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A Novel Frequency Offset Estimation Scheme for OFDM Systems

Youngpo Lee and Seokho Yoon

Abstract—In this paper, we propose a novel frequency offset estimation scheme for orthogonal frequency division multiplexing (OFDM) systems. By correlating the OFDM signals within the coherence phase bandwidth and employing a threshold in the frequency offset estimation process, the proposed scheme is not only robust to the timing offset but also has a reduced complexity compared with that of the conventional scheme. Moreover, a timing offset estimation scheme is also proposed as the next stage of the proposed frequency offset estimation. Numerical results show that the proposed scheme can estimate frequency offset with lower computational complexity and does not require additional memory while maintaining the same level of estimation performance.

Keywords—OFDM, frequency offset estimation, threshold.

I. Introduction

In orthogonal frequency division multiplexing (OFDM) modulation, the data is transmitted on the multiple orthogonal subcarriers. Due to its high spectral efficiency and immunity to multipath fading, OFDM has attracted much attention in the field of wireless communications. For example, OFDM has been adopted as the transmission method of many standards in wireless communications, including European digital audio and video broadcasting (DAB/DVB), and IEEE 802.11a and European Hiper-LAN II for wireless local area network (WLAN) [1], [2]. However, the OFDM is very sensitive to the frequency offset caused by the mismatch of the oscillators. To alleviate these problems, various techniques have been proposed [3]-[7]. Recently, Nogami [8] proposed a frequency offset estimation scheme employing correlation between the known pilot and received symbols. However, it was implemented under the assumption of perfect timing synchronization, and thus, the scheme does not work well in the presence of the timing offset. An efficient frequency offset estimation scheme robust to the timing offset was presented by Bang [9]. Bang's scheme has good performance even though a timing offset exists; however, the complexity in implementation rapidly increases, as the frequency offset range increases.

In this paper, a novel frequency offset estimation scheme based on the coherence phase bandwidth and a threshold is proposed for OFDM systems. The proposed scheme is robust to a symbol timing offset and has lower computational complexity when compared with Nogami's and Bang's schemes. We also propose a timing offset estimation scheme as the next stage of the frequency offset estimation.

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II. The OFDM Signal Model

An OFDM symbol is generated by inverse fast Fourier transform (IFFT). The OFDM symbol in the time domain is expressed as

$$s_n = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} S_l e^{j2\pi nl/N}, \tag{1}$$

where S_l is a phase shift keying (PSK) or quadrature amplitude modulation (QAM) symbol in the lth subcarrier and N is the size of IFFT.

In the presence of frequency and timing offsets, the received OFDM signal r_n can be expressed as

$$r_n = s_{n-n_0} e^{j2\pi f_0 (n-n_0)/N} + w_n, \tag{2}$$

where f_0 and n_0 represent the frequency and timing offsets normalized to the subcarrier spacing $1/N$, respectively and w_n is the zero-mean complex additive white Gaussian noise (AWGN).

A frequency offset f_0 can be divided into a integer part and fractional part, i.e.,

$$f_0 = \epsilon + \Delta f_f, \tag{3}$$

where ϵ is the integer part of f_0 and $f_f \in [-0.5, 0.5)$ is the fractional part of f_0.

Since our focus in this paper is on the estimation of ϵ, we assume that the fractional part f_f is known and perfectly compensated. At the receiver end, the received symbol is demodulated using the FFT operation. The kth FFT output R_k is given by, for $k = 0, 1, 2, \cdots, N-1$,

$$R_k = S_{k-\epsilon} e^{-j2\pi n_0 (k-\epsilon)/N} + W_k, \tag{4}$$

where W_k is the FFT output of w_n.

III. The Effect of Timing Offset on Frequency Offset Estimation

A. Nogami's scheme

In this scheme, to estimates the integer frequency offset by examining the correlation value between the known training symbol and the cyclically shifted version of the received training symbol. The estimated integer frequency offset $\hat{\epsilon}$ is

$$\hat{\epsilon} = \arg\max_{d} \left\{ \left| \sum_{k=0}^{N-1} Z_k^* R_{(k+d)_N} \right| \right\}, \tag{5}$$

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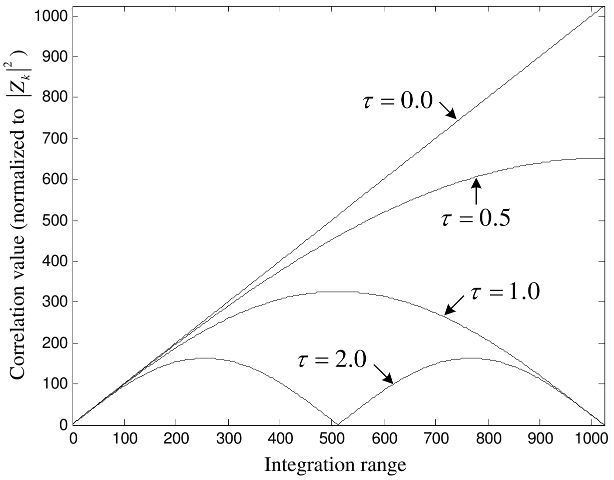

Fig. 1. The partial correlation values from Nogami's scheme versus the integration range for several values of timing offset (when $d = \epsilon$ and $N = 1024$).

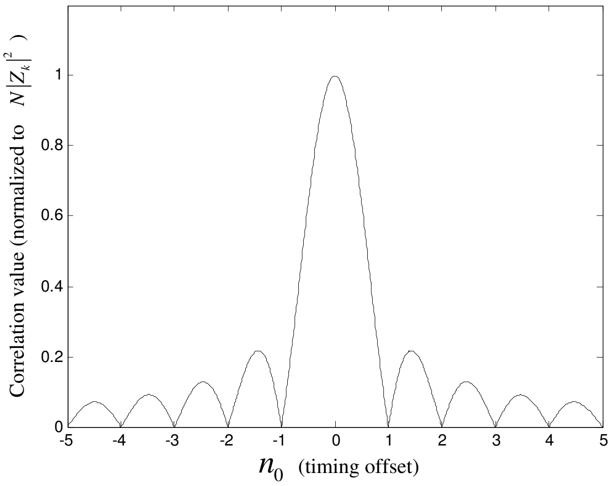

Fig. 2. The partial correlation value from Nogami's scheme plotted as a function of timing offset n_0 (when $d = \epsilon$).

where Z_k is the transmitted training symbol, R_k is the received training symbol, d is the amount of cyclic shift, and $(\cdot)_N$ is the modulo-N operator.

Nogami's scheme was proposed on the assumption of perfect timing synchronization. The actual case that exists timing offset has less reliability.

Fig. 1 shows the correlation values for some value of timing offset where $d = \epsilon$. For a timing offset exist, the correlation value from Nogami's scheme oscillates according to the increment of the integration range.

Fig. 2 shows additional results for Nogami's scheme. As shown in Fig. 2, the correlation value from Nogami's scheme is very small when timing offset is non-zero integer value (note that the large correlation value is essential in the indication of the correct frequency estimation). Therefore, Nogami's scheme has less reliability where timing offset is specific value.

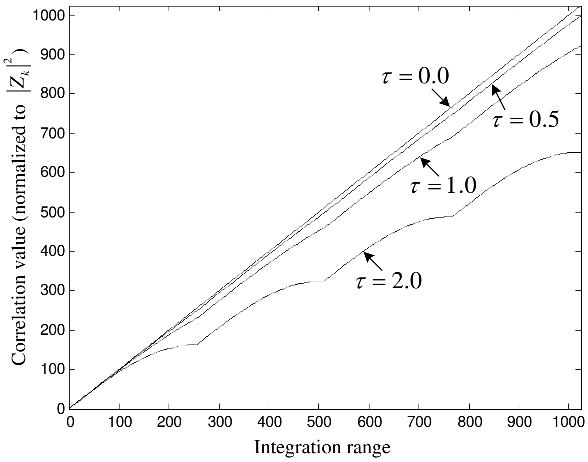

Fig. 3. The partial correlation values from Bang's scheme versus the integration range for several values of timing offset (when $d = \epsilon$, $N = 1024$, and CPB$= 256$).

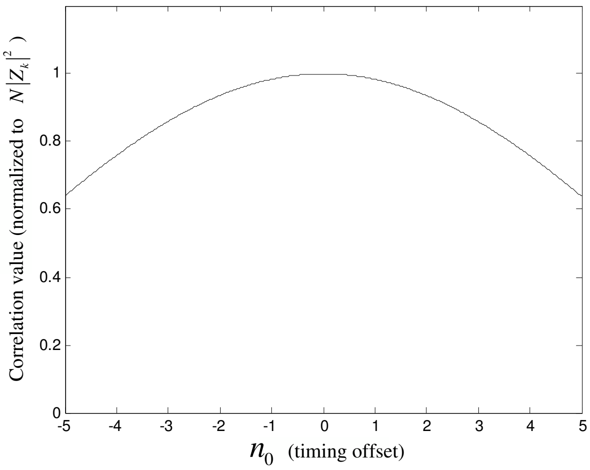

Fig. 4. The partial correlation value from Bang's scheme plotted as a function of timing offset n_0 (when $d = \epsilon$ and CPB $= N/10$).

B. Bang's scheme

In this scheme, the correlation is calculated individually within each coherence phase bandwidth block and then summed. The coherence phase bandwidth (CPB) is related to maximum value of allowed timing offset and expressed as follows:

$$\text{CPB} = \frac{1}{2n_0^t}N, \tag{6}$$

where, n_0^t is a maximum tolerated value of the timing offset. The estimated frequency offset can be expressed as

$$\hat{\epsilon} = \arg\max_{d} \left\{ \sum_{m=0}^{K-1} \left| \sum_{k=0}^{\text{CPB}-1} Z_{k+m\text{CPB}}^{*} R_{(k+m\text{CPB+d})_N} \right| \right\}, \tag{7}$$

where $K = N/n_0^t$.

As shown in Fig. 3, the correlation value from Bang's scheme increases monotonically depending on the increment of the integration range.

Fig. 4 shows additional results for Bang's scheme. As shown in this figure, the correlation value from Bang's scheme has not weak point where the correlation value is zero unlike Nogami's scheme. Therefore, Bang's scheme has good accuracy of estimation even if a timing offset exists. However Bang's scheme has high computational complexity because of the correlation value is calculated for all possible values of d. This is the very effect on the hardware cost and speed.

IV. Proposed Scheme

A. Frequency offset estimation

If we calculate one CPB block in (7), correlation value C can be expressed as

$$C = \left| \sum_{k=0}^{\text{CPB}-1} Z_k^* R_{(k+d)_N} \right| . \tag{8}$$

If we assume $d = \epsilon$ and ignore AWGN in (8), the correlation value C is

$$C = |Z_k|^2 \left| \sum_{k=0}^{\text{CPB}-1} e^{-j2\pi n_0 k/N} \right| . \tag{9}$$

The value C in (9) is minimum when $n_0 = n_0^t$. Therefore, the minimum correlation value $C_{\min}$ can be expressed as

$$C_{\min} = |Z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_0^t}{N}\right) \right| . \tag{10}$$

The minimum value of full correlation is acquired in similar manner as in (10)

$$C_{\min}^{full} = 2n_0^t |Z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_0^t}{N}\right) \right| . \tag{11}$$

In this paper, we use a threshold value η one half of (11).

$$\eta = n_0^t |Z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_0^t}{N}\right) \right| . \tag{12}$$

As the value of the threshold becomes smaller, the detection probability for the correct estimate of ϵ and the false alarm probability increase, and vice versa. Thus the threshold chosen as one half of (11) is one of the reasonable choices in view of the detection and false alarm probabilities, which decide the overall system performance.

The proposed scheme calculate the η and CPB using n_0^t, and then, the correlation value is calculated using the CPB. If the correlation value exceeds η, the d is decided to be the correct estimate of ϵ. Otherwise, the received signal is cyclically shifted by d and the above procedure is repeated. The operation of the proposed scheme is described in Fig. 5.

If expected frequency offset range L is adequately big, the proposed scheme will generally have about a half computational complexity compare to the others. Another important result is that the proposed scheme does not require additional memory for correlation values unlike Nogami's and Bang's schemes.

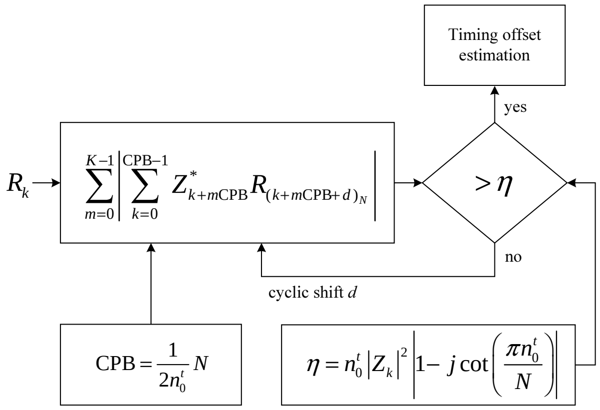

Fig. 5. Block diagram of the proposed frequency offset estimation scheme

B. Timing offset estimation

If we assume that frequency offset is completely recovered, the received symbol is

$$R_k = H_k Z_k e^{j2\pi n_0 k/N}, \tag{13}$$

where H_k is a frequency response of the channel. The argument of correlation between the transmitted and received training symbol is expressed as

$$\Phi_k = \angle\left(Z_k^* R_k\right) = 2\pi n_0 \frac{k}{N} + \angle\left(H_k\right), \tag{14}$$

where $\angle$ denotes the argument of a complex number.

Let us assume that the channel is time invariant during one OFDM symbol duration, then the difference of two successive sample can be written as (15) and the effect of channel is removed.

$$\Delta_k = \Phi_k - \Phi_{k-1} = 2\pi n_0 \frac{1}{N}. \tag{15}$$

Then, the timing offset can be estimated as

$$\hat{n}_0 = \frac{N}{2\pi} \text{avg}\left(\Delta_k\right), \quad \text{for } k = 1, 2, \cdots, N-1, \tag{16}$$

where $|\Delta_k| \leq 2\pi n_0^t/N$ and $\text{avg}(\cdot)$ denotes the average. The average of Δ_k where $|\Delta_k| \leq 2\pi n_0^t/N$, is used for improve the operation's accuracy, since the allowed range of $\angle$ operator is $[-\pi, \pi)$.

V. Numerical Results

A. Frequency offset estimation

We can see from Table 1, for $L \gg 1$, where L is the frequency offset range, the proposed scheme has about a half computational complexity compared with the Nogami's and Bang's schemes (computational complexity of proposed scheme is average). Another important result is that the

TABLE I
COMPLEXITY COMPARISON FOR THE SCHEMES

Scheme	Number of complex multiplication	Number of comparison operation	Size of memory for correlation value
Nogami's scheme	LN	$L-1$	L
Bang's scheme	LN	$L-1$	L
Proposed scheme	$\frac{L+1}{2}N$	$\frac{L+1}{2}$	-

proposed scheme does not require any memory for correlation value unlike Nogami's and Bang's schemes.

In this paper, we use two channel model for performance comparisons. The first one is AWGN channel model and the other is multipath channel model. The signal to noise ratio (SNR) of AWGN channel model is 5 dB and multipath channel model was specified in Table 2.

TABLE II
THE MULTIPATH CHANNEL MODEL

Path	1	2	3	4
Time delay (sample)	0	5	10	15
Amplitude attenuation (dB)	0	4	8	12
Phase	Random			
AWGN	10 dB			

The multipath channel model has four path and each signal through a multipath has 5, 10, 15 samples time delay based on first path signal. And each signal has 4, 8, 12 dB attenuation in the amplitude and random phase jitter. Also 10 dB AWGN was added on multipath model.

TABLE III
ACCURACY COMPARISON OF THE FREQUENCY OFFSET ESTIMATION SCHEMES IN THE AWGN CHANNEL MODEL.

Timing offset (samples)	Nogami's scheme	Bang's scheme	Proposed scheme
0	100	100	100
1	0	100	100
2	0	100	100
5	0	100	100

TABLE IV
ACCURACY COMPARISON OF THE FREQUENCY OFFSET ESTIMATION SCHEMES IN THE MULTIPATH CHANNEL MODEL.

Timing offset (samples)	Nogami's scheme	Bang's scheme	Proposed scheme
0	100	100	100
1	0	100	100
2	0	100	100
5	0	100	100

The simulation results for accuracy comparison which was simulated under AWGN (5 dB) and multipath model

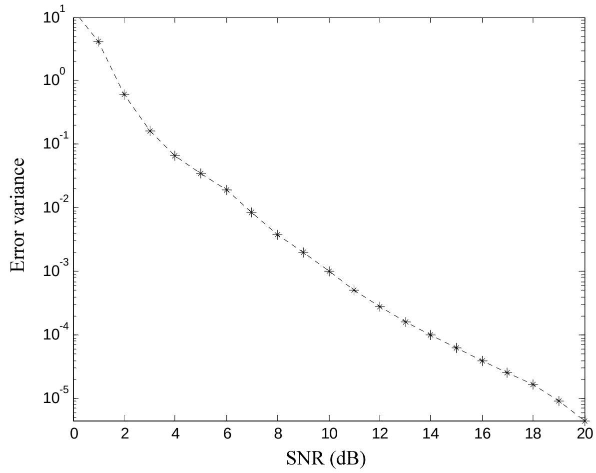

Fig. 6. Error variance of timing offset estimator in AWGN channel

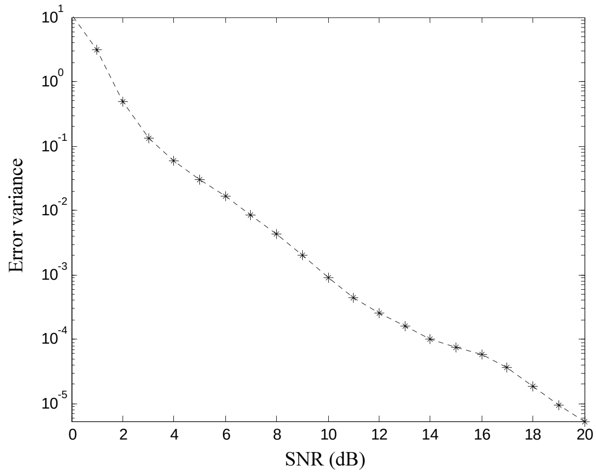

Fig. 7. Error variance of timing offset estimator in multipath channel

(specified in Table 2), are shown in Table 3 and 4, respectively. The frequency offset used in this simulation is an integer value, where the range of [0, 500], $N = 1024$, and CPB= $N/32$. As shown in this results, the proposed scheme exhibits significantly improved frequency offset estimation performance over Nogami's scheme in the presence of the timing offset.

B. Timing offset estimation

Fig. 6 shows the error variance versus some level of SNR. This simulation runs 10,000 iterations at each SNR level. We can show that the error variance was decreased with SNR level.

Fig. 7 shows the error variance versus some values of SNR in multipath channel model (using the parameters in table 2 and the SNR varies from 0 to 20 dB) and runs 10,000 iterations. Since we assume the channel is time-invariant during one symbol time, the error variance of proposed scheme on multipath channel is similar to Fig. 6.

VI. Conclusion

In this paper, we have proposed a novel OFDM frequency offset estimation scheme with a reduced complexity compared with the conventional scheme by employing the CPB and correlation threshold. In addition, we have proposed a timing offset estimation scheme as the next stage of the frequency offset estimation. From numerical results, we have shown that the proposed scheme exhibits much lower complexity than the conventional schemes while maintaining the same level of estimation performance.

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