# Space Vector PWM Simulation for Three Phase DC/AC Inverter 

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#### Abstract

Space Vector Pulse Width Modulation SVPWM is one of the most used techniques to generate sinusoidal voltage and current due to its facility and efficiency with low harmonics distortion. This algorithm is specially used in power electronic applications. This paper describes simulation algorithm of SVPWM \& SPWM using MatLab/simulink environment. It also implements a closed loop three phases DC-AC converter controlling its outputs voltages amplitude and frequency using MatLab. Also comparison between SVPWM \& SPWM results is given.


Keywords- DC-AC Converter, MatLab, SPWM, SVPWM, Vref - rotating frame.

## I. Introduction

THE DC-AC converters are one of the most popular domains for research and development, because of its large utilization in industrial applications. As a result many algorithms in this domain was developed such as SVPWM, dq-rotating frame control, direct torque control DTC.......etc. These algorithms are able to control output frequency and amplitude of the power module of DC-AC converter.

## II. Space Vector Modulation Theory

The main idea behind SVPWM [1] , [2] is to divide the 2Dplane into six equal areas each of them is called sector. As shown in Fig.1. Each sector is determined by four vectors $V_{i}, V_{i+1}$ where $i \in\{1, . ., 5\}$ these vectors are called active vectors because when these vectors are applied to the power module Fig. 2 the output voltage of the power module, will be greater than zero i.e. one of the switches $\mathrm{S}_{\mathrm{a}}, \mathrm{S}_{\mathrm{b}}, \mathrm{S}_{\mathrm{c}}$ will not be off. The other two vectors $\mathrm{V}_{0}$ and $\mathrm{V}_{7}$ are called inactive vector, because all switches will be off or on. These two vectors allocate in the center of the circle C of Fig. 1.

The desired output assigned $\mathrm{V}_{\text {ref }}$ will rotate in angular velocity equal to desired output AC voltage frequency.

Rotating $\mathrm{V}_{\text {ref }}$ vector will scan all sectors with the time. For every sample time we can determine the sector containing $\mathrm{V}_{\text {ref }}$, and calculate the time period for each vector of the determined sector.
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Fig. 1 SVPWM 2D plot
To each active vector corresponds a switch sequence Fig.2, $S_{a} \mathrm{~S}_{\mathrm{b}} \mathrm{S}_{\mathrm{c}}$ [3], for example the sequence:

$$
\begin{align*}
& S_{a} \mathrm{~S}_{\mathrm{b}} \mathrm{~S}_{\mathrm{c}}=010  \tag{1}\\
& \mathrm{~S}_{\mathrm{a}}^{\prime} \mathrm{S}_{\mathrm{b}}^{\prime} \mathrm{S}_{\mathrm{c}}^{\prime}=101 \tag{2}
\end{align*}
$$

Corresponds to vector $\mathrm{V}_{3}$.


Fig. 2 Three legs power module
As we know that:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{An}}=-\frac{\mathrm{U}_{\mathrm{dc}}}{3}, \mathrm{~V}_{\mathrm{Bn}}=\frac{2 \mathrm{U}_{\mathrm{dc}}}{3}, \mathrm{~V}_{\mathrm{Cn}}=-\frac{\mathrm{U}_{\mathrm{dc}}}{3} \tag{3}
\end{equation*}
$$

The three phase output voltage can be transformed from switch sequence to $\alpha \beta 0$ using this matrix equation:

$$
\left[\begin{array}{l}
V_{\alpha}  \tag{4}\\
V_{\beta}
\end{array}\right]=\frac{U_{d c}}{3}\left[\begin{array}{ccc}
2 & -1 & -1 \\
0 & \sqrt{3} & -\sqrt{3}
\end{array}\right]\left[\begin{array}{c}
S_{a} \\
S_{b} \\
S_{c}
\end{array}\right]
$$

For $\mathrm{V}_{3}$ :

$$
\begin{gather*}
{\left[\begin{array}{l}
V_{\alpha} \\
V_{\beta}
\end{array}\right]_{(010)}=\frac{U_{d c}}{3}\left[\begin{array}{ccc}
2 & -1 & -1 \\
0 & \sqrt{3} & -\sqrt{3}
\end{array}\right]\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right]}  \tag{5}\\
{\left[\begin{array}{l}
V_{\alpha} \\
V_{\beta}
\end{array}\right]_{(010)}=U_{d c}\left[\begin{array}{c}
\frac{-1}{3} \\
\frac{1}{\sqrt{3}}
\end{array}\right]} \tag{6}
\end{gather*}
$$

From equation (6) we can write:

$$
\begin{gather*}
\theta=\tan ^{-1}\left[\frac{V_{\beta}}{V_{\alpha}}\right]=\frac{2 \pi}{3}  \tag{7}\\
V_{\text {ref }}=\sqrt{V_{\alpha}^{2}+V_{\beta}^{2}}=\frac{2 U_{d c}}{3} \tag{8}
\end{gather*}
$$

Thus the vector corresponds to the sequence 010 is the vector $\mathrm{V}_{3}$. The other switch sequence can be found similarly [4], as shown in Fig. 3.


Fig. 3 SVPWM Vectors sequence
For each sector we can determine an ON/OFF time period to each switch. As an example let's take the sector number one see Fig. 4, this sector is defined by two active vectors $\mathrm{V}_{1}$ during the period $t_{a}$ and $V_{2}$ during the period $t_{b}$ while inactive vectors $\mathrm{V}_{0}$ and $\mathrm{V}_{7}$ are applied during $2 \mathrm{t}_{0}$.


Fig. 4 Sector 1 Vectors
So the space vector $\mathrm{V}_{\text {ref }}$ is given by the following formula:

$$
\begin{gather*}
\int_{0}^{T_{c h}} V_{r e f} d t=\int_{0}^{t_{a}} V_{1} d t+\int_{t_{a}}^{t_{a_{+}} t_{b}} V_{2} d t+\int_{t_{a+} t_{b}}^{t_{a+} t_{b+} t_{0}} V_{0} d t+ \\
\int_{t_{a+} t_{b+} t_{0}}^{t_{c h}} V_{7} d t \tag{9}
\end{gather*}
$$

In $\alpha \beta 0$ coordination and from the above equation we can find:

$$
\begin{gather*}
t_{a}=T_{c h} \frac{3\left|V_{r e f}\right|}{2 U_{d c}} \frac{\sin \left(\frac{\pi}{3}-\theta\right)}{\sin \left(\frac{\pi}{3}\right)}  \tag{10}\\
t_{b}=T_{c h} \frac{3\left|V_{r e f}\right|}{2 U_{d c}} \frac{\sin (\theta)}{\sin \left(\frac{\pi}{3}\right)}  \tag{11}\\
t_{0}=t_{7}=\frac{T_{c h}-t_{a}-t_{b}}{2} \tag{12}
\end{gather*}
$$

For sector number k we can generalize the above equations as follow:

$$
\begin{gather*}
t_{a}=T_{c h} \frac{3\left|V_{r e f}\right|}{2 U_{d c}} \frac{\sin \left(\frac{\pi}{3} k-\theta\right)}{\sin \left(\frac{\pi}{3}\right)}  \tag{13}\\
t_{b}=T_{c h} \frac{3\left|V_{r e f}\right|}{2 U_{d c}} \frac{\sin (\theta)}{\sin \left(\theta-\frac{\pi}{3}(k-1)\right)}  \tag{14}\\
t_{0}=t_{7}=\frac{T_{c h}-t_{a}-t_{b}}{2} \tag{15}
\end{gather*}
$$

According to the sector which contains $\mathrm{V}_{\text {ref }}$ and the switch state we can determine the ON/OFF period time for each switch. As shown in Table I.

TABLE I
Switch on Time for Each Period

| Sector | State | Time Period of $\mathrm{T}_{\mathrm{ch}}$ |
| :---: | :---: | :---: |
| 1 | $\mathrm{S}_{\mathrm{a}}=0$ | $\mathrm{t}_{\mathrm{a}}+\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{b}}=0$ | $\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{c}}=1$ | $\mathrm{t}_{0}$ |
| 2 | $\mathrm{Sa}_{\mathrm{a}}=0$ | $\mathrm{t}_{\mathrm{a}}+\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{b}}=1$ | $\mathrm{t}_{\mathrm{a}}+\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{c}}=1$ | $\mathrm{t}_{0}$ |
| 3 | $\mathrm{S}_{\mathrm{a}}=0$ | $\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{b}}=1$ | $\mathrm{t}_{\mathrm{a}}+\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{c}}=0$ | $\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
| 4 | $\mathrm{S}_{\mathrm{a}}=1$ | $\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{b}}=1$ | $\mathrm{ta}_{\mathrm{a}}+\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{c}}=0$ | $\mathrm{t}_{\mathrm{a}}+\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
| 5 | $\mathrm{S}_{\mathrm{a}}=1$ | $\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{b}}=0$ | $\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{c}}=0$ | $\mathrm{t}_{\mathrm{a}}+\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
| 6 | $\mathrm{Sa}_{\mathrm{a}}=1$ | $\mathrm{ta}_{\mathrm{a}}+\mathrm{t}_{\mathrm{b}}+\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{b}}=0$ | $\mathrm{t}_{0}$ |
|  | $\mathrm{S}_{\mathrm{c}}=1$ | $\mathrm{ta}_{\mathrm{a}}+\mathrm{t}_{0}$ |

## III. Over Modulation

Over-modulation occurs when the user tries to get output voltage with magnitude more than of that allocated inside the cotangent circle of hexagon. This is determined by active vector, as it depicted in Fig. 5:


Fig. 5 Over Modulation region

## IV. SVPWM Simulation

The "SVPWM" simulation has been done using MatLab/Simulink environment [5], [6] which it is easy to implement such algorithm. Another reason for using MatLab is its ability to simulate power module such as DC-AC converter.

The block diagram of "SVPWM" simulation system, shown in Fig. 6, consists of five sections. The first section defines the inputs commands. These commands are the desired amplitude, frequency of the output voltage. Additional inputs
are the chopping time Tz and the input rail dc voltage. The second part is the modulation algorithm which is named "SVPWM". The third part which is "Embedded MatLab Function" contains a code (.m file). In this file we assign the threshold gate voltage to the upper and lower ON/OFF switches of the next block. Part four contains three Legs Bridge, LC filter and the load.


Fig. 6 Block diagram schematic of the overall system
The block diagram of SVPWM constructed using MatLab/Simulink is shown in Fig. 7:


Fig. 7 Block diagram schematic of the SVPWM block

Part four of the overall simulated system is shown in Fig. 8. This part consists of three legs MOSFET transistor. MatLab/simulink environment permits us to enter the real MOSFET parameters [5], [6]. The simulation was done using IRFPC60 parameters. The cut off frequency of the LC filter shown in Fig. 9, Was $\mathrm{f}_{\mathrm{c}}=1600 \mathrm{~Hz}$. So $\mathrm{L} \approx 0.01 \mathrm{H}, \mathrm{C}=1 \mu \mathrm{~F}$, while the load is three variable resistance.


Fig. 8 Block diagram of the Power module, filter and load block


Fig. 9 LC Low Pass Filter and resistive load

## V. Simulation Results

After we have executed the overall simulink design of space vector PWM AC/DC converter using variable resistive load shown in Fig. 10, the simulation results of the output voltages ABC are shown in Fig. 11, which are three phases $120^{\circ}$ out of phase pure sin wave.


Fig. 10 Load Block diagram


Fig. 11 Simulation Result
The SVPWM simulation results plot in $\alpha \beta 0$ coordination of the output voltages ABC are shown in Fig. 11:


Fig. $12 \alpha \beta 0$ simulation result of SVPWM

From Fig. 12, we notice that the overall system gain is greater than one.

## VI. SPWM Simulation

The block diagram of the simulation system of Sine Pulse Width Modulation "SPWM" is indicated in Fig. 13. The SPWM contains two blocks, the first one, Fig. 14, is a SPWM generated by comparison between desired sine wave signal and triangle high frequency signal. The other block is the converter with a resistive load. The simulation results of the output voltages ABC are shown in Fig. 15:


Fig. 13 First block of SPWM


Fig. 14 Second block of SPWM
The output simulation results for SPWM as ABC phases are shown in Fig. 15:


Fig. 15 ABC Simulation Result of SPWM

The SPWM simulation results plot in $\alpha \beta 0$ coordination of the output voltages ABC are shown in Fig. 16:


Fig. $16 \alpha \beta 0$ Simulation results of SPWM

## VII. COMPARISON BETWEEN SPWM AND SVPWM

We plot the amplitude of vector $\mathrm{V}_{\text {ref }}$ versus time of the SVPWM and SPWM algorithms in Fig. 17 and Fig. 18. The comparison between these two plots shows that: for SPWM there are high fluctuations with respect to $\mathrm{V}_{\text {ref }}$ which can produce high harmonics distortion in the output voltages of the DC/AC converter. While in SVPWM the total harmonic distortion is smaller than SPWM. And the gain of overall system for SVPWM is greater than one.


Fig. $17 \mathrm{~V}_{\text {ref }}$ plot of SPWM DC/AC Converter


Fig. $18 \mathrm{~V}_{\text {ref }}$ plot of SVPWM DC/AC Converter

## VIII. Closed Loop Controlling Output Voltage Amplitude of DC/AC Converter

The control algorithm is based on dq rotating frame PI corrector. This control procedure is widely used to control the amplitude voltages of DC/AC converter [8]. In our simulation work we used a similar procedure, ( $\mathrm{V}_{\text {ref }}$ Rotating frame control) the block diagram of closed loop control algorithm is shown in Fig. 19. It contains PI compensator, hard limiter, abc $\rightarrow \alpha \beta 0 \rightarrow V_{\text {ref }}$ transformers and comparators. In this method the compensator pursues $V_{\text {ref }}$ directly.


Fig. $19 \mathrm{~V}_{\text {ref }}$ Rotating frame control block diagram
The Simulink result of this procedure is shown in Fig. 20, in $\alpha \beta 0$, which indicates a good tracking of output voltage with $V_{\text {ref }}$.


Fig. 20 Simulation Result of plot of $\mathrm{V}_{\text {ref }}$ Rotating frame control

## IX. Conclusion

The simulation of DC/AC converter with SVPWM algorithm is verified using MatLab/Simulink. And the $\mathrm{V}_{\text {ref }}$ rotating frame algorithm is done in this algorithm. The simulation results have low harmonics distortion and good stability [7], as a function of the frequency and amplitude. After such simulation it is easy to implement [8], [9] this converter using hard-ware circuit such as FPGA or TMS, which is our next work.

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