

# Influence of Hydraulic Hysteresis on Effective Stress in Unsaturated Clay

Anuchit Uchaipichat

**Abstract**—A comprehensive program of laboratory testing on a compacted kaolin in a modified triaxial cell was performed to investigate the influence of hydraulic hysteresis on effective stress in unsaturated soils. The test data are presented on a range of constant suction shear tests along wetting and drying paths. The values of effective stress parameter  $\chi$  at different matric suction were determined using the test results. The effect of hydraulic hysteresis phenomenon on the effective stress was observed. The values of effective stress parameter  $\chi$  obtained from the experiments were compared with those obtained from the expressions proposed in literature.

**Keywords**—Unsaturated soils, Hydraulic hysteresis, Effective stress

## I. INTRODUCTION

SEVERAL elasto-plastic models for unsaturated soils have been developed using effective stress approach [1]-[5]. The effective stress converts a soil with multiple fluid phases to an equivalent single-phase material with single-stress state continuum. Thus, the elasto-plastic constitutive equations of solid phase can be applied using the effective stress to unsaturated soils. Another advantage of the effective stress approach is that soil properties can be determined using a conventional laboratory equipment with a few modification for testing unsaturated soils.

The effective stress for unsaturated soils can be expressed as a function of the externally applied stresses and the internal fluid pressures and defined as [6],

$$p' = (p - u_a) + \chi(u_a - u_w) \quad (1)$$

where  $p'$  is the mean effective stress,  $p$  is the mean total stress,  $u_a$  is the pore air pressure,  $u_w$  is the pore water pressure,  $u_a - u_w$  is the matric suction, and  $\chi$  is the effective stress parameter attaining a value of unity for a saturated soil and zero for a dry soil. The parameter  $\chi$  is strongly related to the soil structure [7] and suction ratio, defined as the ration of matric suction over the air entry suction [8],

$$\chi = \begin{cases} [s/s_e]^{-\Omega} & \text{for } s \geq s_e \\ 1 & \text{for } s \leq s_e \end{cases} \quad (2)$$

in which,  $s = u_a - u_w$  is the matric suction,  $s_e$  is suction value marking the transition between saturated and unsaturated states, and  $\Omega$  is a material parameter, with a best-fit value of 0.55. For the main wetting path,  $s_e = s_{ex}$ , and for the main drying path  $s_e = s_{ae}$ , in which  $s_{ex}$  is the air expulsion value and  $s_{ae}$  is the air entry value.

Furthermore, the use of the degree of saturation as parameter  $\chi$  has been applied in a few models [1], [3]. Thus, the influence of hydraulic hysteresis on the effective stress has been introduced through the parameter  $\chi$ .

The main objective of this paper is to present effect of hydraulic hysteresis on effective stress in unsaturated soils through results from a comprehensive program of laboratory testing on a compacted sample of kaolin in a triaxial cell. The test data are presented on a range of constant suction shear tests along wetting and drying paths.

## II. EXPERIMENTAL EQUIPMENT

### A. Equipment Preparation

The tests were performed using a conventional triaxial cell with a few of modifications for testing unsaturated soils as shown in Fig. 1. The modified cell was capable of independent measurement and control of pore air pressure and pore water pressure at the top and the bottom boundaries of the specimen. The pore air pressure was controlled through a coarse porous stone placed at the top of the specimen. The pore water was controlled at the bottom of the specimen through a high air entry value ceramic disc. The ceramic disc was attached to the pedestal base using epoxy glue to prevent a flow of air to the water compartment through its surroundings. The suction in the sample was controlled using axis-translation technique [9].

Prior to each test, the ceramic disc and the pore water control system were saturated using a technique similar to that used by [10]. The empty cell was filled with de-aired water and pressurized to a cell pressure of 600 kPa. The water was allowed to flow through the ceramic disc into the water compartment while maintaining the water pressure at the base of ceramic disc at the atmospheric level. After collecting 200 cm<sup>3</sup> of water flow, the drainage valve was closed for at least 2 hours to dissolve any air trapped within the ceramic disc. The

A. Uchaipichat is with the Civil Engineering Department, Vongchavalitkul University, Nakhonratchasima THAILAND (corresponding author to provide phone: 66-44-203778; fax: 66-44-203785; e-mail: anuchitu@yahoo.com).

addition flushing process was required to ensure that the ceramic disc was fully saturated.

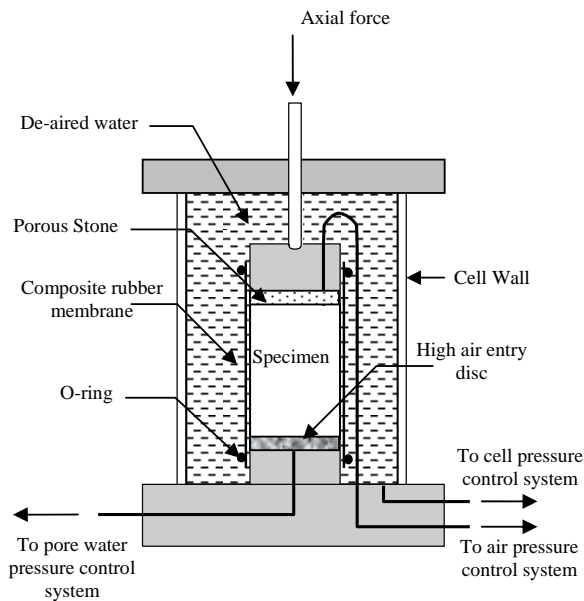


Fig. 1 modified triaxial cell

### B. Specimen Volumetric Strain Measurement

A digital image-processing technique was used for measuring volumetric strains of the specimens. The images of the specimens were taken during testing using digital imaging equipment. A Nikon D50 digital single lens reflex camera with 3008 pixels by 2000 pixels resolution was used to capture the images of specimens. A Nikon AF-Nikkor 85 mm f/1.8D lens was used together with the Nikon D50 camera. To produce clear specimen boundaries on the images, the specimens were lighted to provide sufficient illumination for their shadow areas.

The process of a determination of specimen volume from the digital images was performed using technique similar to that of [12]. The specimen height was measured along the axis of specimen appearing in the digital images. For diameter measurement, the specimen was assumed as a series of stacked discs with variation in diameter. The diameter of the specimens was obtained through averaging all assumed discs. Finally, the specimen volume and the volumetric strains were calculated from the equivalent cylindrical shape of specimen with the measured height and the average diameter.

## III. SAMPLE PREPARATION AND SET-UP

### A. Sample Properties and Preparation

The experiments were carried out on a laboratory-compacted-kaolin. The index properties of test soil are given in Table 1.

TABLE I  
INDEX PROPERTIES OF KAOLIN

Property	Values
Liquid limit (%)	52
Plastic limit (%)	31
Specific gravity	2.72
Maximum dry unit weight* (kN/m <sup>3</sup> )	14.1
Optimum moisture content (%)	27.5

\* Obtained using standard Proctor test.

To make identical specimens with a matrix amenable to stiffening with increasing matric suction, the samples were prepared dry of optimum as in [11]. The kaolin sample with a water content of 25% was statically compacted to a dry unit weight of approximately 11.8 kN/m<sup>3</sup>. Prior to compaction, the kaolin sample was carefully wetted to a water content of 25% and cured for 24 hours in a sealed plastic bag for moisture equalization. The compaction was carried out in nine equal layers in a greased split mold of 38 mm diameter.

The soil-water characteristic curve (SWCC) of the compacted specimens, presented in Fig. 2, was obtained by keeping the compacted specimens in the modified triaxial cell at the different values of constant matric suction for 3 days. For wetting portion of the curve, the matric suction was first increased to 300 kPa and then reduced to the target value. The values of air entry ( $s_{ae}$ ) and air expulsion ( $s_{ex}$ ) obtained from the SWCC are 80 and 35 kPa respectively.

### B. Specimen Set-up

The specimen was covered by two rubber membranes. To minimize diffusion of air through the membranes, the silicon grease was placed between the two membranes. The covered specimen was then placed on the ceramic disc and sealed by fitting O-rings around the base pedestal and the top cap as shown in Fig. 1. After assembling the apparatus, the cell was filled with de-aired water and an axial load of about 5 kPa was applied to secure the specimen in place. The free air in the specimen was flushed through the top cap by applying the water pressure of 50 kPa through the cell based while maintaining the pressure at the top at the atmospheric level. To prevent failure of the specimen, a cell pressure of 75 kPa was applied throughout the flushing process. After flushing free air in the specimen for three days, the top drainage line was closed for 24 hours to equalize pore water pressure within the specimen.

## IV. TEST PROGRAM AND PROCEDURE

A total of 12 shear tests were performed using the modified triaxial equipment and were divided into two groups for saturated and unsaturated specimens. The main purpose of the tests on saturated specimens was to determine the slope of the critical state line in the plane of deviator stress against mean effective stress. For the tests on unsaturated specimens, the purpose was to investigate the effect of hydraulic hysteresis

on stress-strain characteristics. All the test results were carefully analyzed and utilized to determine the parameter  $\chi$ .

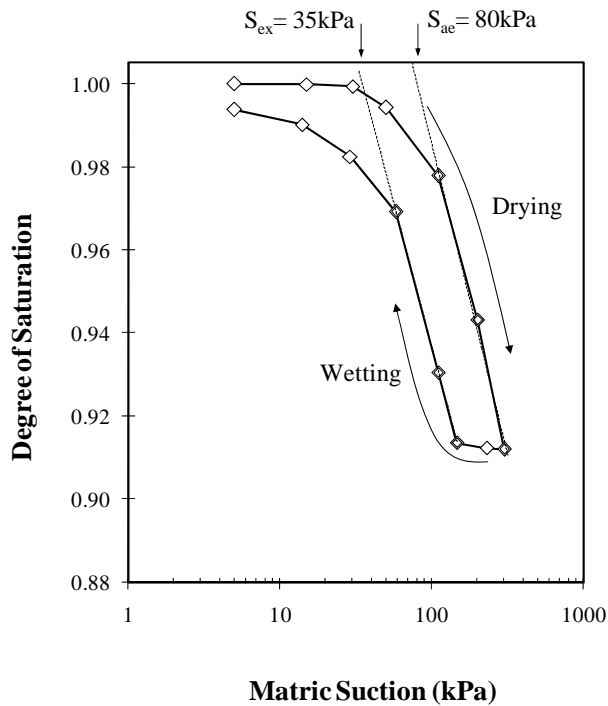


Fig. 2 Soil water characteristic curve of compacted kaolin

#### A. Shear Tests on Saturated Specimens

A conventional consolidated drained triaxial tests procedure was followed in experiments on saturated specimens. Three specimens were consolidated to different isotropic effective stress of 100, 200 and 300 kPa. At the end of consolidation, the axial stress was applied while maintaining the cell pressure constant.

#### B. Shear Tests on Unsaturated Specimens

The shear tests were performed on both dried and wetted specimens. Prior to each shear test, the specimens were consolidated to an isotropic stress of 200 kPa. Then the specimens were unloaded to the isotropic stress of 100 kPa. Thus, all specimens in this testing group were subjected to identical stresses state before applying matric suction. The specimens were then subjected to the different values of matric suction ranged from 0 to 300 kPa. For the dried specimens, the matric suction was increased from 0 kPa (saturated state) to the testing value. For the wetted specimens, the matric suction was first increased to 300 kPa and then reduced to the testing value. The drained shear tests were then performed by increasing axial stress while maintaining cell pressure, pore air pressure and matric suction constant.

### V. RESULTS AND DISCUSSION

The test results in terms of standard notations for triaxial tests are presented throughout. Two pairs of the work

conjugate variables are mean effective stress and volumetric strain ( $p'$  and  $\varepsilon_v$ ), and deviator stress and shear strain ( $q$  and  $\varepsilon_s$ ). These variables are defined as,

$$p' = \frac{\sigma'_a + 2\sigma'_r}{3} \quad (2)$$

$$q = \sigma'_a - \sigma'_r$$

$$\varepsilon_v = \varepsilon_a + 2\varepsilon_r$$

$$\varepsilon_s = \frac{2}{3}(\varepsilon_a - \varepsilon_r)$$

in which,  $\sigma'_a$  and  $\sigma'_r$  are the axial and radial effective stresses.  $\varepsilon_a$  and  $\varepsilon_r$  are the axial and radial strains. Compressive stresses and strains are assumed positive.

The tests results are presented in Figs. 3 to 6. It can be seen that the critical state, in which the deviator stress and the volumetric are constant and independent of the shear strain, was approached in all the tests.

The stress-strain curves and stress path for the saturated specimens subjected to different values of isotropic stress are shown in Figs. 3 and 4, respectively. The slope of the critical state line in the plane of deviator stress against mean effective stress of the saturated specimen is 0.88.

The stress-strain curves for the dried and wetted specimens at different values of matric suction are shown in Figs 5 and 6. The strain-hardening response was observed in all the matric suction values. Furthermore, at the same level of matric suction, the stress-strain curves of the dried specimens always lies above that of the wetted specimens.

Making use of the observation of the critical state lines at different matric suction by [13], the parameter  $\chi$  can be determined by assuming that the slope of the critical state line in the plane of deviator stress against mean effective stress is independent of matric suction. Thus, using the effective stress expression in (1), the parameter  $\chi$  can be expressed as,

$$\chi = \frac{\left(\frac{q_f}{M}\right) - (p_f - u_a)}{u_a - u_w} \quad (3)$$

where  $p_f$  and  $q_f$  are the mean total stress and the deviator stress at the critical state,  $M$  is the slope of the critical state line in the plane of deviator stress against mean effective stress.

The values of the parameter  $\chi$  calculated using relation (3) and the shear test data at the critical state at the different values of matric suction are shown in Figs 7 and 8. It is obvious that the hydraulic hysteresis phenomenon affects the effective stress through the parameter  $\chi$ . Moreover, the expression in (2) for parameter  $\chi$  proposed by [8] gives a good trend over the range of testing matric suction on both drying and wetting paths. The results also show that the use of degree of saturation as parameter  $\chi$  is not suitable for

calculating the effective stress.

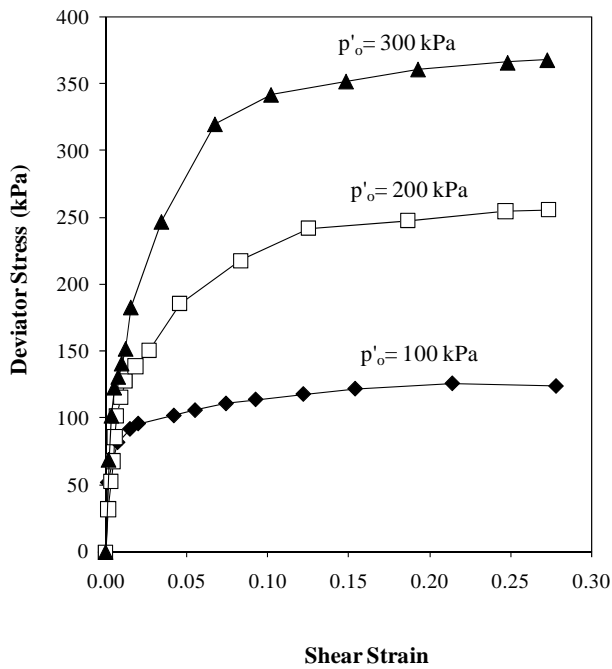


Fig. 3 Consolidated drained triaxial tests on saturated specimens

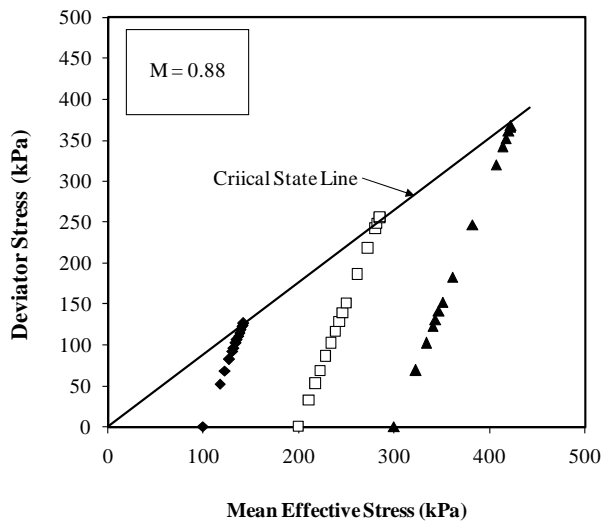


Fig. 4 Effective stress paths for triaxial tests on saturated specimens

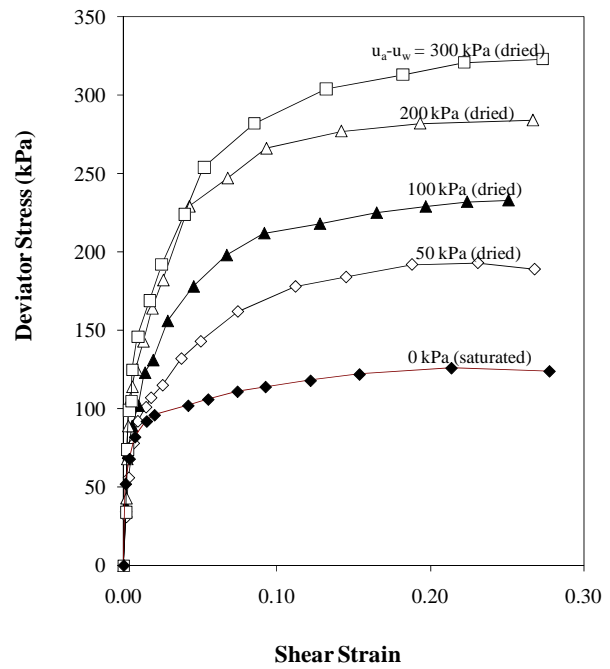


Fig. 5 Suction-controlled triaxial shear tests on dried specimens

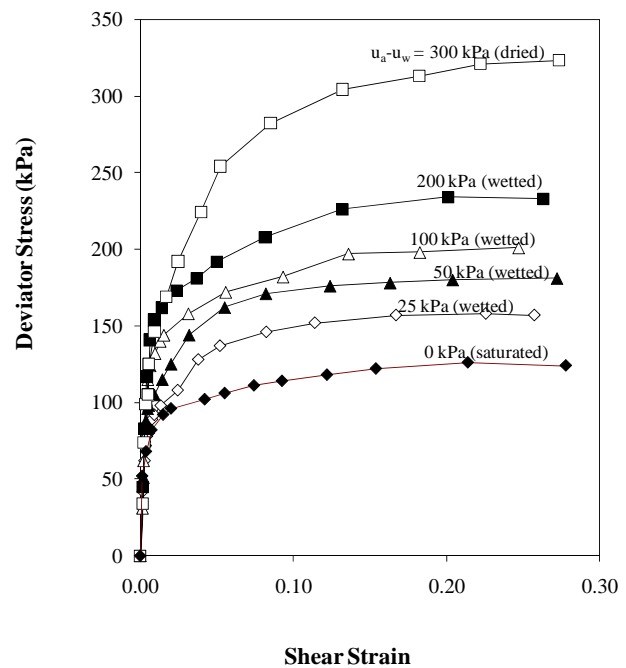
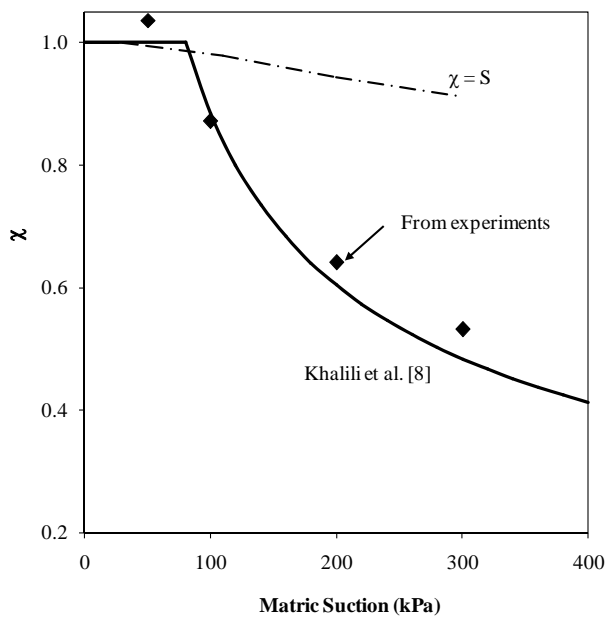
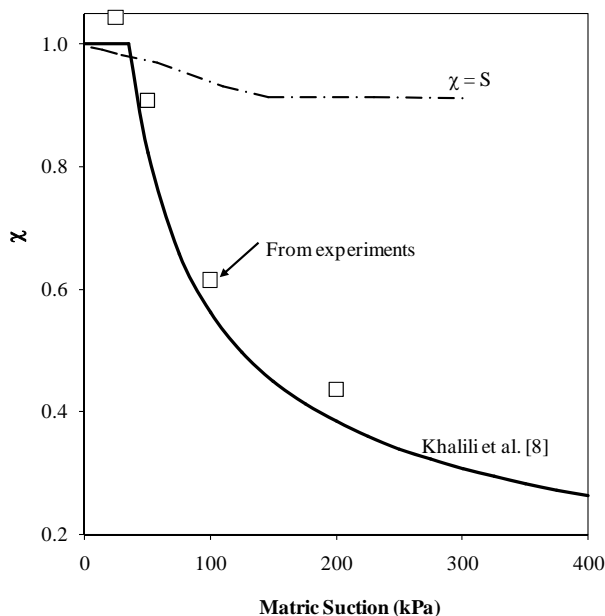


Fig. 6 Suction-controlled triaxial shear tests on wetted specimens

Fig. 7 Parameter  $\chi$  for dried specimensFig. 8 Parameter  $\chi$  for wetted specimens

## VI. CONCLUSION

An experimental program of shear tests was conducted on a compacted kaolin using a conventional triaxial equipment modified for testing unsaturated soils. The modified triaxial cell was capable of independent measurement and control of pore air pressure and pore water pressure at the top and the bottom boundaries of the specimen. The effect of hydraulic

hysteresis on the stress-strain characteristic was observed. The values of effective stress parameter  $\chi$  at different matric suction were determined using the test results and compared with those obtained from the expressions proposed in literature.

## ACKNOWLEDGMENT

This work was supported by the Thailand Research Fund under Grant MRG5080028

## REFERENCES

- [1] R. W. Lewis, and B. A. Schrefler, *The Finite Element Method in the Deformation and Consolidation of Porous Media*, John Wiley and Sons, UK, 1987.
- [2] Y. Kohgo, M. Nakano, and T. Miyazaki, "Theoretical aspects of constitutive modelling for unsaturated soils," *Soils and Foundations*, Vol. 33, No. 4, 1993, pp. 49-63.
- [3] D. C. Sheng, S. W. Sloan, and A. Gens, "Finite element formulation and algorithms for unsaturated soils. Part I: Theory," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 27, no. 9, 2003, pp. 745-765.
- [4] B. Loret, and N. Khalili, "An effective stress elastic-plastic model for unsaturated porous media," *Mechanics of Materials*, vol. 34, 2002, pp. 97-116.
- [5] D. Gallipoli, A. Gens, R. Sharma, and J. Vaunat, "An elasto-plastic model for unsaturated soil incorporating the effects of suction and degree of saturation on mechanical behaviour," *Geotechnique*, vol. 53, no. 1, 2003, pp. 123-135.
- [6] A. W. Bishop, *The Principle of Effective Stress*, Teknisk Ukebladk, Norway, 1959, pp. 859-863.
- [7] J. D. Coleman, "Stress-strain relations for partly saturated soils," *Geotechnique*, Vol. 12, No. 4, 1962, pp. 348-350.
- [8] N. Khalili, F. Geiser, and G. E. Blight, "Effective stress in unsaturated soils, a review with new evidence," *International Journal of Geomechanics*, vol. 4, no. 2, 2004, pp. 115-126.
- [9] J. W., Hilf, *An Investigation of Pore Pressure in Compacted Cohesive Soils*, Technical Memorandum 654. Denver, CO: US Bureau of Reclamation, 1956.
- [10] D. G., Toll, *The Behaviour of Unsaturated Compacted Naturally Occurring Gravel*, PhD Thesis, The University of London, Imperial College of Science and Technology, London, UK, 1988.
- [11] Y. J. Cui, and P. Delage, "Yielding and plastic behavior of an unsaturated compacted silt," *Geotechnique*, vol. 46, no. 2, 1996, pp. 291-311.
- [12] E. J. Macari, J. K. Parker, and N. C. Costes, "Measurement of volume changes in triaxial tests using digital imaging techniques," *Geotechnical Testing Journal*, Vol. 20, No. 1, 1997, pp. 102-109.
- [13] A. Uchaipichat, *Experimental Investigation and Constitutive Modelling of Thermo-Hydro-Mechanical Coupling in Unsaturated Soils*, PhD Thesis, The University of New South Wales, NSW, AUSTRALIA, 2005.

**Anuchit Uchaipichat** was born in Nakhonratchasima, Thailand. He obtained Phd in Geotechnical Engineering from the University of New South Wales, Australia in 2005. He is currently an assistant professor at Vongchavalitkul University in Thailand. His major fields of study are unsaturated soil behaviors, ground improvements and geoenvironmental engineering.