

Optimization and GIS-Based Intelligent Decision Support System for Urban Transportation Systems Analysis

Mohamad K. Hasan, and Hameed Al-Qaheri

Abstract—Optimization plays an important role in most real world applications that support decision makers to take the right decision regarding the strategic directions and operations of the system they manage. Solutions for traffic management and traffic congestion problems are considered major problems that most decision making authorities for cities around the world are looking for. This review paper gives a full description of the traffic problem as part of the transportation planning process and present a view as a framework of urban transportation system analysis where the core of the system is a transportation network equilibrium model that is based on optimization techniques and that can also be used for evaluating an alternative solution or a combination of alternative solutions for the traffic congestion. Different transportation network equilibrium models are reviewed from the sequential approach to the multiclass combining trip generation, trip distribution, modal split, trip assignment and departure time model. A GIS-Based intelligent decision support system framework for urban transportation system analysis is suggested for implementation where the selection of optimized alternative solutions, single or packages, will be based on an intelligent agent rather than human being which would lead to reduction in time, cost and the elimination of the difficulty, by human being, for finding the best solution to the traffic congestion problem.

Keywords—Multiclass simultaneous transportation equilibrium models, transportation planning, urban transportation systems analysis, intelligent decision support system.

I. INTRODUCTION

THE world, including transport, was changing fast at the turn of the century. We still encounter many of the same transport problems of the past: congestion, pollution, accident, financial deficits and so on. Fuel shortages are (temporarily) not a problem but the general increase in the road traffic and transport demand has resulted in congestion, delay, accidents and environmental problem well beyond what has been considered acceptable so far. These problems have not been restricted to roads and car traffic alone. Economic growth seems to have generated levels of demand exceeding the capacity of most transport facilities. Long periods of underinvestment in some modes and regions have resulted in fragile supply systems which seem to break down whenever something differs slightly from average conditions.

However, we have learned a good deal from long periods of

weak transport planning, limited investment, emphasis on the short term and mistrust in strategic transport modelling and decision making. We have learned, for example, that old problems do not fade away under the pressure of mild attempts to reduce them through better traffic management; old problems reappear with even greater vigour, pervading wider areas, and in their new forms they seem more complex and difficult to handle.

By the end of the century, the world had entered a stage of greater confidence in technical solutions than during the previous thirty years. This is not the earlier confidence in technology as the magic solution to economic and social problems; we have also learned that this is a mirage. However, Information and Communication Technologies (ICT) have advanced so much as to make possible new conceptions of transport infrastructure (e.g. road transport informatics) and movement systems (e.g. automated driverless trains). Of particular interest to the subject is the advent of low-cost and high-capacity computing; this has practically eliminated computing power as a bottleneck in transport modelling. The main limitations are now human and technical: contemporary transportation planning requires skilled professionals and theoretically sound modelling techniques with competent implementations in software.

Developing countries are suffering serious transport problems as well. These are no longer just the lack of roads to connect distant rural areas with markets. Indeed, the new transport problems bear some similarities with those prevalent in the industrialized world: congestion, pollution, and so on. However, they have a number of very distinctive features deserving a specific treatment: low incomes, fast urbanization and change, high demand for public transport, scarcity of resources including capital, sound data and skilled personnel.

The birth of the twenty-first century was dominated by two powerful trends affecting most aspects of life and economic progress. The stronger trend is globalization, supported and encouraged by the other trend, cheap and high-capacity telecommunications. The combination of the two is changing the way we perceive and tackle many modern issues, and their influence in transport planning is starting to be felt. Some of these influences are the role of good transport infrastructure in enhancing the economic competitiveness of modern economies; a wider acceptance of the advantages of involving the private sector more closely in transport supply and operations; the possible role of telecommunications in reducing the need to travel.

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Important technical developments in transport modeling have taken place since the mid-1970s, in particular at major research centers; these developments have been improved and implemented by a small group of resourceful consultants. However, many of these innovations and application have received limited attention outside the more academic journals. After these years of experimentation there is now a better recognition of the role of modeling in supporting transport planning.

The main objective of this review paper is to develop an urban transportation system analysis framework in a form of a intelligent decision support system, using the state of the art developments in urban transportation system modeling, that can be useful decision support tool to transportation planners and decision makers to analysis and evaluate strategic transportation plans include transportation projects and policies.

A transportation system can be define as the combination of elements and their interactions, which produce the demand for travel within a given area and the supply of the transportation service to satisfy this demand. This definition is general and flexible enough to be applied to different context. The specific structure of the system is defined by the problem itself (or class of problem) for whose solution is employed.

Almost all of the components of a social and economic system in a given geographical area interact with different levels of intensity. However, it is practically impossible to take into account every interacting element to solve a transportation problem. The typical system approach is to isolate the most relevant element in the problem. These elements, and the relation among them, make up the analysis system. The remaining elements which belong to the external environment are taken into account only in term of their interaction with the analysis system. The transportation system of a given area can also be seen as a sub-system of a wider territorial system with which it strongly interacts. The extent, to which these interactions are included in the analysis system, or in the external environment, depends on the specific problem.

Transportation is the process of transferring people, good and information from one place to another. To perform its function, any transportation system consists of several components which together act as a single unit that is designed and developed to provide a suitable technology for the "objects" to be transported.

Any transport technology must provide mobility, control, protection, and land access for the objects. Perhaps the most widespread transport technology is the one used for inland transport. That is vehicles and containers operating on highway or railway networks.

Components of such a transport system may be divided into two categories. The first category includes the physical components such as the network infrastructural fixed elements (i.e., road and rail links, intersections, terminals, parking spaces, railway yards, maintenance shops, stations, etc.), and moving elements (i.e., vehicles and containers). The second

category includes the human components such as users, operators, owners, and regulators of the system, government and the society at large. The key players in system analysis are defined as follows:

Users: are those traveler and shipper who represent demand on the transport system.

Operators: are those who own the fleet of vehicles and hence, are responsible for their operation, maintenance and investment.

Owners: are those who own the network elements and hence, are concerned with construction, upgrading, operation and maintenance of the infrastructure.

Regulators: are responsible for traffic laws, ordinances and regulations.

Government: is responsible for creating the master plan for the city transportation systems and policies or projects that affect the transportation systems.

It should be emphasized that these components should interact all together in order to provide effective and efficient "transportation". Furthermore, a transport system may be viewed as one of several components of more complex socio-economic system of the society. The interaction between the transport system and its surrounding socio-economic environment is, again, evident. Transport demand is a function of the magnitude and spatial distribution of socio-economic activities which in turn are greatly influenced by the characteristics of transport systems. Therefore, the actual performance of any transport system is a function of several interacting and interdependent factors within the system and those outside the system.

Like any other complex system, transport system may not always perform as desired and there are often problems and issues to be addressed and resolved. Traffic congestion, limited parking space, high accident rate, weak connectivity between major development centers, freight movements, public transit, and air pollution are but a few examples to mention. The range of possible remedies is enormous. Construction of new highways, building multistory garages, introducing new transport technologies, creating new organizational structures and traffic regulations are examples of actions which would be undertaken.

The "best" action (or set of actions) to be implemented in particular situation, is a question that is, often, not a simple one to resolve immediately and usually requires a systematic process of analysis that takes into consideration the interacting affects on the system, that is, transportation planning. Formally defined, planning is "a systematic analytical process that assists decision makers of a given system to achieve a specific set of goals and objectives within a given socio-economic environment in an optimum fashion."

II. TRANSPORTATION PLANNING PROCESS

The transportation planning is a continuous process consists of the following steps as shown in Fig. 1.

A. Problem Definition

It is the most important stage in the planning process. In fact, without a precise definition of the problem (s) to be addressed in the analysis, one should suspect that any meaningful results would ever be reach. Therefore, it is safe to say that more than 50% of efforts spent on the solution of a given problem may be associated with the detailed and comprehensive definition of that problem including the identification of its symptoms, causes and consequences.

B. Generation of Alternatives

This step involves the generation of possible alternative solutions to the defined problem (s). Each alternative should satisfy the previously stated objectives (at least partially) and should be "feasible" in the practical sense. There is no standard approach for generating alternative solutions. The quality of such alternatives will mainly depend upon the professional and practical experience of planners particularly in relation to the problem under consideration.

C. Building and Calibration of Transportation Models

This step can be performed parallel to the previous step (B). Transportation models are mathematical representation of the users travel behavioral that can simulate the exiting situation and can be used later in the prediction of the future situation under any alternatives that represent the changes in the socio-economic and future urban development for the city.

D. Application of the Calibrated Transportation Models and Analysis of Alternatives

After the generation of all alternatives and building and calibrating the suitable transportation models, the analysis of each alternative can be performed by applying the previous calibrated transportation models to the given alternative. This involves predicting the impacts of alternative solutions on the users, operators, owners, and regulators of the transportation system as well as on the society at large. In other words, this analysis should result in a clear description of the expected performance of each alternative previously generated. Such a description would include the expected level of service of different components of the system, resources consumed in producing these services measured in terms of operating, maintenance and investment costs for operators and owners, and environmental impacts.

E. Evaluation and Choice

Once the analysis of alternatives has been performed, it is essential to evaluate the characteristics of each alternative and consequently, to choose the "best" solution. This stage involves qualitative and quantitative valuation of the impacts of each alternative on the users, operators, owners, and regulators of the transportation system as well as on the society at large. The alternative which would have the highest positive impacts, the lowest negative impacts or some "optimal" combination of both would then have more chances of being chosen for implementation.

F. Implementation

Implementation is the real test of the chosen plan. Monitoring the stage of implementation should reveal the strengths and weaknesses of the plan, and hence, should assist planners to identify any unresolved or new problems. The process then continues to address these new set of problems, and so on.

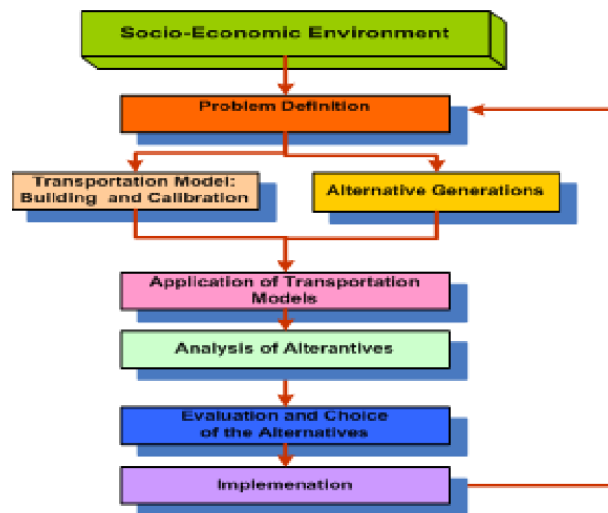


Fig. 1 The Transportation Planning Process

III. TRANSPORTATION SYSTEMS ANALYSIS

A. Basic Premises

As indicated earlier any transport system consists of several physical as well as human components which interact together in order to produce "transportation." The transport system itself is, again, one of several components in the socio-economic activity system of the society when all interact produce what we call the "development" of that society. Therefore, the analysis of transportation systems should be based on two basic premises:

- 1- The total transport system within a given socio-economic environment must be considered and viewed as a single multimodal system.
- 2- The interaction between the total transport system and the surrounding socio-economic activity system must be taken into account in the analysis.

B. Basic Variables and Relationships

Based on the above premises, the resultant effect of the interactions between the activity and transport systems is manifested in the flow pattern distributed on the different elements of the transport system. Therefore, we can define three basic variables for the analysis:

T : The Transport System,

A : The Activity System

F : The Flow Pattern

The interrelationships among these three variables are shown on Fig. 2 and may be described as follows:

Relationship I: T and A determines F

Relationship II: F cause changes over time in A (e.g., an increase of flow on a given route may induce more activities to shift along that route).

Relationship III: F causes changes over time in T (e.g., a congestion of flow may influence the decision to build a new road or to modify existing one).

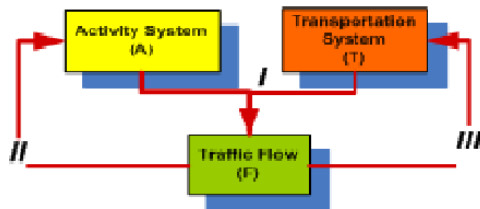


Fig. 2 Basic variables and relationships in transportation system analysis

In order to complete the description of the basic framework of the analysis it is essential to identify the major individuals, groups or institutions whose decision could influence and change any of the three basic variables of the analysis T , A and F . Five major groups can be identified: user, operators, owners, regulators and society at large (a brief description of each group has been introduced earlier). A very important set of institutions, that is the government, has not been defined as a separate group because any governmental institution should essentially belong to one or more of the above five groups.

C. Basic Issues

Having defined the basic framework of analysis let us turn our attention to the basic issues involved. The first basic issue is related to the available "options" through which the five major groups can influence the system. The second basic issue is concerned with predicting the possible "impacts" of these "options" on the same groups. Thus the analysis can be described by firstly identifying the available options, secondly identifying the possible impacts, and thirdly explaining the process of predicting the set of impacts resulting from a given set of options.

D. Options

Options or "decision variables" are those aspects of the transport and activity systems which can directly be changed by the decision(s) of one or more individuals or groups. It is, therefore, natural to divide the available options into two categories, the first includes those related to the transport system and the second includes those related to the activity system.

Transportation options:

Transportation options are those decisions which can mainly influence the transport system's performance by

changing aspects of the network, technology, operating policies, and/or institutional structures as follows:

- 1- The network may be influenced (by the owners) through the geometric and structural design of different links, intersections and terminals, the network topology and hierarchy, traffic signals, signs, markings, parking facilities, etc.
- 2- Technological options may include the use of electric or solar power for vehicles, the use of containers, the introduction of a new rapid transit system, etc.
- 3- Operating policies may include vehicle routing, scheduling, pricing, exit and entry regulations, financial regulations, laws, ordinances, etc.
- 4- Institutional options may include the number and types of institutions, the functions of different institutions, the domain of responsibilities, channels of communications, coordination, control, etc.

Activity options:

Activity options are those decisions which can mainly influence transport demand and which are, in general, not controlled by the decision makers of the transport system. People in the society have a wide range of options about how, when and where they would conduct their activities. Two types of decision should be considered:

1- Long term decisions:

- The location of residence
- Employment
- Scale and pattern of activities

These decisions determine the spatial distribution of socio-economic activities and land uses in a given area. Within this context, the actual transport demand will be influenced by

2- Short term decisions (travel options):

- Trip purpose
- Time of trips
- Frequency of trips
- Trip mode
- Trip route

E. Impacts

Impacts are those aspects of the transportation and activity systems that would be influenced by the implementation of alternative options and would consequently influence the five major groups of the system: users, operators, owners, regulators and the society at large.

In order to predict and evaluate these impacts it is useful to categorize them according to the affected groups:

1- User impacts

Users are mainly influenced by the level of service of the transport system, and hence, their impacts variables would include:

- Travel time
- Travel cost
- Safety

- Comfort

2- Operators impacts

Operators are mainly concerned with maximizing their share of demand and/or their net revenues, and hence, their impact variables would include:

- The costs of several resources consumed in the operation
- Maintenance and investment of their vehicle fleets
- Their shares of transport demands

3- Owners impacts

Owners are mainly interested in the usage and condition to their infrastructural elements, and hence their impacts would be reflected through the investment, maintenance and upgrading costs which are, in general, functions of the usage of the transport network.

4- Regulators impacts

Regulators are mainly interested in the safety aspects of operation.

5- Society impacts

The impacts on the society at large include the effects caused by the physical presence of transport facilities, such as:

- Noise
- Pollution
- Relocation of residents and activities.

6- Governmental impacts

Governmental impacts will depend upon whether the given government institution is a user, operator, owner, regulator or representing the society at large.

It should be obvious that the above impacts will, in general, have differential effects on different groups, or even subgroups of the above five major ones. That is, some will gain while others may lose.

F. Prediction Process

As indicated earlier, the core of the analysis is the process of predicting the “impacts” of alternative sets of “options”. A central aspect of this process is the prediction of an “equilibrium” flow pattern on the transport system resulting from a given set of options. Once this equilibrium flow is predicted, the different impacts variables can be estimated through a set of impact models. The prediction process may, then, be described as follows:

- 1-Specification of the transport system options T establishes performance functions, J . The function J indicates how the level of service S varies as a function of the transport options T and the volume of flow V . That is,

$$S = J(T, V)$$

- 2-Specification of the activity system options A establishes demand function, D . The function D indicates the

volume of flow V as a function of the activity options A and the level of service S , that is,

$$V = D(A, S)$$

- 3-The flow pattern F consists of the volume V and the level of service S on the different elements of the total transport system. That is,

$$F = (V, S)$$

- 4-For a given set of options T^* and A^* , the resulting equilibrium flow pattern, F^* , can be computed by solving the performance and demand function. That is,

$$\left. \begin{array}{l} S = j(T^*, V) \\ V = D(A^*, S) \end{array} \right\} F^* = (V^*, S^*)$$

More details about different approaches for urban transportation network equilibrium will be given in the Section V.

IV. THE METHODOLOGICAL FRAMEWORK FOR URBAN TRANSPORTATION SYSTEMS ANALYSIS

Fig. 3 shows the main components of the methodological framework for urban transportation systems analysis that we will described in this review paper and which depends on the transportation planning steps described in Section II. This framework can be described as follow:

A. Socio-Economic Environment Description

This part of the frameworks represents the diagnoses and the analysis of the socio-economic environment characteristics and factors of the urban area that should be considered in the analysis and developments of the transportation system. These factors may include the population distribution, income, car ownership, and land use of the study area. In this part, we can also define the transportation problems that should be addressed and clearly and specifically determined the symptoms, cause, and consequences for each problem in addition to the objectives and goals that should be satisfied by solving these problems. Then different alternative can be generated through different options of the five groups, users, operators, owners, regulator, and society at large as in next parts (B) and (C). A related data collection should be performed in this part. This is considered as step (A) in transportation planning steps in Section II.

B. Users Characteristics and Behavior

This part represents the transportation demand side where there are, for example, the following alternatives (options or decision variables):

- Residence Locations
- Economic& Social Activities Locations

- Frequency of trip
- Destination of Trip
- Trip Purpose
- Travel Mode
- Time of Trip
- Work Locations
- Trip Route

C. Owners, Operators and Regulators Behavior

This part represents the transportation system performance side where there are, for example, the following alternatives (options or decision variables):

- Network Configuration (e.g., links, nodes, intersection, traffic signals location, waiting areas)
- Investments and maintenance Policies for the transportation network and facilities
- Vehicle fleets characteristics
- Routing, scheduling and pricing policies
- Laws, regulations and controls
- Institutions

D. Transport Demand

As a result of a combination of alternatives (options) of users characteristics and behavioral in part (B), a set transport demand functions (in fact a set of transportation demand models) can be defined. These demand functions will be functions of user behavior and system's performance.

E. Transportation System Performance

As a result of a combination of alternatives (options) of owners, operators and regulators behavior in part (C), a set transport performance functions (in fact a set of transportation performance models) can be defined. These performance functions will be functions of transportation supply and transportation demand.

A combination of alternatives from part (B) and part (C) together is considered a complete scenario (alternative) that would developed (generated) to solve the problem under consideration in part (A). This is considered as step (B) in transportation planning steps in Section II.

Parts (D) and (E) required building transportation demand and performance models and calibrating these models using the data collected in part (A). This is considered as part of step (C) in transportation planning steps in Section II.

F. Transportation Network Equilibrium

To analysis any completed scenario (alternative) generated by part (B) and (C), an equilibrium between the built and calibrated transportation demand models ($V = D(A, S)$) and transportation performance models ($S = J(T, V)$) should be performed. The output of this equilibrium process is the traffic pattern ($F = (V, S)$) that can be used in estimate the impacts

of the given scenario on users, owners, operators, regulators, and society at large (see part (G) next). Part (D) represents, in general, the activity system A while part (E) represents, in general, the transportation system T that interacting together to produce the traffic pattern F . This represents **Relation I** in Fig. 2 This part represents part of step (C) in transportation planning steps of Section II.

G. Impacts on Users, Owners, Operators, Regulators, Government, and Society at large

The traffic pattern result from part (F) can be used in different impact models that should be developed for the five groups: users, owners, operators, regulators, and society at large (see Section III part (E)). This completes step (C) in transportation planning steps of Section II.

H. Evaluation and Choice

As a result of the impacts on the five groups in part (G), each alternative can be evaluated using specified evaluation criteria and the best alternative (scenario) should be chosen. (step (D) in transportation planning steps of Section II).

I. Implementation

This step is same as step (E) in transportation planning steps of Section II.

The traffic pattern F that used in the alternatives evaluation, choice and implementation will cause changes over time in activity system, A , through part (B), this represents **Relation II** in Fig. 2. This traffic pattern will also cause changes over time in transportation system, T , through part (C), this represents **Relation III** in Fig. 2.

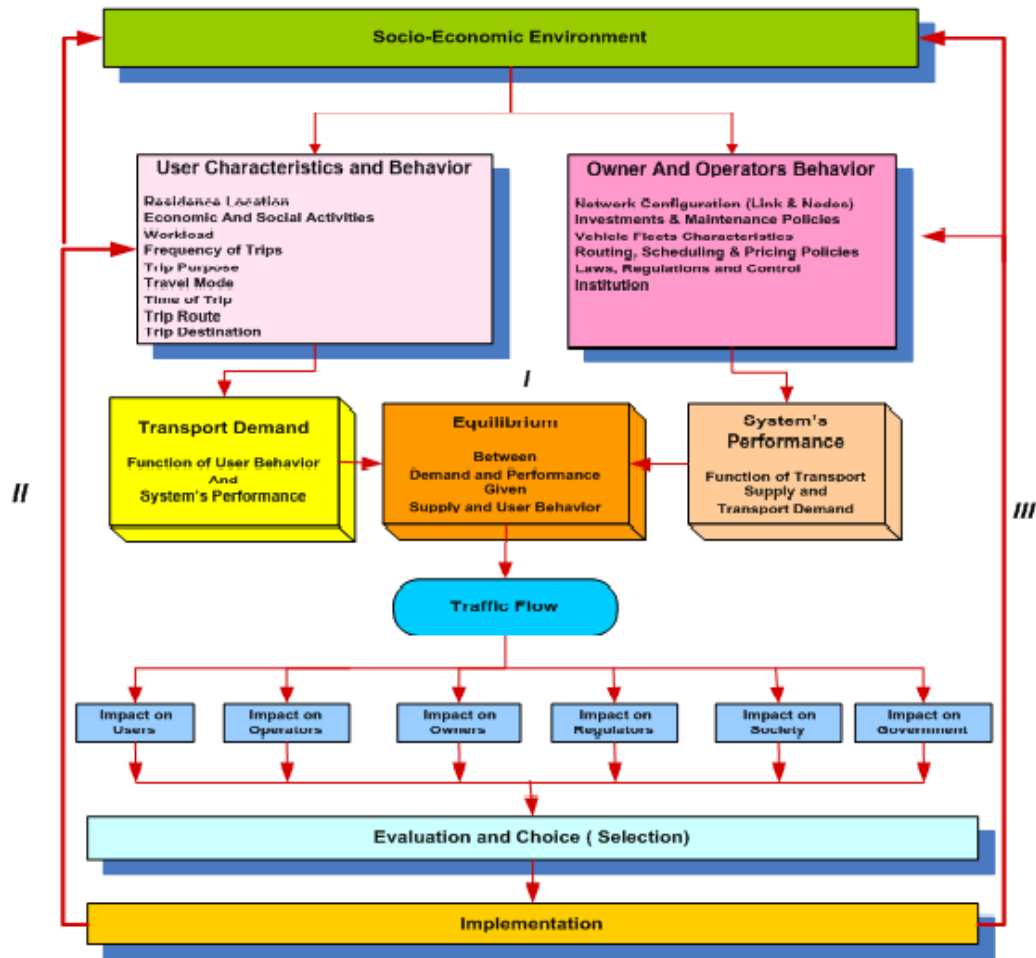


Fig. 3 A framework for urban transportation system analysis

V. URBAN TRANSPORTATION NETWORK EQUILIBRIUM MODELS

The main objective of the study of traffic network equilibria is the determination of traffic patterns characterized by the property that, once established, no user or potential user may decrease his travel cost or disutility by unilaterally altering his travel arrangements. This is known as the user-optimized problem governed by Wardrop's [1] equilibrium principle, which states that all utilized paths connecting an origin and destination pair incur the same travel cost which must be minimal among travel costs on all paths joining the pair.

Most of the literature on network equilibrium has concentrated on steady-state condition, although recently, attempts to understand dynamic network equilibrium have been made. The two most widely studied steady-state network equilibrium concepts are user equilibrium and spatial price equilibrium. The former is generally employed to model urban passenger networks and the latter to model inter regional commodity networks. We will consider the user equilibrium concept in this review paper.

In discussing this equilibrium concept we will use the following notation:

(N, A) , a directed graph (i.e., any transportation network) consisting of a set N of nodes and a set A of links;

i , an origin node in the set N ;

j , a destination in the set N ;

p , a simple (i.e., no node repeated) path in the network (N, A) ;

a , a link in the set A ;

I , set of origin nodes ($N \supseteq I$);

J , set of destination nodes ($N \supseteq J$);

m , a transportation mode;

M , set of transportation modes;

D_i , set of destination nodes that are accessible from a given origin i ($N \supseteq D_i$).

R , set of origin-destination (O-D) pairs;

P_{ij} , set of simple paths from origin i to destination j ;

P , set of simple paths in the network;

f_a , flow on link a ;

\mathbf{f} , vector of all link flows $\mathbf{f} = (\dots, f_a, \dots)$;

h_p , flow on path p

where $f_a = \sum \delta_{ap} h_p$

\mathbf{h} , vector of all path flows, $\mathbf{h} = (\dots, h_p, \dots)$;

$\delta_{ap} = \begin{cases} 1, & \text{if link } a \text{ belongs to path } p \\ 0, & \text{otherwise} \end{cases}$

$C_a(f)$, the average cost of transportation on link a ;

$\mathbf{C}(\mathbf{f})$, vector of all link costs;

$\mathbf{C}(\mathbf{f}) = (\dots, C_a(f), \dots)$;

$C_p(h)$, the average cost of transportation on path p where

$C_p = \sum \delta_{ap} C_a$

u_{ij} , minimum cost of transportation between origin i and destination j ;

\mathbf{u} , vector of minimum costs between all O-D pairs;

$\mathbf{u} = (\dots, u_{ij}, \dots)$;

$T_{ij}(\mathbf{u})$, the demand for transportation between origin i and destination j ;

\mathbf{T} , vector of demands between all O-D pairs;

$\mathbf{T} = (\dots, T_{ij}(\mathbf{u}), \dots)$

$u_{ij}(\mathbf{T})$, the inverse transportation demand function between origin i and destination j ;

$\mathbf{U}(\mathbf{T})$, vector inverse of inverse demand functions

between all O-D pairs, $\mathbf{U} = (\dots, u_{ij}(\mathbf{T}), \dots)$

S_i , accessibility variable that measure the expected maximum utility of travel from origin i

\mathbf{S} , vector of accessibility for all origins, $\mathbf{S} = (\dots, S_i, \dots)$.

Definition: User Equilibrium (UE)

A flow pattern satisfying the following conditions is user equilibrium:

$$h_p(C_p(h) - u_{ij}) = 0 \quad \forall p \in P_{ij}, \forall ij \in R \quad (1)$$

$$C_p(h) - u_{ij} \geq 0 \quad \forall p \in P_{ij}, \forall ij \in R \quad (2)$$

$$\sum_{p \in P_{ij}} h_p - T_{ij}(\mathbf{u}) = 0 \quad \forall ij \in R \quad (3)$$

$$f_a - \sum_p \delta_{ap} h_p = 0 \quad \forall a \in A \quad (4)$$

$$\mathbf{h} \geq 0 \quad (5)$$

$$\mathbf{T} \geq 0 \quad (6)$$

Expressions (1) and (2) above are readily recognized as equivalent to Wardrop's first principle; expression (3) is, of course, a statement of flow conservation; expression (4) is definitional; expressions (5) and (6) are non-negativity conditions.

A. Sequential Approach

Earlier procedures, which have been applied to hundreds of transportation studies throughout the world for the past 50 years and still are in use today, have viewed transportation planning as a sequential process, often with four stages: Trip Generation, Trip Distribution, Modal Split, and Trip Assignment (Detroit Metropolitan Area Traffic Study [2]; Chicago Urban Transport Study [3]; Cairo Urban Transportation Project [4]; Riyadh Development Authority [5]; United States Federal Highway Administration [6]; United States Urban Mass Transportation Administration [7]).

Unfortunately, the sequential approach has an inherent weakness in several respects:

- 1- The four steps correspond to a sequential decision process. For most trips, this is undoubtedly a highly unrealistic representation of travelers' decision making.
- 2- Its prediction need not be internally consistent. That is, because each step in this type of sequential planning depends upon the others, the performance and demand levels that one needs to assume as input at any step need not agree with those that one determines as outputs from the other steps. Also, the basis for forecasting travel choices, as defined in terms of variables and parameters, is inconsistent across the several models; for example, trip assignment is often based on travel times only, whereas mode split is based on a weighted combination of travel time and operating costs. (Tatineni *et al.* [8])
- 3- Evaluation of alternative transportation policies using the sequential modeling process is complex and time consuming. This does not allow producing results in the time frame of decision makers desire (Tatineni *et al.* [8])

B. Simultaneous (Combined) Approach: Single-Class Models

The above deficiencies have motivated attempt to predict all four steps simultaneously. Research intended to develop simultaneous models and related computational procedures for predicting short-run transport equilibria has proceeded in three directions. One line of investigation, Equivalent Optimization approach, has significant computational advantages; the others, the Variational Inequality and Stochastic Equilibrium approaches, permit richer modeling of user behavior. The first of such is the elastic demand traffic assignment problem which appeared in the work of Beckmann *et al.* [9]. In this problem the number of trips between each origin-destination pair is a function of the travel time between that pair. Beckmann's model was cast as an equivalent optimization problem that when solved yields the desired transport equilibrium solution. Beckmann *et al.* did not follow up on

their own accomplishment. Instead, the task of extending Beckmann's model to the multiple-class case was taken up by Dafermos [10]. From her thesis 1968 until her death in 1990, Dafermos, later in collaboration with Nagurney, established a wide-ranging theory of traffic equilibrium, including contributions to models with variable as well as fixed demand, treatment of multiple user classes and asymmetric cost functions, and perhaps most importantly extensions and applications of the theory of variational inequalities to transportation network equilibria.

Beckmann Traffic Assignment (UE-TA) Model:

The first of the simultaneous approaches, which originates with the early and seminal research of Beckmann *et al.* [9], views the equilibrium model as an Equivalent Optimization Problem (EOP) which when solved, yields the desired equilibrium solution. The most general form for this problem is given by:

$$\text{Minimize } \sum_a \int_0^{f_a} C_a(w)dw + \sum_i \sum_j \int_0^{T_{ij}} u_{ij}(w)dw$$

Subject to:

$$\sum_{p \in P_{ij}} h_p = T_{ij}(u_{ij}) \quad \forall ij \in R$$

$$f_a = \sum_p \delta_{ap} h_p \quad \forall a \in A$$

$$\mathbf{h} \geq \mathbf{0}$$

$$\mathbf{T} \geq \mathbf{0}$$

$$\text{where } C_p \begin{cases} = u_{ij} & \text{if } h_p > 0 \\ \geq u_{ij} & \text{if } h_p = 0 \end{cases}; \forall p \in P_{ij}, ij \in R,$$

provided that $\nabla \mathbf{C}(\mathbf{f})$ and $\nabla U(\mathbf{T})$ are symmetric matrices (where ∇ is the gradient operator). This formula is a mathematical program with a unique global minimum when $\nabla \mathbf{C}(\mathbf{f})$ and $-\nabla U(\mathbf{T})$ are positive definite (i.e., $\nabla \mathbf{C}(\mathbf{f})$ and $-\nabla U(\mathbf{T})$ are strictly monotone increasing). Actually, Beckmann's original formula dealt only with separable functions so that the symmetry restrictions necessary for writing down its EOP are satisfied trivially. Originally, Beckmann derived the ECP by first postulating its validity and then showing that the associated Kuhn-Tucker conditions are identical to the equilibrium conditions. Since the above EOP is a convex optimization problem (assuming monotonicity of demand and performance), it can be solved efficiently by any of several convergent algorithms (e.g., Bruynooghe *et al.* [11]; Bertsekas and Gafni [12]; Leblanc [13]; Nguyen [14], [15], [16]; Golden [17]; and Florian and Nguyen [18]). Florian [19] provides a comprehensive review of algorithms for this problem class and their application to traffic equilibrium problems. The main disadvantage of this equivalent

optimization formulation is behavioral. It requires strong modeling assumptions that frequently are unrealistic. In particular, it assumes that the demand between each origin-destination (O-D) pair depends solely upon the performance between that O-D pair.

Evans [20] examined how to combine trip distribution and traffic assignment models in a single formulation. She succeeded to formulate one version of the combined model as an optimization problem with a convex objective function consisting of two parts, one related to route choice, similar to the objective function in Beckmann's formulation, and the other related to travel demand or trip distribution. In the direct application of the iterative solution method of Frank and Wolfe [21], each iteration solves a subproblem created by full linearization of the objective function around the current solution, and an averaging of the subproblem with current solution. Evans proposed a partial linearization algorithm, in which only auto route choice part of the objective function is linearized in the subproblem.

Evans Trip Distribution and Traffic Assignment (UE-TD/TA) Combined Model:

The following summaries are quoted from Miller [22] with some modifications to the notations that we described earlier

Model Assumptions

- i) one mode;
 - ii) separable cost functions $C_a = C_a(f_a)$;
 - iii) non-negative cost functions $f_a \geq 0 \Rightarrow C_a(f_a)$;
 - iv) increasing cost functions $\frac{\partial C_a(f_a)}{\partial f_a} > 0$;
 - v) total outflows from origins O_i and total inflows to destinations D_j fixed and exogenous;
 - vi) TD component is a separable demand function in the form of a spatial interaction ("gravity") type function with an exponential cost function $T_{ij} \propto \exp\left(\frac{-u_{ij}}{\gamma_{ij}}\right)$;
- where γ_{ij} a parameter that can be obtain by a calibration process.

Model Optimization Problem

$$\text{Minimize } \sum_a \int_0^{f_a} C_a(w)dw + \sum_i \sum_j \gamma_{ij} T_{ij} [\ln(T_{ij}) - 1]$$

Subject to:

$$\sum_{p \in P_{ij}} h_p = T_{ij}(\mathbf{u}) \quad \forall ij \in R \quad (7)$$

$$f_a = \sum_p \delta_{ap} h_p \quad \forall a \in A \quad (8)$$

$$\sum_j T_{ij} = O_i \quad \forall i \in I \quad (9)$$

$$\sum_i T_{ij} = D_j \quad \forall j \in J \quad (10)$$

$$\mathbf{h} \geq \mathbf{0} \quad (11)$$

$$\mathbf{T} \geq \mathbf{0} \quad (12)$$

Model structure

The Evans [20] TD/TA model extends the UE-NA model to include a TD component. O-D flows are influenced by the minimum route cost between each pair through a spatial interaction or “gravity”-type TD component. This TD component is not explicit in the equivalent optimization problem but rather is implied by the optimality conditions for that problem. Evans [20] main contribution was to combine foundational work by Wilson [23] and [24] on “entropy-maximizing,” doubly constrained spatial interaction models within the UE-TA optimization problem developed by Beckmann *et al.* [9]. The joint optimization problem combines in a consistent manner the flow-related costs associated with the network equilibrium, a TD based on the route costs and the TD’s influence on the network flow levels. Evans [20] UE-TD/TA model equilibrium requires solving a constrained minimization problem similar to the UE-TA problem. The objective function consists of two components:

- 1- a arc-flow cost component equivalent to the TA objective function;
- 2- an entropy term that corresponds to the trip distribution model.

The decision variables to be solved when minimizing this function are the flow levels on each arc and the aggregate flows between each O-D pair. The TD term of the objective function allocates flows according to entropy-maximizing principles. In brief, this requires the flow pattern to be the most likely or highest probability pattern consistent with known aggregate information about the system (see Fotheringham and O’Kelly [25]; Stuart *et al.* [26]). In this case, the known information include:

- 1- total outflows from each origin O_i ;
- 2- total inflows to each destination D_j ;
- 3- the minimum travel costs between each O-D pair.
- 4- The flow variable values that minimize the TD component

of the objective function generate the most likely TD pattern given this information.

Constraints on the Evans [20] minimization program generally correspond to standard flow totaling and non-negativity conditions. These include:

- 1- flows on all routes between an O-D pair must sum to the total flow between that pair equation (7);
- 2- flows on all routes that use an arc must sum to the total flow on that arc equation (8);
- 3- outflows from each origin to all destination must sum to the known outflows from that origin equation (9);

- 4- the flows entering each destination from all origins must sum to the known total inflows to that destination equation (10);

- 5- all path flows and aggregate O-D flows must be non-negative equations (11) and (12).

Evans [20] provides a rigorous proof that the TD/TA objective function is convex and therefore has a unique minimum. From an intuitive perspective, we can note that the TA component’s convexity is ensured by the same arc cost function assumptions as in the TA optimization problem (separable, non-negative and increasing). Also, the TD component is a convex function. Since the sum of two convex functions is also convex, we know the overall objective function is convex.

Boyce initiated an implementation of the Evans formulation and algorithm for the Chicago region in 1979. During the next 15 years he and his students, initially in the collaboration with LeBlanc, implemented a single-class, two-mode combined model with 389 zones and about 3000 road links. Model parameters were first borrowed from other studies, but later estimated in a way that is self-consistent with the model solution (Boyce [27]; Boyce *et al.* [28]; and Boyce and Zhang [29]). A textbook review paper synthesizing these developments was offered by Boyce and Daskin [30].

The Centre for Research on Transportation at the University of Montreal, founded in 1972, embarked early on both theoretical and model implementation and testing. Initially led by Florian, successive generation of faculty and students at Montreal have made sustained contributions to transportation network modelling of several types, including traffic equilibrium.

Contributions toward solving the general problem of transportation network equilibrium with variable demand, including mode choice, were made by Florian and Nguyen during the 1970’s. Florian [31] developed a two mode (private car and transit) network equilibrium model where the most important features are the distinction between the flow of vehicles and flow of transit passengers and the means of modeling the interaction between both types of vehicles that use same road links of the network. In this case, non-separable demand functions are used to more realistically capture demand-side interdependence. The equilibrium is found by solving a sequence of problems like the one proposed by Beckman *et al.* for one mode (car), while parametrically varying the equilibrium travel costs of the other mode (transit) whose assignment is determined by an all-or-nothing technique.

Using the fact that an entropy distribution model implies a Logit modal split model, Florian and Nguyen [32] extended the formulation to include modal split. They formulated a combined trip distribution, modal split and trip assignment model, considering two (car and transit) independent modes, which results quit unrealistic for modeling systems where car and transit vehicles share the same road infrastructure. Their model formulation can be cast as a convex optimization problem which can be solved using Frank-Wolfe algorithm

(Frank and Wolfe [21]) or the partial linearization algorithm proposed by Evans [20].

Florian and Nguyen Trip Distribution, Mode Split and Trip Assignment (UE-TD/MS/TA) Combined Model:

The following summaries are quoted from Miller [22] with some modifications to the notations that we described earlier

Assumptions

- i) two modes, automobile (au) and public transit (tr);
- ii) separable cost functions for automobile mode, $C_a^{au} = C_a^{au}(f_a^{au})$
- iii) non-negative cost functions for automobile mode, $f_a^{au} \geq 0 \Rightarrow C_a^{au}(f_a^{au})$;
- iv) increasing cost functions for automobile mode, $\frac{\partial C_a^{au}(f_a^{au})}{\partial f_a^{au}} \succ 0$;
- v) public transit arc costs are fixed and exogenous;
- vi) TD component is a separable demand function in the form of a spatial interaction ("gravity") type function with an exponential cost function $T_{ij}^{au} \propto \exp(\frac{-u_{ij}^{au}}{\gamma})$ and

$$T_{ij}^{tr} \propto \exp(\frac{-u_{ij}^{tr}}{\gamma});$$

- vii) MS component is a binomial Logit model

$$prob(au) = \frac{\exp(\frac{-u_{ij}^{au}}{\gamma})}{\exp(\frac{-u_{ij}^{au}}{\gamma}) + \exp(\frac{-u_{ij}^{tr}}{\gamma})}$$

where $prob(au)$ is the probability of choosing automobile mode.

Model Optimization Problem

$$\begin{aligned} \text{Minimize } & \gamma \sum_i \sum_j T_{ij}^{au} \ln(T_{ij}^{au}) + \sum_i \sum_j T_{ij}^{tr} [\gamma \ln(T_{ij}^{tr}) + u_{ij}^{tr}] \\ & + \sum_a \int_0^{f_a} C_a(w) dw \end{aligned}$$

Subject to:

$$\sum_j (T_{ij}^{au} + T_{ij}^{tr}) = O_i \quad \forall i \in I \quad (13)$$

$$\sum_i (T_{ij}^{au} + T_{ij}^{tr}) = D_j \quad \forall j \in J \quad (14)$$

$$\sum_{p \in P_{ij}} h_p^{au} = T_{ij}^{au} \quad \forall ij \in R \quad (15)$$

$$f_a = \sum_p \delta_{ap} h_p^{au} + f_a^{tr} \quad \forall a \in A \quad (16)$$

$$\mathbf{h} \geq \mathbf{0} \quad (17)$$

$$\mathbf{T} \geq \mathbf{0} \quad (18)$$

Model structure

Florian and Nguyen [32] combine the UE-TA model with a combined entropy-maximizing TD/MS component (see Ortuzar and Willumsen [33]). The TD/MS component combines a binomial Logit model (MS) with a doubly constrained spatial interaction model (TD). The two models share the same parameter to control the cost function effect in the spatial interaction model as well as the modal split dispersion.

The objective function in the Florian and Nguyen [32] consists of three components

- i) an entropy component that determines TD and MS for the automobile mode;
- ii) a modified entropy component that determined TD and MS for the public transit mode; and,
- iii) the standard UE-TA cost component. The modification of the public transit entropy component accounts for the fixed travel costs assumed for that mode. Components i) and ii) together comprise the combined TD/MS for both modes. The decisions variables to be determined when minimizing this objective function include: i) the aggregate travel demand for the automobile mode between each O-D pair; ii) the aggregate travel demand for the public transit mode between each O-D pair; iii) the route flows for the automobile mode; and,
- iv) the arc flows for the public transit mode.

Constraints on the Florian and Nguyen [32] TD/MS/TA model comprise the standard aggregate travel demand constraints, albeit modified to account for the particulars of their "two modes with fixed costs for one mode" model. These constraints include: i) flows for both modes leaving an origin must sum to the known (exogenous) total outflow from that origin (13); ii) flows for both modes entering a destination must sum to the known (exogenous) total inflow to that destination (14); iii) route flows for the automobile mode between an O-D pair must sum to the aggregate automobile travel demand for that O-D pair (15); iv) the total flow on an arc is equal to the automobile flows on routes that use that arc plus the public transit flow contribution to that arc (this latter quantity may be zero if routes are separated) (16), and; v) aggregate travel demands and route flows for both modes must be non-negative (17), (18).

The Florian and Nguyen [32] model offers some practical advantages with respect to parameter estimation and computational tractability. However, these advantages require some theoretical costs, particularly with respect to assumptions regarding mode behavior and modal interactions. First, note that the model only allows two modes; this can be a drawback when analyzing travel demand in large urban areas

with multiple modes. Second, note that travel costs (including travel time) for public transit are fixed, meaning that these costs are not affected by congestion. Thus, the model assumes that public transit travel times remain constant even when the network is highly congested. This is not a problem if the public transit mode is separate from the automobile network (e.g., subways) but can be a problem when public transit shares the automobile network. This problem is mitigated to some degree if the public transit schedules are accurate reflections of actual travel times, although these schedules may become less accurate when forecasting more congested conditions in the future. Also note that although public transit is not affected by congestion, public transit can affect automobile congestion. The total flow on an arc is comprised of the automobile flow plus any contribution made by public transit; this can be modified by flow equivalency factors (16).

Safwat and Magnanti [34] further enriched the behavioral features of the equivalent optimization approach to include trip generation. In their model (i.e., the Simultaneous Transportation Equilibrium Model, STEM) trip generation can depend upon the system's performance through an accessibility measure that is based on the random utility theory of users' behavior (instead of being fixed), and trip distribution is given by a more flexible Logit model based on the random utility theory (instead of being given by a less flexible entropy model). In practice, STEM has been applied to a few real-world transportation systems. Earlier applications covered intercity passenger travel in Egypt (Safwat [35] and [36]) and the urban transportation network of Austin, Texas, in the United States (Safwat and Walton [37]). More recently the model was applied to the urban transportation networks of Riyadh, Saudi Arabia (Hasan and Al-Gadhi [38]), Tyler, Texas, (Hasan and Safwat [39]), ESCWA countries integrated transport system (Safwat and Hasan [40]).

Safwat and Magnanti Trip Generation, Trip Distribution, Mode Split and Trip Assignment (TG/TD/MS/TA) Combined Model (STEM):

The following summaries are quoted from Miller [22] with some modifications to the notations that we described earlier

Model Assumptions

- i) a separate subnetwork represents each transportation mode in the study area

$$N = \bigcup_{m=1}^M N^m, A = \bigcup_{m=1}^M A^m$$

where N^m, A^m are the nodes and arcs of the network for mode m ;

- ii) separable cost functions $C_a^m = C_a^m(f_a^m) \quad \forall m \in M$;
 iii) non-negative cost functions
 iv)

$$f_a^m \geq 0 \Rightarrow C_a^m(f_a^m) \quad \forall m \in M;$$

increasing cost functions

$$\frac{\partial C_a^m(f_a^m)}{\partial f_a^m} > 0 \quad \forall m \in M;$$

- v) TD component is in the format of a Logit model whose utility function consists of the minimum travel cost between the O-D pair u_{ij} and a non-transportation-related destination attractiveness measure A_j and it given by

$$T_{ij} = G_i \frac{\exp(-\theta u_{ij} + A_j)}{\sum_{k \in D_i} \exp(-\theta u_{ik} + A_k)} \quad (19)$$

- vi) TG is a linear function of each origin's accessibility to destinations and other, non-transportation relative "propulsiveness" factors E_i and it is given by

$$G_i = \alpha_i S_i + E_i \quad (20)$$

where the accessibility S_i is given by

$$S_i = \max \{0, \ln(\sum_{j \in D_i} \exp(-\theta u_{ij} + A_j))\} \quad (21)$$

Model Optimization Problem

$$\begin{aligned} \text{Minimize } & \frac{1}{\theta} \sum_{i \in I} [\frac{\alpha_i}{2} S_i^2 + \alpha_i S_i - (\alpha_i S_i + E_i) \ln(\alpha_i S_i + E_i)] \\ & + \frac{1}{\theta} \sum_i \sum_j T_{ij} \ln(T_{ij}) - A_j T_{ij} - T_{ij}] + \sum_a \int_0^{f_a} C_a(w) dw \end{aligned}$$

Subject to:

$$\sum_{j \in D_i} T_{ij} = \alpha_i S_i + E_i \quad \forall i \in I \quad (22)$$

$$\sum_{p \in P_{ij}} h_p = T_{ij} \quad \forall ij \in R \quad (23)$$

$$f_a = \sum_p \delta_{ap} h_p \quad \forall a \in A \quad (24)$$

$$\mathbf{S} \geq \mathbf{0} \quad (25)$$

$$\mathbf{T} \geq \mathbf{0} \quad (26)$$

$$\mathbf{h} \geq \mathbf{0} \quad (27)$$

Model structure

The simultaneous transportation equilibrium model (STEM) encompasses all four components of a travel demand analysis (Safwat [35] and [36]; Safwat and Magnanti [34]; Safwat and Walton [37], Safwat and Hasan [40], Hasan and Safwat [39], Safwat and Hasan [41]). The STEM objective function combines the UE- TA component with two entropy components, specifically a TD and TG component. STEM incorporates MS by assuming that separate subnetworks represent each mode in the study area. Therefore, the UE

paths though the overall multimodal network are simultaneous MS/TA for travelers. An advantage of this approach is it can accommodate mixed-mode trips, e.g., “park and ride” transit situations.

STEM formulates the TG and TD components through a random utility decision process at the individual traveler level. The observed utility component consists of two variables:

1- the minimum average travel cost between the O-D pair u_{ij} and,

2- a composite variable reflecting the non transportation-related attractiveness of that destination A_j . The destination attractiveness composite variable is exogenous; this can be the result of an external, separate model (e.g., a regression analysis of inflows against variables such as the amount of retail or office space). The travel cost variable has an associated negative parameter to reflect the disutility of travel. The unobserved or random utility component is assumed to have a “type I extreme value distribution,” in other words, the typical error assumption used to derive a Logit choice model. Some additional comments regarding this assumption are below. The TG component generates flow from origins based on two factors:

1- a composite variable that takes into account non-transportation-related factors on origin outflows E_i and,

2- the accessibility provided to that origin by the transportation system S_i . Similar to the destination attractiveness composite variable A_j , the origin composite variable E_i is exogenous and can result from an external model (e.g., a regression model of the observed trips against residential population density in the particular origin). The second TG component measures the “accessibility” as the *expected maximum utility* of that origin. The “expected maximum utility” measures the benefit of travel from the origin assuming random utility-maximizing decisions. The accessibility variable can assume any positive or negative value; however, the STEM equivalent optimization program includes a constraint that requires this variable to assume non-negative values (25)-(27) since negative accessibility (and negative origin outflows) are nonsensical.

The first component of the STEM equivalent optimization program’s objective function reflects the TG theoretical basis at the aggregate level. The TD component uses the utility function to distribute flows generated from an origin among the destinations. Logit model-generated destination choice probabilities are multiplied by the number of travelers leaving each origin to estimate the flow from the origin to each destination (19). The second component of the STEM objective function generates entropy-maximizing O-D flow estimates consistent with the Logit TD model.

The main strengths of Logit-based foundation of the TG and TD STEM components are its robustness and tractability. With respect to robustness, Safwat and Magnanti [34]

demonstrate that STEM can approximate any doubly constrained spatial interaction model with fixed and known origin outflows and destination inflows. This occurs by defining the origin propulsiveness variable E_i and the destination attractiveness variable A_j as functions of the known outflows and inflows (respectively) and by restricting certain STEM parameter values (see Safwat and Magnanti [34] Appendix B). Thus, STEM can accommodate a wide range of data for defining factors that affect TG and TD. This can allow the model to adapt to changes in available data and relevant policy variables. With respect to tractability, the Logit choice model only requires very basic calculations and therefore can be applied to very large choice problems without undue computational burden.

In his 1991 doctoral dissertation, Hasan [42] addressed the trade-offs between computational and the behavioral aspects of modeling and predicting short-run transportation equilibrium on large-scale real-world networks by performing a formal comparison between the variational inequality, equivalent optimization, and traditional (sequential) approaches to the problem. He generalized the STEM to the Generalized Simultaneous Transportation Equilibrium Model (GSTEM) that explicitly combines trip generation, trip distribution, modal split and traffic assignment for a general class of behaviorally sound demand models, and general asymmetric cost functions and can be cast as a Variational Inequality (VI). Implementation programs for comparative analysis of computational and behavioral issues had developed for the Tyler, Texas urban transportation network. Safwat and Hasan [41] investigate the relative computational efficiency of LDT algorithm as a function of demand, performance, and network parameters. Hasan [42] used the relaxation (diagonalization) algorithm to solve the VI of GSTEM where at each iteration of the algorithm a subproblem can be solved by the LDT algorithm.

Fernandez et al. [43] proposed a mathematical formulation of a supply-demand network equilibrium model with sequential (hierarchical) rather than simultaneous destination and mode choices. In this paper the authors present several approaches for formulating network equilibrium models with combined modes. One of these formulations considers a nested demand structure to model mode choice (car or car/metro) and the transfer point choice, which allows the calibration of different parameters for the transfer point and mode choice functions.

Abrahamsson and Lundqvist [44] developed nested combined models for trip destination, mode and route choices and implemented these models in the context of the Stockholm region. They consider a simple problem where the transit network and the road network are independent and no congestion effects exist over the transit network, whose travel impedance can be exogenously determined. They proposed three different models: the traditional nested (distribution, modal split and assignment), the reverse nested (modal split,

distribution and assignment) and the simultaneous (trip distribution and modal split and assignment).

Each of the above proposed models concerned a single-class model, in the sense that all travelers by purpose or socioeconomic group are represented as one homogeneous group.

C. Simultaneous (Combined) Approach: Multiclass Models

Simple travel forecasting models, as mentioned above, assume that all travelers are similar in their travel-decision characteristics, such as their money-value of the time and their sensitivity to travel times in choosing their origin, destination and mode of travel, etc. To obtain more realistic models, travelers are often divided into classes, either by socio-economic attributes or by the purpose of their travel (work, shop, etc.), assuming that travel-decision characteristics are the same within each class, but differ among classes.

Notation

Let $G = (N, A)$ be a multimodal network consisting of a set of N nodes and a set of A links that can represent any mode of transport m in an urban area. These modes can be grouped into different nests n that could be multiple pure and combined (combination of pure) modes. A typical user of class l with trip purpose o traveling from a given origin i at a specific departure time period t to any destination j that is accessible from i can use any of these modes for his journey. We will use the following notation for the multiclass models:

$G = (N, A)$ A multimodal network consisting of a set of N nodes and a set of A links
 l = User class (e.g., income level, car availability, etc.)
 L = Set of all user classes
 o = Trip purpose (e.g., home-based-work, home-based-shopping, etc.)
 O = Set of all trip purpose
 I^{lo} = Set of origin nodes for user class l and trip purpose o
 i = An origin node in the set I^{lo} for user class l with trip purpose o
 D_i^{lo} = Set of destination nodes that are accessible from a given origin i for user class l with trip purpose o
 j = A destination node in the set D_i^{lo} for user class l with trip purpose o
 R^{lo} = Set of origin-destination pairs ij for user class l with trip purpose o , i.e., the set of all origins $i \in I^{lo}$ and destinations $j \in D_i^{lo}$
 m = Any transportation mode in the urban area
 n = Nest of transportation modes m that has a specific characteristics (e.g., pure modes including private and public or combined modes) that are available for user

class l with trip purpose o travel between origin-destination pairs ij

Λ_{ij}^{lo} = Set of all nests of modes n that are available for user class l with trip purpose o travel between origin-destination pairs ij

M_n^{lo} = Set of all transportation modes m in the nest n for user class l with trip purpose o travel between origin-destination pairs ij

t = Departure time period for user class l with trip purpose o using mode m in the nest n to travel between origin-destination pairs ij

K_m^{lo} = Time horizon of the departure time periods t for users of class l with trip purpose o using mode m between origin-destination pairs ij

p = A simple (i.e., no node repeated) multimodal path (i.e., it may include links with combined modes m) in the multimodal network (N, A)

P_{ij}^{lonmt} = Set of simple paths for travel from the origin node i to destination node j in the multimodal network (N, A) for users of class l with trip purpose o depart at time $t \in K_m^{lo}$ using mode $m \in M_n^{lo}$ from the nest of modes $n \in \Lambda_{ij}^{lo}$.

a = A link in the set A in the multimodal

u_{ij}^{lonmt} = the perceived minimum (generalized) cost of travel for users of class l with trip purpose o depart at time t using mode m in the nest n from the origin node i to destination node j in the set D_i^{lo} ,

S_i^{lo} = the accessibility of origin $i \in I^{lo}$ as perceived from user of class l with trip purpose o traveling from that origin

G_i^{lo} = the number of trips generated from origin i for users of class l with trip purpose

A_{wj}^{lo} = the value of the w^{th} socio-economic variable that influences trip attraction at destination j for users of class l with trip purpose o ,

$g_w^{lo}(A_{wj}^{lo})$ = a given function specifying how the w^{th} socio-economic variable A_{wj}^{lo} influences trip attraction at destination j for users of class l with trip purpose o , and

A_j^{lo} = a composite measure of the effect that socio-economic variables, which, are exogenous to the transport system, have on trip attraction at destination j for users of class l with trip purpose o .

θ_i^{lo} and θ_{iw}^{lo} for $w=1, 2, \dots, W$ are coefficients to be estimated, where $\theta_i^{lo} > 0$.

E_{oi}^{lo} = the value of the ω^{th} socio-economic variable that influences the number of trips generated from origin i for users of class l with trip purpose o ,

$q_{\omega}(E_{oi}^{lo})$ = a given function specifying how the ω^{th} socio-economic variable, E_{oi}^{lo} , influences the number of trips generated from origin i for users of class l with trip purpose o , and

E_i^{lo} = a composite measure of the effect the socio-economic variables, which are exogenous to the transport system, have on the number of trips generated from origin i for users of class l with trip purpose o

α^{lo} and α_{ω}^{lo} for $\omega = 1, 2, \dots, \Omega$ are coefficients to be estimated $\forall l \in L, \forall o \in O$

T_{ij}^{lonmt} = the number trips of users of class l with trip purpose o traveling from the origin node $i \in I^{lo}$ to the destination node $j \in D_i^{lo}$ and whose already chose the mode of transport $m \in M_n^{lo}$ from the nest of modes $n \in \Lambda_{ij}^{lo}$ and start their trip at the time $t \in K_m^{lo}$

T_{ij}^{lonm} = the number trips of users of class l with trip purpose o traveling from the origin node $i \in I^{lo}$ to the destination node $j \in D_i^{lo}$ and whose already chose the mode of transport $m \in M_n^{lo}$ from the nest of modes $n \in \Lambda_{ij}^{lo}$.

T_{ij}^{lon} = the number trips of users of class l with trip purpose o traveling from the origin node $i \in I^{lo}$ to the destination node $j \in D_i^{lo}$ and whose already chose the nest of modes $n \in \Lambda_{ij}^{lo}$.

T_{ij}^{lo} = the number trips of users of class l with trip purpose o traveling from the origin node $i \in I^{lo}$ to the destination node $j \in D_i^{lo}$.

The introduction of multiple classes increases the mathematical complexity of travel forecasting models (see Boyce and Bar-Gera [45], De Cea et al. [46] for the most recent multiclass combined models reviews that will be mostly summarized here). Travel costs in single class models are often separable and symmetric, allowing for convex optimization formulation. In multiclass models travel costs of

one class are affected by decisions of other classes; hence the cost structure is not separable, and in general it is not symmetric and does not allow a convex optimization formulation (Altman and Wynter [47], Patriksson [48]).

In 1986 researchers in Chile began to implement multiclass combined models emphasizing route choices in a congested transit network with several combinations of transit modes, as found in Santiago (De Cea et al. [46]). This research led to the development of ESTRAUS and related software, which has been extensively applied to Santiago as well as other Chilean cities. ESTRAUS is currently developed and distributed by Modelos Computacionales de Transporte Limitada. Florian et al. [49] proposed a variant of ESTRAUS intended to be more efficient computationally.

In 1997, Boyce and Bar-Gera [50] and [51], with several collaborators, set out to implement, estimate and validate a multiclass, multimodal combined model at the same level of detail used by transportation planning professional in the Chicago region. The result of this research effort was a three-class model, with provision for expansion to ten classes, estimated on a 1990 household travel survey, and validated on the 1990 Census travel-to-work survey. The model was solved by a generalized Evans algorithm.

Lam and Huang [52], [53] and [54] were the first to describe an optimization formulation for the multiclass version, which was implemented for Hong Kong. Lam and Huang [52] offered a classification of multiclass models, in part based on Abdulaal and LeBlanc [55] and LeBlanc and Abdulaal [56]. The following classification corresponds to their classes plus an extension to consider types of classes other than mode-based classes:

Case 1: An O-D matrix is available for each mode. The objective is to obtain the user-optimal (UO) route and link flows, where the link costs are flow-dependent, that is the link costs depend on the flow of each mode.

Case 2: The total O-D matrix is known, together with a function of the modal travel costs for allocating each O-D flow to the modes. As above, the objective is to obtain the UO route and link flow, where the link costs are flow-dependent, and there can be mode switching through the mode choice function. As a special case, the mode choice function may allocate all flow for an O-D pair to the lowest cost mode, resulting in UO mode and route flows.

Case 3: Only the originating and terminating flows for each mode are known, but not the mode's O-D matrix. The objective is to find the modal O-D matrices, such that the route and link flows are UO, where the link costs are flow-dependent.

Case 4: The total originating and terminating flows are known, but not the total O-D matrix, or the O-D matrix by mode. As above, the problem is to find the O-D-mode matrices and the UO route and link flows, where the link costs are flow-dependent.

Case 5: Finally as an extension of the above case, the total originating and terminating flows are known by classes, such as trip purpose or socio-economic group, but not by mode. As in case 4, the problem is to determine the O-D-mode flows by class, as well as the UO route and link flows.

Case 1 corresponds to the models studies by Dafermos [10], as well as van Vliet *et al.* [57]. Dafermos identified the integrality condition on the objective function that the effects of the classes on each other must be symmetric. For certain formulations, such as autos and buses on the same link, this condition appears to be unrealistic.

Case 2 was examined by Florian [31] and Abdulaal and LeBlanc [55], as well as LeBlanc and Farhangian [58] found an equivalent optimization formulation that avoids the symmetry restriction on the link cost functions. However, the resulting mathematical model is not convex and appears to require route enumeration for its solution.

Lam and Huang [52], [53], and [54] consider case 3 where they consider private auto, truck and franchised bus as the modal-based user classes. They attribute the generalized cost function to van Vliet *et al.* [57].

The models of Boyce and his collaborators up to 1997, as well as the models proposed by Evans [20] and Florian and Nguyen [32], belong to case 4, with the simplification that the mode flows occur on separate networks defined for each mode. That is, there is no interaction among the mode flows at the route or link level.

The models of De Cea and Fernandez [59] and Florian *et al.* [49] belong to case 5. The model of Boyce and Bar-Gera [51] also belong to case 5, with the restriction that auto and transit flows occur on separate networks.

The STEM model developed by Safwat and Magnanti [34] can be considered as a special case between case 4 and case 5 where the problem is to find the originating flow, the O-D-mode matrices, and the UO route and link flows, where link costs are flow independent.

Boyce and Bar-Gera [45] presented a good detailed comparison of the implementation of the following four multiclass combined models:

1- Lam and Huang were evidently the first to describe in the literature a multiclass model with origin-destination as well as route choices. As just noted, their model defines classes in terms of modes, and does not include a mode choice function. Accordingly, origins and destinations must be mode-specific, a serious limitation for a model intended for travel forecasting practice. Autos, trucks and buses do interact on the road network, albeit in a somewhat limited manner: bus flows are pre-specified by link and trucks are evidently converted to auto equivalents. This model was implemented for a portion of the Hong Kong region, and the results compared with sequential travel forecasting procedure.

2- De Cea and Fernandez [59] and de Cea *et al.* [46] describe the formulation and solution of ESTRAS, in many respects the most detailed multiclass, combined model implemented to date. The model is formulated as a variation

inequality problem with capacity constraints for all public transit modes, an important consideration for modeling congested systems. Interaction of autos, taxis and buses on the road network are explicitly considered. In addition, metro and exclusive bus lanes are modeled as an independent network. The implementation for Santiago has 13 user classes, 3 trip purposes, 7 pure transit modes and 4 combined modes. The nested Logit structure of the O-D-mode choice model has three levels of choice, perhaps the most detailed attempted in an equilibrium framework.

3- Boyce and Bar-Gera [50] and [51], with several collaborators, implemented, estimated and validated a detailed model for the Chicago region, that is simpler in structure, but is the largest multiclass, combined model solved to date in terms of the number of zones (1790) and road network size (12,092 nodes; 39,018 links). The O-D-mode choice model is implemented for two user classes; a fixed truck matrix is also assigned to the road network.

4- A related model was implemented by Marshall and Boyce (Resource Systems Group [60]). This model has four user classes plus trucks. The O-D choice model is novel in the use of a compound deterrence function consisting of a negative exponential function times a negative power function, both depending on auto generalized travel cost. Mode choices depend on auto vs. transit generalized costs; transit cost is defined as the best transit sub mode cost at the O-D level of detail. The solution method uses the method of successive averages, since no objective function corresponding to the O-D choice function is variable; however, it is based on the Evans algorithm.

All of the above four models reviewed are solved with algorithms based on partial linearization method proposed by Evans [20]. Parameters of three of the models were estimated from travel survey data and validated in various ways. Taken together, they represent the state of the art of multiclass, combined models intended for travel forecasting practice.

De Cea and Fernandez Trip Distribution, Mode Split and Trip Assignment (TD/MS/DT/TA) Multiclass Combined Model (ESTRAS):

Model Assumptions and Structure

- 1- The model considers a simultaneous equilibrium formulation for trip distribution, modal split and assignment, in order to ensure consistency of the levels of service in the system for the different submodels. In this way, the levels of service used to estimate demands (i.e. total trips and trips by mode among zones) must be the same as the levels of service obtained when the O/D matrices by mode are loaded over their corresponding subnetworks (road network and transit networks).
- 2- The trip generation stage is exogenous; that is, trip productions and attractions are given as inputs of the model.
- 3- The model considers multiple pure and combined (combination of pure) model.

- 4- The congestion interactions between all the vehicles using the road network are explicitly considered (car, taxis and vehicles offering public transport services), given that all of them compete for the same common road capacity. The exceptions are the case of the exclusive bus lanes (separate links are coded in this case), the metro lines that operate over an independent network, and the interactions of the car part of the combined mode car passenger/metro trips with other vehicles using the road network.
- 5- Demands is modeled using aggregate gravity functions (singly and doubly constrained) for trip distribution and disaggregate Logit expressions (simple and hierarchical) for modal split. The specific formulation to be used would depend on the trip purpose modeled.
- 6- Capacity constraints are considered for vehicles of all public transport modes represented by De Cea and Fernandez [61] transit equilibrium assignment model.
- 7- The interactions between the car part of a trip by mode car-driver/metro with the other vehicles using the road network are taken into account.
- 8- The demand side of the equilibrium model has a hierarchical structure where the destination and mode choices can be modeled simultaneously or sequentially (distribution first and mode choice second), depending on the values obtained for the calibration parameters of the demand models.
- 9- The hierarchical structure of the demand choices allows the introduction of other choices like departure time and transfer points for combined modes.

i) Networks and Cost Functions

The link's average operating cost $C_a^{lo\bar{m}t}$ (operating time or generalized a cost), for users of class l , with trip purpose o , of private transportation mode \bar{m} (e.g., car, taxi, etc.) depart from his origin at time t . This is a function of the summation of vehicle flows over all private transportation modes, user classes and trip purposes ($f_a^{lo\bar{m}t}$), as well as the fixed flow of public transportation vehicles (\bar{F}_a^t) on link a at time period t , all measured in equivalent vehicles (e.g., p.c.u.):

$$C_a^{lo\bar{m}t} = C_a^{lo\bar{m}t} \left(\sum_l \sum_o \sum_{\bar{m}} f_a^{lo\bar{m}t}, \bar{F}_a^t \right) \quad (28)$$

Although the Jacobian of the cost functions vector is not diagonal, it does turn out to be symmetrical, given the functional form supposed for cost functions $C_a^{lo\bar{m}t}$ (every vehicle, from whichever user class, trip purpose and private transportation mode, produces the same impact on congestion). Nevertheless, this "symmetry" of the cost functions for private modes, which is a simplification, could be relaxed without changing the problem formulation and its solution algorithm. If more general cost functions are used, considering, for instance, that different classes of users of

private modes produce different impacts on road congestion, the Jacobian of these cost functions will be asymmetric just like the one associated to the public transport cost functions. Then the combined problem is asymmetric, independent of the particular characteristics of the cost functions for private modes.

For every pure public transportation mode \bar{m} , the pure service networks can be defined as $G_{\bar{m}} = (N_{\bar{m}}, S_{\bar{m}})$ where $N_{\bar{m}}$ is the set of nodes ($N_{\bar{m}} = N$ for ground services that use the road network, such as buses, and $N_{\bar{m}} = N'$ where $N \cap N' = \emptyset$ for independent public transportation services, (e.g., metro)) and $S_{\bar{m}}$ is the set of transit links (route sections) that belong to mode \bar{m} .

The generalized time (cost) functions of the public transportation links, considering the vehicle capacity constraints as in De Cea and Fernández [61], (sum of travel time, waiting time, transfer time, fare, etc.) depend on the vehicle flow over the road network as well as the passenger flow in the existing services, as follows:

$$\begin{aligned} C_s^{lo\bar{m}t} = & \varphi^{lo\bar{m}t} \left(\sum_l \sum_o \sum_{\bar{m}} f_a^{lo\bar{m}t}, \bar{F}_a^t, \forall a \in \ell, \ell \in B_{s_{\bar{m}t}} \right) \\ & + (P_{TAR})^{lo\bar{m}t} \cdot (TAR)_s^{\bar{m}t} \\ & + (P_{WAIT})^{lo\bar{m}t} \left[\frac{\alpha^{\bar{m}t}}{d_s^{\bar{m}t}} + \beta^{\bar{m}t} \left(\frac{V_s^{lo\bar{m}t} + \tilde{V}_s^{lo\bar{m}t}}{(CAP)_s^{\bar{m}t}} \right)^{\nu^{\bar{m}t}} \right] \end{aligned} \quad (29)$$

where

$C_s^{lo\bar{m}t}$: average or generalized cost on link s for users of class l , with trip purpose o , of public transportation mode \bar{m} (e.g. bus, subway, etc.) at time period t .

$(P_{TAR})^{lo\bar{m}t}$: fare multiplier for users of class l , with trip purpose o , of public transportation mode \bar{m} at time period t .

$(TAR)_s^{\bar{m}t}$: fare related to public transportation link s of mode \bar{m} at time period t .

$(P_{WAIT})^{lo\bar{m}t}$: waiting time multiplier for users of class l , with trip purpose o , of public transportation mode \bar{m} at time period t .

$\alpha^{\bar{m}t}$, $\beta^{\bar{m}t}$, $\nu^{\bar{m}t}$: calibration parameters of waiting time function, for public transportation mode \bar{m} at time period t .

$d_s^{\bar{m}t}$: vehicle frequency of public transportation mode \bar{m} over the public transportation link s at time period t .

$(CAP)_s^{\bar{m}t}$: capacity of public transportation link s of mode \bar{m} at time period t .

$V_s^{l\bar{o}mt}$: passenger flow of class l , with trip purpose o , belonging to public transportation mode \bar{m} at time period t , and which use public transportation link s .

$\tilde{V}_s^{l\bar{o}mt}$: passenger flow that competes with $V_s^{l\bar{o}mt}$ for the capacity of transit lines belonging to B_s (flow with the same trip purpose, user class, mode, and time period t that belong to other public transportation links that compete or reduce the B_s line's capacity, plus flow from other purposes, classes, modes, and time periods that also compete for the B_s lines capacity).

It is easily seen, in this case, that the Jacobian of the cost function vector is non-diagonal and asymmetric. In addition, the model considers the existence of combined modes, for example car/metro (private transportation/public transportation) or bus/metro (public transportation/public transportation). In each case, the union of the pure mode networks that compose them forms the combined mode network. The combined modes (m^c) are considered to be formed by two public transportation modes. Nevertheless, it is important to stress that this does not limit the model's general use, since there is no problem in representing combined modes such as car-public transportation (as in the application of ESTRAUS for the city of Santiago considers combined modes like car driver-metro and car passenger-metro).

i) Trip Distribution, Nest/Mode Split, and Departure Time Logit Models (TD/MS/DT)

In this combined problem where trip generations and attractions are fixed for a given period of time (i.e. morning peak period), within this period and based on the levels of service existing over the private and public networks during alternative sub-periods, users choose the time (sub-period) in which they travel, the mode used and the origin-destination pair of their trips. Distribution is represented by a doubly-constrained entropy maximizing model, mode and departure time choices are modeled with a hierarchical Logit structure and assignment over each modal network in each alternative sub-period is consistent with Wardrop's first principle. Within a particular sub-period, travelers of different classes and trip purposes interact. So, congestion due to the physical capacity of the road network and the physical capacity of the public transport vehicles exists. Nevertheless, given the static nature of the model, traffic interactions between travelers belonging to different time subperiods is not considered. Trip distribution is given by:

$$T_{ij}^{lo} = (FA)_i^{lo} (PRO)_i^{lo} (FB)_j^o (ATR)_j^o \cdot \exp(-\beta^{lo} \zeta_{ij}^{lo}) \quad (30)$$

where FA and FB are balancing factors and

$(PRO)_i^{lo}$ = total trip production from zone i for user class l and trip purpose o

$(ATR)_j^o$ = total trip attraction to zone j for user class l and trip purpose o

The nest/mode split and departure time is given by the following hierarchical Logit models:

$$T_{ij}^{lon} = T_{ij}^{lo} \frac{\exp(-\lambda^{lo} \eta_{ij}^{lon})}{\sum_{n' \in N_{ij}^{lo}} \exp(-\lambda^{lo} \eta_{ij}^{lon'})} \quad (31)$$

$$T_{ij}^{lom} = T_{ij}^{lon} \frac{\exp(-\delta^{lon} \mu_{ij}^{lom})}{\sum_{m' \in n} \exp(-\delta^{lon} \mu_{ij}^{lom'})} \quad (32)$$

$$T_{ij}^{lomt} = T_{ij}^{lom} \frac{\exp(-\gamma^{lom} u_{ij}^{lomt})}{\sum_{t' \in TH} \exp(-\gamma^{lom} u_{ij}^{lomt'})} \quad (33)$$

where

$$\begin{aligned} \zeta_{ij}^{lo} &= -\frac{1}{\lambda^{lo}} \ln \left(\sum_n \exp(-\lambda^{lo} \eta_{ij}^{lon}), \eta_{ij}^{lon} \right) \\ &= -\frac{1}{\delta^{lon}} \ln \left(\sum_{m \in n} \exp(-\delta^{lon} \mu_{ij}^{lom}), \mu_{ij}^{lom} \right) \\ &= -\frac{1}{\gamma^{lom}} \ln \left(\sum_t \exp(-\gamma^{lom} u_{ij}^{lomt}) \right) \end{aligned}$$

TH = Time horizon and β^{lo} , λ^{lo} , δ^{lon} , and γ^{lom} are parameter need to be estimated.

ii) Trip Assignment

The model's basic assumption, with respect to network flow equilibrium, is that for every mode, over its corresponding network at time period t , each user of class l and trip purpose o chooses his/her route according to Wardrop's first principle (i.e., every individual tries to minimize his/her average operating cost or generalized average trip cost). This gives place to the following equilibrium conditions:

$$C_p^{lomt} \begin{cases} = u_{ij}^{lomt} & \text{if } h_p^{lomt} > 0 \\ \geq u_{ij}^{lomt} & \text{if } h_p^{lomt} = 0 \end{cases} ; \forall p \in P_{ij}^{mt}, ij \in R, l, o \quad (34)$$

The above means that at equilibrium, routes with flow will have an equal (minimum) cost, while those without flow, will have an equal or greater cost than the minimum u_{ij}^{lomt} .

P_{ij}^{mt} represents the set of routes between origin-destination pair ij for mode m at time period t , which can be a pure mode (private or transit mode) or any combined mode.

According to the definitions of the link cost functions for both private and public transport networks and the above equilibrium conditions (34), the assignment part of the model is consistent with a deterministic user equilibrium.

Variational Inequality Formulation for ESTRAS Model

From the previous assumption, the link cost functions of private modes are not diagonal but symmetric, while the transit links cost functions are asymmetrical (i.e., Jacobian of the cost function vector is asymmetric). As a result of this, no equivalent optimization formulation of the Beckman type exists. Therefore, the ESTRAS model cannot be cast as an equivalent optimization program. Instead, it can be formulated as the following variational inequality (VI).

$$\mathbf{C}(\mathbf{f}^*)^T (\mathbf{f} - \mathbf{f}^*) - \mathbf{U}(\mathbf{T}^*)^T (\mathbf{T} - \mathbf{T}^*) \geq \mathbf{0} \quad \forall \text{ feasible } \mathbf{f}, \mathbf{T} \quad (35)$$

Where

\mathbf{f} : vector of flow on links of the multimodal network

\mathbf{f}^* : vector of equilibrium flow on links of the multimodal network

\mathbf{T} : vector of trips between origin-destination pairs of the multimodal network $\mathbf{T} = (\mathbf{T}^{i,l,o} : i \in I^{lo}, l \in L, o \in O)$

\mathbf{T}^* : vector of equilibrium trips between origin-destination pairs of the multimodal network

$\mathbf{C}(\mathbf{f}^*)$: column-vector of network link's cost functions (with non-diagonal and asymmetric Jacobian)

$\mathbf{U}(\mathbf{T}^*)$: column-vector of inverse demand functions (with non-diagonal and symmetric Jacobian),

$$\mathbf{U} = (\mathbf{u}^{i,l,o} : i \in I^{lo}, l \in L, o \in O)$$

Many different algorithms have been proposed in order to solve variational inequality problems (35). ESTRAS uses the diagonalization approach, which as it was already mentioned is one of the most widely used methods for solving these types of problems. Dafermos [62], who referred to the diagonalization procedure as "relaxation algorithm" developed a global convergence criterion and established that a linear convergence rate occurs when the demand and cost functions are strongly monotonic and the cost functions yield a Jacobian matrix which is only mildly asymmetric.

At each iteration of the diagonalization algorithm, the cost functions $C_a^{lo\tilde{m}t}$ and $C_a^{lo\bar{m}t}$ result in as diagonalized cost functions $\hat{C}_a^{lo\tilde{m}t}$ and $\hat{C}_a^{lo\bar{m}t}$, which depend only on their own flows, and the following variational inequality must be solved:

$$\hat{\mathbf{C}}(\mathbf{f}^*)^T (\mathbf{f} - \mathbf{f}^*) - \mathbf{U}(\mathbf{T}^*)^T (\mathbf{T} - \mathbf{T}^*) \geq \mathbf{0} \quad \forall \text{ feasible } \mathbf{f}, \mathbf{T} \quad (36)$$

Problem (36) can be formulated as the following an Equivalent Optimization Problem (EOP).

Equivalent Optimization Problem (EOP) for DT/MS/DT/TA ESTRAS Model

$$\begin{aligned} \text{Min } Z = & \sum_l \sum_o \sum_m \sum_t \sum_a \int_0^{f_a^{lo\tilde{m}t}} \hat{C}_a^{lo\tilde{m}t}(x) dx \\ & + \sum_l \sum_o \sum_m \sum_t \sum_s \int_0^{V_s^{lo\bar{m}t}} \hat{C}_s^{lo\bar{m}t}(x) dx + \sum_l \sum_o \frac{1}{\beta^{lo}} \sum_{ij} T_{ij}^{lo} (\ln T_{ij}^{lo} - 1) \\ & - \sum_l \sum_o \frac{1}{\lambda^{lo}} \sum_{ij} T_{ij}^{lo} (\ln T_{ij}^{lo} - 1) + \sum_l \sum_o \sum_n \frac{1}{\lambda^{lo}} \sum_{ij} T_{ij}^{lon} (\ln T_{ij}^{lon} - 1) \\ & - \sum_l \sum_o \sum_n \frac{1}{\delta^{lon}} \sum_{ij} T_{ij}^{lon} (\ln T_{ij}^{lon} - 1) \\ & + \sum_l \sum_o \sum_n \sum_m \frac{1}{\delta^{lon}} \sum_{ij} T_{ij}^{lom} (\ln T_{ij}^{lom} - 1) \\ & - \sum_l \sum_o \sum_n \sum_m \frac{1}{\gamma^{lom}} \sum_{ij} T_{ij}^{lom} (\ln T_{ij}^{lom} - 1) \\ & + \sum_l \sum_o \sum_n \sum_m \frac{1}{\gamma^{lom}} \sum_{ij} \sum_h T_{ij}^{lomt} (\ln T_{ij}^{lomt} - 1) \end{aligned}$$

Subject to:

$$T_{ij}^{lo} = \sum_n T_{ij}^{lon} \quad , \quad \forall ij, l, o \quad (37)$$

$$T_{ij}^{lon} = \sum_{m \in n} T_{ij}^{lom} \quad , \quad \forall ij, l, o, n \quad (38)$$

$$T_{ij}^{lom} = \sum_t T_{ij}^{lomt} \quad , \quad \forall ij, l, o, m \in n \quad (39)$$

$$T_{ij}^{lomt} = \sum_{p \in P_{ij}^{pmt}} h_p^{lomt} \quad , \quad \forall ij, l, o, m \in n, t \quad (40)$$

$$(PRO)_i^o = \sum_j T_{ij}^{lo} \quad , \quad \forall i, l, o \quad (41)$$

$$(ATR)_j^o = \sum_l \sum_i T_{ij}^{lo} \quad , \quad \forall j, o \quad (42)$$

$$f_a^{lo\tilde{m}t} = \sum_{ij} \sum_{p \in P_{ij}^{pmt}} \delta_{ap} h_p^{lo\tilde{m}t} \quad , \quad \forall a, l, o, \tilde{m}, t \quad (43)$$

$$V_s^{lo\bar{m}t} = \sum_{ij} \sum_{p \in P_{ij}^{pmt}} \delta_{sp} h_p^{lo\bar{m}t} \quad , \quad \forall s, l, o, \bar{m}, t \quad (44)$$

$$\mathbf{h} \geq \mathbf{0} \quad (45)$$

$$\mathbf{T} \geq \mathbf{0} \quad (46)$$

Although several works concerning departure time choices are reported in the technical literature, this travel decision has not been integrated yet to supply-demand equilibrium models. Only quite recently, Dekock [63] and Dekock *et al.* [64] have described simultaneous equilibrium model considering trip distribution, modal split and departure time choices. The basic idea of this model is that even if trip generations (and attractions) are fixed for a given period of time, users can choose the sub-period in which they travel, according to a Logit model. ESTRAS combined model considers a doubly constrained entropy-maximizing model, while modal split and

departure time choices are modeled with a hierarchical Logit model. Based on these works, two different models, depending on the relative values of the calibration parameters of the Logit model, were developed but not used yet in any implementation of the model.

D. The Multiclass Simultaneous Transportation Equilibrium Model (MSTEM).

All the multiclass combined models reviewed in part (C), except STEM model, consider that the total originating and terminating flows are known, i.e., the trip generation step of transportation planning process is exogenous to the combined prediction process. This deficiency is accounted in the STEM model which is the only model that combined the trip generation in the prediction process, but it is not a multiclass model as those of Case 5 models. This encourage the development of STEM model to be multiple user classes model in term of socio-economic group (income level, care availability, etc.), trip purpose, as well as pure and combined transportation modes, interacting over a physically unique network. The developed Multiclass Simultaneous Transportation Equilibrium Model (MSTEM) (Hasan and Dashti [65] also combine explicitly the departure time as one of the main components of the prediction process for the first time and be considered as a new generation of new Case 6 of the multiclass model classification. The MSTEM includes all the features of ESTRAUS in addition to the others features mentioned above and more flexible structure for demand models where the trip generation can depend upon the system's performance through an accessibility measure that is based on the random utility theory of users' behavior (instead of being fixed as in ESTRAUS), trip distribution is given by a more flexible single constrained Multinomial Logit (MNL) model based on the random utility theory (instead of being given by a less flexible doubly constrained entropy maximization model in ESTRAUS), and modal split and departure time are given by Multinomial Logit (MNL) models based on the random utility theory (instead of hierarchical Logit for modal split only in ESTRAUS).

The developed MSTEM can be considered the state of the art of the multiclass combined models that include the most recent features of others multiclass combined models in addition to new others features.

Hasan and Dashti Trip Generation, Trip Distribution, Mode Split, Departure Time, and Trip Assignment (TG/TD/MS/DT/TA) Multiclass Combined Model (MSTEM):

Model Assumptions and Structure

i) Travel cost functions in single class models are often separable and symmetric, allowing for convex optimization formulation. In multiclass models travel costs of one class are affected by decisions of other classes; hence the cost structure is not separable, and in general it is not symmetric and does not allow a convex optimization formulation. We assume the following:

1. For each link $a \in A$, the link cost function $C_a^{lonmt}, \forall t \in K_m^{lo}, \forall m \in M_n^{lo}, \forall n \in \Lambda_{ij}^{lo}, \forall ij \in R^{lo}, \forall l \in L, \forall o \in O$, will depend, in general, upon the flow over all links, the vector \mathbf{f} , in the multimodal network (N, A) for all user class $l \in L$, trip propose $o \in O$, transport mode nest $n \in \Lambda_{ij}^{lo}$, transport mode $m \in M_n^{lo}$, and departure time period $t \in K_m^{lo}$, that is

$$C_a^{lonmt} = C_a^{lonmt}(\mathbf{f}) \quad \forall t \in K_m^{lo}, \forall m \in M_n^{lo}, \quad \forall n \in \Lambda_{ij}^{lo}, \forall ij \in R^{lo}, \quad \forall l \in L, \forall o \in O \quad (47)$$

$$\mathbf{f} = (f_a^{lonmt} : a \in A, t \in K_m^{lo}, m \in M_n^{lo}, n \in \Lambda_{ij}^{lo}, ij \in R^{lo}, l \in L, o \in O)$$

We will also assume that the perceived cost of travel on any multimodal route (path) $p \in P_{ij}^{lonmt}$, is the sum of travel costs on the links that comprise that path, that is:

$$C_p^{lonmt} = \sum_{a \in A} \delta_{ap}^{lonmt} C_a^{lonmt}(\mathbf{f}), \quad \forall p \in P_{ij}^{lonmt}, \quad \forall t \in K_m^{lo}, \quad \forall m \in M_n^{lo}, \quad \forall n \in \Lambda_{ij}^{lo}, \quad \forall ij \in R^{lo}, \quad \forall l \in L, \quad \forall o \in O \quad (48)$$

$$\delta_{ap}^{lonmt} = \begin{cases} 1 & \text{if link } a \text{ belongs to path } p \\ 0 & \text{otherwise} \end{cases}$$

$$\forall p \in P_{ij}^{lonmt}, \quad \forall t \in K_m^{lo}, \quad \forall m \in M_n^{lo}, \quad \forall n \in \Lambda_{ij}^{lo}, \quad \forall ij \in R^{lo}, \quad \forall l \in L, \quad \forall o \in O$$

2. The Jacobian $\nabla \mathbf{C}(\mathbf{f})$ of the link cost functions

$$\mathbf{C}(\mathbf{f}) = (C_a^{lonmt}(\mathbf{f}) : t \in K_m^{lo}, m \in M_n^{lo}, n \in \Lambda_{ij}^{lo}, ij \in R^{lo}, l \in L, o \in O)$$

is asymmetric.

The function form specification of the link cost function in (47) depends on the application of the model and how well this link cost function represents the transport system supply in the urban area of the study. For example, if we consider only three nests of transport modes $n \in \Lambda_{ij}^{lo}$ named n_1, n_2 , and n_3 and define them as follows:

$n_1 = \tilde{m}$ as pure private transportation modes (e.g., car, taxi, etc.)

$n_2 = \bar{m}$ as pure public transportation modes (e.g., bus, subway, metro, etc.)

$n_3 = m^c$ as combined transportation modes (e.g., car/metro, bus/metro, etc.) and used the specified link cost functions for each of the modes \tilde{m} , \bar{m} , and m^c that used in ESTRAUS (de Cea *et al.* [46]) in its application to Chilean city Santiago, MSTEM model will have all the advantages of ESTRAUS from the transport system supply side representation, especially the capacity constraints for vehicles of all public transport modes, in addition to, the advantages of MSTEM model over the ESTRAUS from the transport system demand side.

ii) for TG component, following the same line of thought of Safwat and Magnanti [34], the accessibility S_i^{lo} of origin $i \in I^{lo}$ as perceived from user of class l with trip purpose o traveling from that origin can be defined as follows:

$$S_i^{lo} = E \left[\max_{j \in D_i^{lo}} \max_{n \in \Lambda_{ij}^{lo}} \max_{m \in M_n^{lo}} \max_{t \in K_m^{lo}} v_{ij}^{lonmt} \right] \quad \forall i \in I^{lo}, \forall l \in L, \forall o \in O$$

where E = the expectation operator.

$$S_i^{lo} = \max \left\{ 0, \ln \sum_{j \in D_i^{lo}} \sum_{n \in \Lambda_{ij}^{lo}} \sum_{m \in M_n^{lo}} \sum_{t \in K_m^{lo}} \exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo}) \right\} \quad \forall i \in I^{lo}, \forall l \in L, \forall o \in O \quad (49)$$

The number of trips generated from origin i for users of class l with trip purpose o , G_i^{lo} can be expressed by:

$$G_i^{lo} = \alpha^{lo} S_i^{lo} + \sum_{\omega=1}^{\Omega} \alpha_{\omega}^{lo} q_{\omega}(E_{\omega i}^{lo})$$

$$G_i^{lo} = \alpha^{lo} S_i^{lo} + E_i^{lo} \quad \forall i \in I^{lo}, \forall l \in L, \forall o \in O \quad (50)$$

Similar to A_j^{lo} , E_i^{lo} is assumed to be a fixed constant during the time period required to achieve short-run equilibrium, and G_i^{lo} depends solely on the system's performance as measured by the accessibility variable S_i^{lo} .

iii) Trip Distribution, Nest/Mode Split, and Departure Time Logit Models (TD/MS/DT).

Following the same line of thought of Safwat and Magnanti [34], Oppenheim [66] and Ran and Boyce [67] our distribution, nest/mode, and departure time Logit models can be given by:

$$T_{ij}^{lo} = G_i^{lo} \frac{\sum_{n \in \Lambda_{ij}^{lo}} \sum_{m \in M_n^{lo}} \sum_{t \in K_m^{lo}} \exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo})}{\sum_{j \in D_i^{lo}} \sum_{n \in \Lambda_{ij}^{lo}} \sum_{m \in M_n^{lo}} \sum_{t \in K_m^{lo}} \exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo})} \quad \forall ij \in R^{lo}, \forall l \in L, \forall o \in O \quad (51)$$

$$T_{ij}^{lon} = T_{ij}^{lo} \frac{\sum_{m \in M_n^{lo}} \sum_{t \in K_m^{lo}} \exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo})}{\sum_{n \in \Lambda_{ij}^{lo}} \sum_{m \in M_n^{lo}} \sum_{t \in K_m^{lo}} \exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo})} \quad (52)$$

$$\forall n \in \Lambda_{ij}^{lo}, \forall ij \in R^{lo}, \forall l \in L, \forall o \in O$$

$$T_{ij}^{lonm} = T_{ij}^{lon} \frac{\sum_{t \in K_m^{lo}} \exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo})}{\sum_{m \in M_n^{lo}} \sum_{t \in K_m^{lo}} \exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo})} \quad (53)$$

$$\forall m \in M_n^{lo}, \forall n \in \Lambda_{ij}^{lo}, \forall ij \in R^{lo}, \forall l \in L, \forall o \in O$$

$$T_{ij}^{lonmt} = T_{ij}^{lonm} \frac{\exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo})}{\sum_{t \in K_m^{lo}} \exp(-\theta_i^{lo} u_{ij}^{lonmt} + A_j^{lo})} \quad \forall t \in K_m^{lo}, \quad (54)$$

$$\forall m \in M_n^{lo}, \forall n \in \Lambda_{ij}^{lo}, \forall ij \in R^{lo}, \forall l \in L, \forall o \in O$$

iii) Trip Assignment (TA). Based on the previous choices assumption, the given user will choose his or her route according to Wardrop's user equilibrium principle. That is, for all users of class l with trip purpose o traveling from the origin node $i \in I^{lo}$ to the destination node $j \in D_i^{lo}$ and whose already chose the mode of transport $m \in M_n^{lo}$ from the nest of modes $n \in \Lambda_{ij}^{lo}$ and start their trip at the time $t \in K_m^{lo}$, the perceived generalized costs on all used multimodal paths between the given origin-destination pair are equal and not greater than those on unused paths. This gives the following equilibrium conditions:

$$C_p^{lonmt} = \begin{cases} u_{ij}^{lonmt} & \text{if } h_p^{lonmt} > 0 \\ \geq u_{ij}^{lonmt} & \text{if } h_p^{lonmt} = 0 \end{cases}, \quad \forall p \in P_{ij}^{lonmt}, \forall t \in K_m^{lo}, \quad (55)$$

$$\forall m \in M_n^{lo}, \forall n \in \Lambda_{ij}^{lo}, \forall ij \in R^{lo}, \forall l \in L, \forall o \in O$$

where

$$f_a^{lonmt} = \sum_{l \in L} \sum_{o \in O} \sum_{ij \in R^{lo}} \sum_{p \in P_{ij}^{lonmt}} \delta_{ap}^{lonmt} h_p^{lonmt}, \quad \forall a \in A, \forall t \in K_m^{lo},$$

$$\forall m \in M_n^{lo}, \forall n \in \Lambda_{ij}^{lo}, \forall ij \in R^{lo}, \forall l \in L, \forall o \in O$$

Variational Inequality Formulation for MSTEM Model

Because of the asymmetry of the link cost functions, MSTEM cannot be cast as an equivalent optimization program as STEM. Instead, it can be formulated as the following variational inequality (VI).

$$\mathbf{C}(\mathbf{f}^*)^T (\mathbf{f} - \mathbf{f}^*) - \mathbf{U}(\mathbf{T}^*)^T (\mathbf{T} - \mathbf{T}^*) \geq 0 \quad \forall \text{ feasible } \mathbf{f}, \mathbf{T} \quad (56)$$

The VI problem in (56) is Equivalent to MSTEM and can be solved by the relaxation (diagonalization) algorithm (Dafermos [62], Florian and Spiess [68])

At each iteration of the diagonalization algorithm, the cost functions C_a^{lonmt} of (47) result in diagonalized cost function \hat{C}_a^{lonmt} , which depend only on their own flows, f_a^{lonmt} , and the following VI should be solved:

$$\hat{\mathbf{C}}(\mathbf{f}^*)^T (\mathbf{f} - \mathbf{f}^*) - \mathbf{U}(\mathbf{T}^*)^T (\mathbf{T} - \mathbf{T}^*) \geq 0 \quad \forall \text{ feasible } \mathbf{f}, \mathbf{T} \quad (57)$$

This VI can be formulated as the following EOP:

Equivalent Optimization Program (EOP):

$$\begin{aligned} \text{Min } Z(\mathbf{S}, \mathbf{T}, \mathbf{f}) = & \sum_{l \in L} \sum_{o \in O} \sum_{i \in I^l} \sum_{j \in D_i^l} \sum_{n \in \Lambda_{ij}^l} \sum_{m \in M_n^l} \sum_{t \in K_m^l} \sum_{a \in A} \int_0^{f_a^{lonmt}} \hat{C}_a^{lonmt}(x) dx \\ & + \sum_{l \in L} \sum_{o \in O} \sum_{i \in I^l} \frac{1}{\theta_i^{lo}} \left[\frac{\alpha^{lo}}{2} (S_i^{lo})^2 + \alpha^{lo} S_i^{lo} \right. \\ & \quad \left. - (\alpha^{lo} S_i^{lo} + E_i^{lo}) \ln(\alpha^{lo} S_i^{lo} + E_i^{lo}) \right] \\ & + \sum_{l \in L} \sum_{o \in O} \sum_{i \in I^l} \frac{1}{\theta_i^{lo}} \sum_{j \in D_i^l} \sum_{n \in \Lambda_{ij}^l} \sum_{m \in M_n^l} \sum_{t \in K_m^l} [T_{ij}^{lonmt} \ln(T_{ij}^{lonmt}) \\ & \quad - A_j^{lo} T_{ij}^{lonmt} - T_{ij}^{lonmt}] \end{aligned}$$

subject to:

$$\sum_{j \in D_i^l} T_{ij}^{lo} = \alpha^{lo} S_i^{lo} + E_i^{lo} \quad \forall i \in I^l, \forall l \in L, \forall o \in O$$

$$T_{ij}^{lo} = \sum_{n \in \Lambda_{ij}^l} T_{ij}^{lon} \quad \forall i \in I^l, \forall j \in D_i^l, \forall l \in L, \forall o \in O$$

$$T_{ij}^{lon} = \sum_{m \in M_n^l} T_{ij}^{lonm} \quad \forall i \in I^l, \forall j \in D_i^l, \forall n \in \Lambda_{ij}^l,$$

$$\forall l \in L, \forall o \in O$$

$$T_{ij}^{lonm} = \sum_{t \in K_m^l} T_{ij}^{lonmt} \quad \forall i \in I^l, \forall j \in D_i^l, \forall m \in M_n^l,$$

$$\forall n \in \Lambda_{ij}^l, \forall l \in L, \forall o \in O$$

$$T_{ij}^{lonmt} = \sum_{p \in P_{ij}^{lonmt}} h_p^{lonmt} \quad \forall i \in I^l, \forall j \in D_i^l, \forall t \in K_m^l,$$

$$\forall m \in M_n^l, \forall n \in \Lambda_{ij}^l, \forall l \in L, \forall o \in O$$

$$S_i^{lo} \geq 0 \quad \forall i \in I^l, \forall l \in L, \forall o \in O$$

$$T_{ij}^{lo} \geq 0 \quad \forall i \in I^l, \forall j \in D_i^l, \forall l \in L, \forall o \in O$$

$$T_{ij}^{lon} \geq 0 \quad \forall i \in I^l, \forall j \in D_i^l, \forall n \in \Lambda_{ij}^l, \forall l \in L, \forall o \in O$$

$$T_{ij}^{lonm} \geq 0 \quad \forall i \in I^l, \forall j \in D_i^l, \forall m \in M_n^l, \forall n \in \Lambda_{ij}^l, \\ \forall l \in L, \forall o \in O$$

$$T_{ij}^{lonmt} \geq 0 \quad \forall i \in I^l, \forall j \in D_i^l, \forall t \in K_m^l, \forall m \in M_n^l, \\ \forall n \in \Lambda_{ij}^l, \forall l \in L, \forall o \in O$$

$$h_p^{lonmt} \geq 0 \quad \forall i \in I^l, \forall j \in D_i^l, \forall t \in K_m^l, \forall m \in M_n^l, \\ \forall n \in \Lambda_{ij}^l, \forall l \in L, \forall o \in O, p \in P_{ij}^{lonmt}$$

where

$$f_a^{lonmt} = \sum_{l \in L} \sum_{o \in O} \sum_{ij \in R^l} \sum_{p \in P_{ij}^{lonmt}} \delta_{ap}^{lonmt} h_p^{lonmt}, \quad \forall a \in A, \forall t \in K_m^l,$$

$$\forall m \in M_n^l, \forall n \in \Lambda_{ij}^l, \forall ij \in R^l, \forall l \in L, \forall o \in O$$

Existence, convexity and uniqueness of ECP problem as well as equivalence between MSTEM and ECP can be followed as those of Safwat and Magnanti [34].

The Logit Distribution of Trips (LDT) algorithm that developed by Safwat and Brademeyer [69] has been modified as Multiclass Logit Distribution of Trips (MLDT) algorithm to solve the above ECP.

Boyce [70] gives very good reviews and prospects for network equilibrium models. Boyce [71] gives a good view for future research on urban transportation network modeling.

VI. THE ARCHITECTURE OF THE INTELLIGENT DECISION SUPPORT SYSTEM THE SYSTEM

The architecture and the information flow shown in Fig. 4 represent the high level blueprint for the implementation of the framework for an Intelligent Decision Support system for Urban Transportation Systems Analysis. The framework is derived and guided by the methodological framework for urban transportation system analysis discussed in Section IV and depicted in Fig. 3 and the main Decision Support System (DSS) components found in standard DSS textbook such as Turban [72]. A fourth component, the scenario management was added to package the functionally required by scenarios creation, storage, retrieval, analysis, and evaluation and reporting. An Intelligent Agent for supporting scenario creation is also included in the framework. Fig. 4 depicts the main components and the interaction (data and control flow) with the Transportation Object Repository, with each other and the User Interface Management Subsystem (UIMS) directly or indirectly. In the following section a brief description of each component is presented.

A. Urban Transportation Object Repository (UTOR)

The Urban Transportation Object Repository (UTOR) is an object oriented repository storing various transportation objects such as nodes, links, and zones for multimodal urban

networks; equilibrium models; impact models, scenarios and user interfaces that are managed by the four subsystems discussed below. The four UTOR components are distinct but they are integrated object store (base). These are:

1-Urban Transportation GIS-Data Store: This object store contains two distinct components:

- GIS Object Base: contains various GIS Objects
- Transportation Data Warehouse (TDW): TDW is a multidimensional, object-oriented, nonvolatile integrated database containing various current and historical data about the transportation Objects (Inmon, [73]). The TDW and GIS Objects are populated by the Extract Transform and Loading (ETL) component of Data Management subsystem (DMS) mentioned below.

2-Urban Transportation Model Base: contains various transportation equilibrium and impact models

3-Scenarios Base: contains various scenarios objects that are created over times.

4-User Interface Base: contains various User Interface (UI) objects created over time.

B. Data Management Subsystem (DMS)

The data management subsystem (DMS) is responsible for the data administration such as creation, storage, retrieval of node object, links object, and zone object for different modal network. DMS manages the Urban Transportation GIS-Data Store which contain two distinct but integrated data bases:

- a) a GIS database contain the spatial data;
- b) a transportation data warehouse mentioned above.

C. Extract, Transform and Load (ETL) Component

ETL component of the DMS extracts data from multiple sources, cleanse them and transform the data from its original to a form that could be place in TDW guided by the metadata, and then load the data into the TDW. The purpose of ETL is to populate TDW with integrated and cleansed data required by the transportation objects gathered from three main sources, the socio-economic, the demand data and the supply data.

D. Model Management Subsystem (MMS)

MMS is responsible for the creation, storage, retrieval of the transportation object models which are stored in the Model Object Base (MOB) for utilization or reuse. Two types of models are managed by MMS. These are:

- 1- The Transportation Network Equilibrium Model (MSTEM)
- 2- Impacts Models: such as impact on Users, impact on operators, impact on owners, impact on society, and impact on government discussed earlier in Section IV.

E. Scenario Management Subsystem (SMS)

Much like MMS, SMS is responsible for the creation, storage, retrieval, analysis, evaluation of scenarios. An important part of SMS is the impact evaluation component that assesses the impact models and presents the assessment result to the SMS. Scenario Objects are stored in Scenario

Object Base (SOB) for future utilization or reuse by SMS and Intelligent Scenario Creation Assistance Agent (ISCAA) described below. SMS retrieves previously created scenarios and pass them to the User Interface Management Subsystem (UIMS) as initial scenarios on which further what-if analysis could be performed.

F. Intelligent Scenario Creation Assistance Agent (ISCAA)

The complexity of creating the right scenario or retrieving the right scenario from the previous created ones stored in the scenario base is a complex process requires human expertise which is scarce. An intelligent component within the DSS framework that would look at the historical scenario objects and assist and guide the decision maker in choosing the best alternatives from this pool of historical scenario object to be included in the initial scenario setup is extremely valuable. A solution for this problem is to create an Intelligent Scenario Creation Assistance Agent (ISCAA) that would encode and encapsulates the expertise for scenario creation and would provide the necessary assistance for creating the right scenario. ISCAA would be a hybrid intelligent agent containing multiple computational intelligent tools (such as Ann, rough set, fuzzy logic, etc.) as well as a set of scenario creation rules.

G. User Interface Management Subsystem (UIMS)

The UIMS packages and manages the functionalities require for creating a data-rich intensive (maps, graphs, text, and structured data) with various visualization capabilities user interface. Since The DSS is to be used by users with various roles (Transportation Planners, Transportation Engineers, Transportation Decision Makers or Traffic Administrators), the complexity involves in dynamically creating the right graphical user interface (GUI) for the right role lies within the functionality of UIMS. For example, the transportation planner is responsible for creating models and capturing the right data for those models, as such he/she would directly interact with Model Management and Data Management and as such the GUI for this role would configure that would allow for that only. On the other hand, the transportation decision maker role deals with scenarios and as such the UIMS would create the proper GUI allowing various scenario related activities such as scenario creation, retrieval, storage and execution and presenting the result in a dashboard view allowing for a comprehensive, at-a-glance, GIS-Based graphical view of the solution generated. UIMS also provides various analysis tools such as what-if analysis, sensitivity analysis, reporting the result of impact evaluation and providing various Ad hoc queries and reports. The Graphical Interface Objects, that are created, are stored as UI Objects in the UTOR and managed by UIMS.

VII. CONCLUSION

The traffic congestion problem is one of the most important and urgent problem for all cities around the world. Traditional solutions to the problem are no longer capable in providing

acceptable solutions or improvements. A system view for the problem that consider all factors that effluence the problem in a more comprehensive framework using the state of the art transportation network equilibrium modeling that closely represent the travelers behavioral through optimization techniques and GIS-Based intelligent decision support system are needed.

This review paper summaries these needs and give an overview of the field with state of the art in the Static

Transportation Network Equilibrium Models as well as the a new view for an GIS-Based intelligent decision support system that can be implemented and would be useful tool to help and support transportation planners, transportation engineers, and city municipalities decision makers to take the right decision about traffic congestion solutions and new transportation projects evaluation.

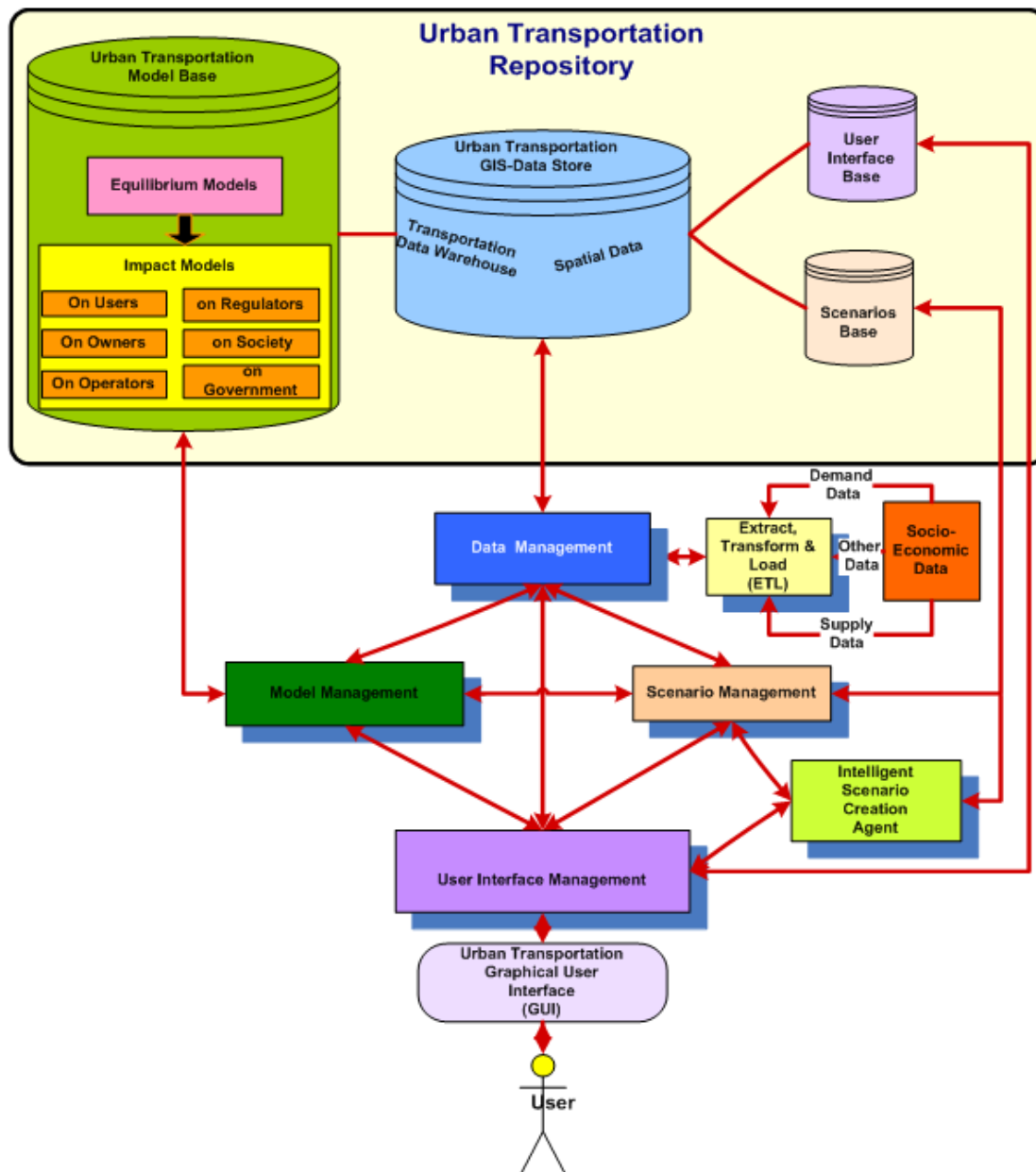


Fig. 4 The architecture and information flow of the intelligent decision support system for urban transportation systems analysis

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