

$$\dot{V}=-e_f^T\left(K_d+k_e\right)e_f \tag{34}$$

Since V is p d. and $\dot{V}\leq 0$ as can be seen from (30) and (34) respectively, the force error e_f and estimated parameter, $\tilde{W}_{xe}$ are bounded. Taking the derivative of $\dot{V}$,

$$\ddot{V}=-2e_f^T\left(k_d+k_e\right)\dot{e}_f \tag{35}$$

Substituting (29) into (35) it can be observed that $\ddot{V}$ is also bounded. From Barbalat theory,

$$\lim_{t\to\infty}\dot{V}=0 \tag{36}$$

which means that

$$\lim_{t\to\infty}e_f=0 \tag{37}$$

Therefore, provided that the target impedance is achieved, with the adaptive impedance control law (21) and the updating law (33) for the robotic finger (5), the actual force exerted on the environment converges to the desired value, $F_e=F_d$ as $t\to\infty$.

C. Impedance Control Strategy for Uncertainties in Dynamic Model

The impedance control law as in [4] is applied to cater for the uncertainties in the plant parameters. In this control strategy, the following properties are utilized [4]

i. $M_x\left(x\right)$ is symmetric and positive definite.

ii. $2C_x\left(x,\dot{x}\right)\dot{x}-\dot{M}_x\left(x\right)$ is a skew- symmetric matrix.

iii. The plant uncertainties is bounded by a known bound $\left\|M_x\left(X\right)\right\| \le k_M, \left\|C_x\left(X,\dot{X}\right)\right\| \le k_C X, \left\|G_x\left(X\right)\right\| \le k_G$ where k_M, k_C, k_G are positive scalars.

The control input is governed by

$$F = F_s + F_f + F_e \tag{38}$$

where

$$F_s = -\left(K_1\left\|\ddot{X}_r\right\| + K_2\left\|\dot{X}\right\|\left\|\dot{X}_r\right\| + K_3\right)\left(\frac{Z}{Z+\delta_z}\right) \tag{39}$$

$$F_f = -KZ \tag{40}$$

K and δ_z are $N\times N$ positive definite diagonal matrices. K_2 and K_3 are also $N\times N$ positive definite diagonal matrices, with the elements

$$k_{1,i} \ge k_M, k_{2,i} \ge k_C, k_{3,i} \ge k_G, \quad i = 1,2....,N \tag{41}$$

The proof for this can be found in [4].

The reference velocity, $\dot{X}_r$ is related to the impedance error by the equation,

$$Z = \dot{X} - \dot{X}_r, \tag{42}$$

and is different for each phase. For position controllable direction in both phases and force controllable direction during non-contact mode, from (13), the impedance error can be written as

$$W = \ddot{E} + M_d^{-1}B_d\dot{E} + M_d^{-1}K_dE + M_d^{-1}E_f \tag{43}$$

Defining Z in this phase as [4]

$$Z = \dot{E} + \lambda E + E_{fl} \tag{44}$$

W can be rewritten as

$$W = \dot{Z} + \gamma Z \tag{45}$$

where $\dot{E}_{fl} + \gamma E_{fl} = M_d^{-1}E_f$, λ and γ are positive definite matrices chosen such that

$$\lambda + \gamma = M_d^{-1}B_d \qquad \dot{\lambda} + \lambda\gamma = M_d^{-1}K_d. \tag{46}$$

Therefore, the control objective is this phase is to drive Z to zero, so that the impedance error converges to zero [4]. In this case, from (42) and (48), $\dot{X}_r$ can be described as

$$\dot{X}_r = \dot{X}_d - \lambda E - E_{fl} \tag{47}$$

During contact phase, from (16), the impedance error for the force controllable direction during non-contact mode is governed by,

$$Z = \dot{E}^{'} + \Omega_1 + B_d^{-1}K_d\left(E^{'} + \Omega_2\right) + B_d^{-1}\Omega_3 + B_d^{-1}K_fE_f \tag{48}$$

From (42) and (48), $\dot{X}_r$ in force controllable direction during non-contact mode is be written as

$$\dot{X}_r = \dot{X}_e^{'} - \Omega_1 - B_d^{-1}K_d\left(E^{'} + \Omega_2\right) - B_d^{-1}\Omega_3 - B_d^{-1}K_fE_f \tag{49}$$

IV. Results

Simulation has been carried out to investigate the effectiveness of the proposed method. The robotic finger is desired to exert a constant desired force, F_d 20 N normal to the surface of the object, in x direction while moving along y direction on a sinusoidal shaped object. The initial position of the robot is defined to be $(x(0), y(0)) = (3.4, 1.5)$. The simulation time is set to be 0.58 and the object stiffness is set to be 40000N/m. In the beginning, the robotic finger is desired to move towards the object and only become in contact with the object after 0.08 seconds. Although the interaction with the environment may occur at any point along the distal phalanx surface, it is assumed that the contact occur at the end of the phalanx for simplicity. It is assumed in the simulation that the robotic finger contains 20% of parameter uncertainties from the true values. The actual values of the parameter can be found in [5]. The true object position or shape is assumed to be unknown and is estimated as a constant. The value of δ_{xe} has been approximated by the first 5 terms of Fourier series (*n*=2) and the controller gains are selected as the following

$$K = diag[4200, 500], K_1 = diag[10, 50], K_2 = diag[20, 20],$$
$$K_3 = diag[7000, 4000], \delta_z = diag[1,1], M_d = diag[1,1],$$
$$B_d = diag[1270, 110], K_d = diag[3000, 3000], K_f = diag[10, 0],$$
$$\hat{W}_{xe}(0) = [0.6\times10^{-2}\ 0.075\times10^{-2}\ 0.075\times10^{-2}\ 0.075\times10^{-2}\ 0.075\times10^{-2}]$$
$$Q_{xe} = diag[10\times10^8, 0.05\times10^8, 0.025\times10^8, 1\times10^8, 0.05\times10^8]$$

Figures 3-7 illustrates the simulation results of the proposed technique with the above control parameters and setting. From Figures 3-4 can be observed that the control system successfully achieve the target impedance in both position and force controllable directions. The force tracking gives a satisfactory performance in which the force error is small as can be seen from Fig. 5. Fig. 6 shows the position tracking of the finger. It can be observed from Fig. 7 that the update law of the adaptive control strategy has successfully approximated the true value of δ_{xe}. The simulation results prove that the proposed technique is successful in controlling the robotic system to exert the desired for under unknown environment position and uncertain plant parameter conditions.

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V. Conclusion

An adaptive impedance control based on Function Approximation Technique (FAT) for a robotic finger operating under uncertain environment position and plant parameter based is presented in this paper. The target impedance during contact mode for the force controllable direction has been modified by including compensators to cater for the imprecise information of the environment position. The environment parameter uncertainty has been expressed using FAT and the update law for the adaptive scheme has been obtained using Lyapunov stability theory. The simulation results shows that the control system has successfully force the robotic finger to exert the desired force while following the reference position trajectory. Future works involves the investigation on the FAT based adaptive impedance control for unknown environment or object stiffness.

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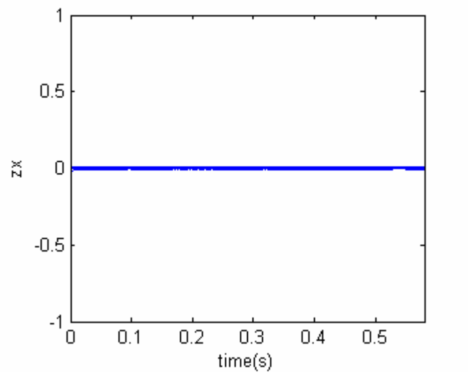

Fig. 3 z_x

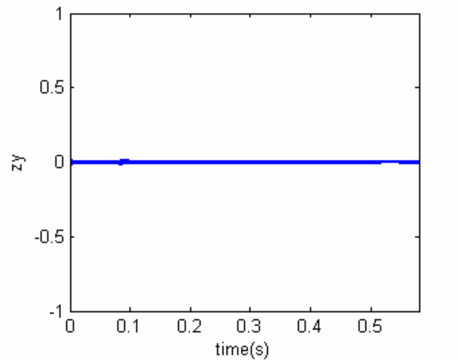

Fig. 4 z_y

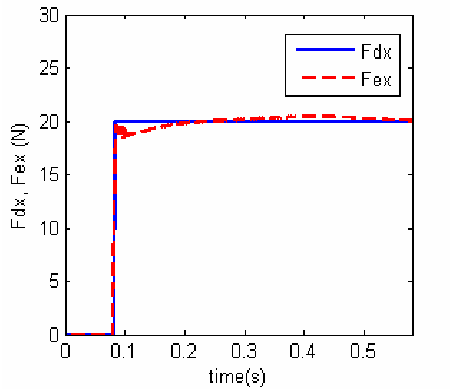

Fig. 5 Force tracking performance

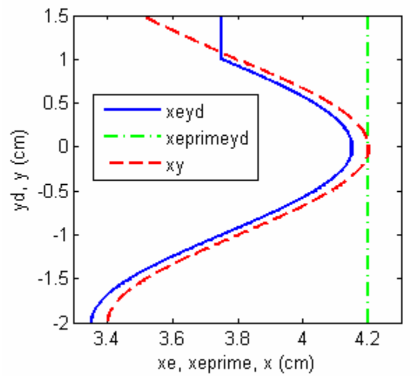

Fig. 6 Position tracking performance

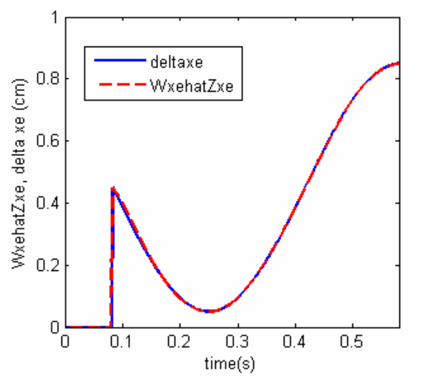

Fig. 7 Approximation of δ_{xe}