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Multiple Crack Identification Using Frequency Measurement

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***Abstract*—**This paper presents a method to detect multiple cracks based on frequency information. When a structure is subjected to dynamic or static loads, cracks may develop and the modal frequencies of the cracked structure may change. To detect cracks in a structure, we construct a high precision wavelet finite element (EF) model of a certain structure using the B-spline wavelet on the interval (BSWI). Cracks can be modeled by rotational springs and added to the FE model. The crack detection database will be obtained by solving that model. Then the crack locations and depths can be determined based on the frequency information from the database. The performance of the proposed method has been numerically verified by a rotor example.

***Keywords*—**Rotor, frequency measurement, multiple cracks, wavelet finite element method, identification.

I. Introduction

ALL metal members that are subjected to vibration and cyclic stresses in more or less localized areas, cracks may occur. Since cracks cannot be easily seen with the naked eyes, the non-destructive testing methods like ultrasonic testing, X-ray, etc. can be used to detect them. However, these methods are costly and time-consuming for complex or large structures. For this reason, the vibration-based structural health monitoring methods, especially those based on the change of modal parameters (frequencies, shape and damping), have been explored for detecting cracks [1-5]. Some results are summarized in [6-8].

However, only the single crack detection methods are well established. These methods involve the prediction of the response of structures with a transverse crack and the detection of transverse cracks by finite element or other numerical methods. Using the linear facture mechanics theory, the local flexibility or stiffness introduced by the crack is evaluated, neglecting the effects that may be incorporated into the mass and damping matrices. There are two procedures to assess the progress of crack detection in structures. The first procedure is forward problem analysis, which considers the construction of a crack stiffness matrix exclusively for the crack section, then the finite element model for crack structures is developed to obtain modal parameters at various crack locations and depths, such as natural frequencies, modal shape, modal damping, etc. These parameters constitute the so-called crack detection database. The second procedure is inverse problem analysis, which considers the measurement of dynamic parameters and searches for crack location from the results of forward problem analysis.

The most popular approach which is particularly well suited for modeling structures is the finite element method. However, the traditional FEM cannot obtain satisfactory results for eigen-value problems. Wavelet finite element method [9-11] is a relatively new numerical method for analyze structural analysis. The numerical simulation precision of this method is higher than that of traditional FEM or boundary element method (BEM).

The purpose of the present work is to establish a method for predicting the normalized locations and depths of multiple transverse cracks in beam-like structures by considering only the few lowest frequencies of the cracked structures. We combine the BSWI wavelet-based beam element with root-mean-square (RMS) to for effective crack detection. The BSWI beam element is employed to obtain a more accurate crack detection database. The model-based inverse problems are solved by computing RMS value between the measurement and calculation frequencies. The application of the proposed method is illustrated by simulating a rotor with two cracks.

II. BSWI Finite Element Model

Goswami et al [12] constructed BSWI functions, and presented unification formulas. The BSWI is defined on the bounded interval [0, 1] and the multilevel interpolating functions on a bounded interval have limited dimension towards every scaling space, which can be regarded as a set of self-contained interpolating basis. Therefore, the BSWI beam elements have been successfully applied to detect single crack in single cantilever beam.Denote m and j as the order and scale of BSWI respectively. The j scale m th order BSWI (simply denoted as BSWIm_j) scaling functions $\phi^j_{m,k}(\xi)$ and the corresponding wavelets $\psi^j_{m,k}(\xi)$ can be evaluated by the following formulas

$$\phi^j_{m,k}(\xi)=\begin{cases}\phi^l_{m,k}(2^{j-l}\xi), & k=-m+1,\cdots,-1\\ \phi^l_{m,2^j-m-k}(1-2^{j-l}\xi), & k=2^j-m+1,\cdots,2^j-1\\ \phi^l_{m,0}(2^{j-l}\xi-2^{-l}k), & k=0,\cdots,2^j-m\end{cases} \quad (1)$$

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$$\psi_{m,k}^{j}(\xi)=\begin{cases}\psi_{m,k}^{l}(2^{j-l}\xi), & k=-m+1,\cdots,-1\\ \psi_{m,2^{j}-2m-k+1}^{l}(1-2^{j-l}\xi), & k=2^{j}-2m+2,\cdots,2^{j}-m\\ \psi_{m,0}^{l}(2^{j-l}\xi-2^{-l}k), & k=0,\cdots,2^{j}-2m+1\end{cases} \quad (2)$$

The wavelet compactly supported intervals are

$$\operatorname{supp}\psi_{m,k}^{j}(\xi)=\begin{cases}[0,\ (2m\text{-}1+k)2^{-j}]\\ [k2^{-j},1]\\ [k2^{-j},(2m-1+k)2^{-j}]\end{cases} \quad (3)$$

The one-dimensional scaling functions $\mathbf{\Phi}$ at the lower resolution approximation space V_j are given by

$$\mathbf{\Phi}=\left\lfloor \phi_{m,-m+1}^{j}(\xi)\ \ \phi_{m,-m+2}^{j}(\xi)\ \ldots\ \phi_{m,2^{j}-1}^{j}(\xi)\right\rfloor \quad (4)$$

The semi-orthonormal wavelets $\mathbf{\Psi}$ at detail space W_j are

$$\mathbf{\Psi}=\left\lfloor \psi_{m,-m+1}^{j}(\xi)\ \psi_{m,-m+2}^{j}(\xi)\ \cdots\ \psi_{m,2^{j}-m}^{j}(\xi)\right\rfloor \quad (5)$$

To illustrate, the scaling functions $\phi_{m,k}^{j}(\xi)$ for order $m=4$ at scales j=5 and 6 are shown in Fig.1(a) and (b) respectively.

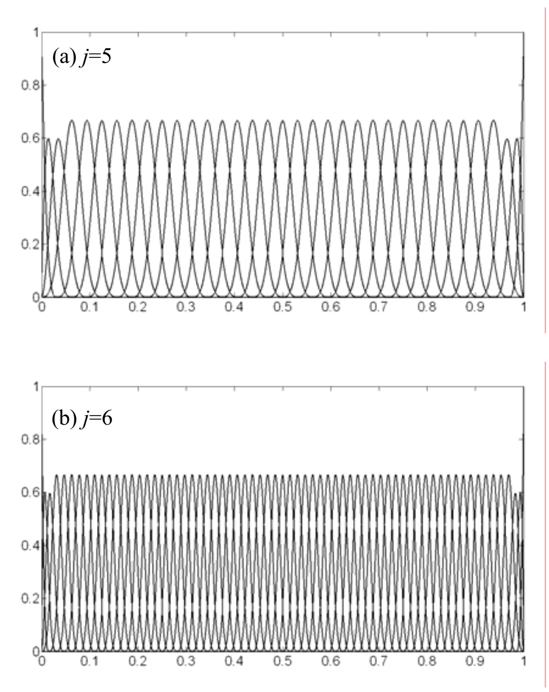


Fig. 1 BSWI scaling functions

Applying the BSWI beam element to the discrete beam-like structures, the free vibration frequency equation can be obtained [13]

$$\left|\overline{\mathbf{K}}-\omega^{2}\overline{\mathbf{M}}\right|=0\,, \quad (6)$$

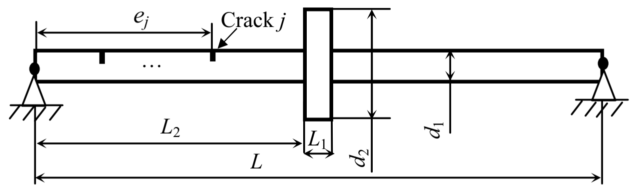
where $\overline{\mathbf{K}}$ and $\overline{\mathbf{M}}$ are the global stiffness and mass matrices and the detailed expressions are shown in [13].

III. Detection of Multiple Cracks

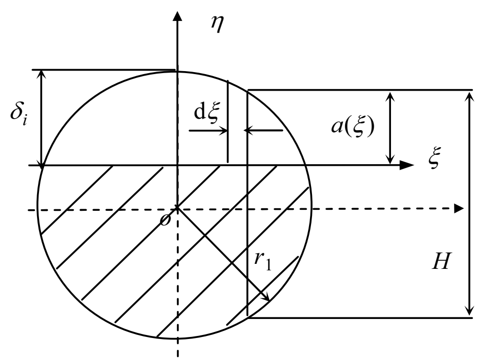
As the modal frequencies can be easily and inexpensively acquired by frequency measurement and the linear rotational spring model can effectively describe open cracks, we develop our method based on the open cracks in rotor.

A Forward problem

Fig. 2 shows a simply supported rotor system with n cracks in the left shaft. The geometry and the cross-section of the cracked shaft are shown in Figs. 2(a) and (b) respectively. L, $L_1$and L_2 are the shaft length, the disc width and the right shaft length respectively. $e_i\,(i=1,2,\cdots,n)$ denote crack locations, h is the height and b is the width of cross-section, $c_i\,(i=1,2,\cdots,n)$ represent crack depths, d_1 is the shaft diameter, δ_i the depth of the ith crack, and n the number of cracks. Referring to Fig. 2, the relative crack location and crack depth can then be denoted by $\beta_i=e_i/L_2$ and $\alpha_i=\delta_i/d_1$, respectively



(a) The geometry of the rotor system



(b) The cross-section of the cracked shaft

Fig. 2 Simply supported rotor system with two cracks in the left shaft

The continuity conditions at crack position indicate that the left node and right node have the same transverse displacement while their rotations are connected through the crack stiffness submatrix $\mathbf{K}_S$ as follows [13]

$$\mathbf{K}_S = \begin{bmatrix} K_t & -K_t \\ -K_t & K_t \end{bmatrix} \quad (7)$$

For a cracked shaft with circular cross-section, K_t is calculated by combination of a series of thin strips as [13]

$$k_t = \frac{\pi E r_1^{8}}{32(1-\mu)} \frac{1}{\int_{-r_1\sqrt{1-(1-2\alpha_i)^2}}^{r_1\sqrt{1-(1-2\alpha_i)^2}} (r_1^2-\xi^2)[\int_0^{a(\xi)} \eta F^2(\eta/H)\mathrm{d}\eta]\mathrm{d}\xi} \quad (8)$$

where δ_i is the depth of the ith crack, r_1 radius of the shaft, μ the Possion's ratio, $\alpha_i = \delta_i / 2r_1$ denotes normalized crack depth, $a(\xi) = 2r_1\alpha_i - (r_1 - \sqrt{r_1^2 - \xi^2})$ and $H = 2\sqrt{r_1^2 - \xi^2}$ are respectively the crack depth and height of a thin strip (Fig. 2(b)), and $F(\eta / H)$ is stress intensity function which is given by the following experimental formula [14]

$$F(\eta/H) = 1.122 - 1.40(\eta/H) + 7.33(\eta/H)^2 - 13.08(\eta/H)^3 + 14.0(\eta/H)^4 \quad (9)$$

According to the crack location β_i, we can assemble stuffiness submatrix of the cracked structure into the global stuffiness matrix in the corresponding place. The global mass matrix of cracked rotor system is the same as the uncracked one.

To construct an accurate crack detection database, the wavelet-based element proposed herein is applied to the forward problem analysis. The functions of the lowest frequencies of crack locations and depths are obtained as follows:

$$f_j = F_j(\alpha_1,\cdots,\alpha_n,\beta_1,\cdots,\beta_n),(j=1,2,\cdots,2n) \quad (10)$$

where n is the number of cracks in the shaft.

B Solving the inverse problem

To detect n cracks in a structure, inverse problem analysis is necessary, which considers the measurement of several lowest frequencies and searches for locations and depths of the cracks from crack detection databases obtained by forward problem analysis. Based on the studies of Dilena and Morassi [15], at least $2n$ frequencies are required as the inputs in order to detect n cracks. Therefore, the first $2n$ frequencies should be measured to obtain optimum crack parameters.

Based on Eq.(11), we have

$$(\alpha_1,\cdots,\alpha_n,\beta_1,\cdots,\beta_n) = F_j^{-1}(f_j),(j=1,2,\cdots,2n) \quad (11)$$

From Eq.(12), we can see clearly that the inverse problem of multi-crack detection is essentially a discrete optimization problem. To evaluate the errors of the input frequencies obtained by experimental measurement of real structures, Euclidean length (EL) is adopted as

$$EL = \sqrt{(f_1-\breve{f}_1)^2 + (f_2-\breve{f}_2)^2 + \cdots (f_{2n}-\breve{f}_{2n})^2} \quad (12)$$

where $f_1, f_2 \cdots, f_{2n}$ are the $2n$ frequencies in the crack detection database, whereas $\breve{f}_1, \breve{f}_2 \cdots, \breve{f}_{2n}$ stand for the measured frequencies by the experimental modal analysis (EMA) or operational modal analysis (OMA).

The commonly used root-mean-square (RMS) value obtained from EL is defined by

$$RMS = EL / \sqrt{2n} \quad (13)$$

From Eq.(14), we can search the optimization value from the crack detection database.

The procedure for multi-crack detection is presented in Fig. 3.

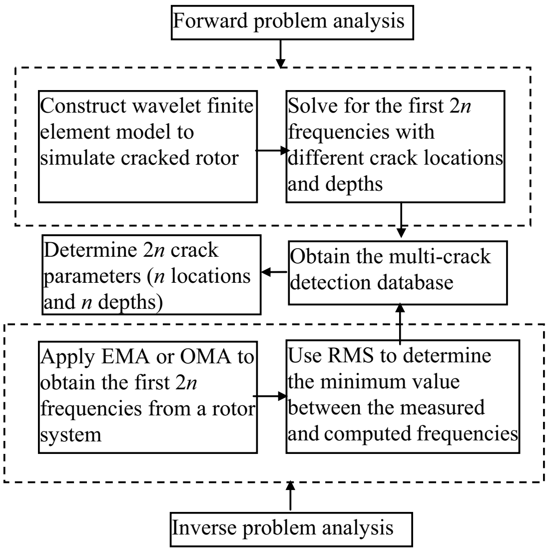


Fig.3 The multi-crack detection procedure

IV. Numerical Simulation

To examine the performance of the proposed method, we present the following numerical simulation analysis. Consider a rotor system shown in Fig.(2). Suppose the rotor dimensions and the material properties are: L = 1000 mm, L_1 = 50 mm, L_2 = 500 mm, d_1 = 20 mm, d_2 = 100 mm, Young's modulus E = 2.06×10^{11}N/m^2, material density ρ = 7860 kg/m^3, Poisson's ratio μ = 0.3. The crack cases are shown in Table 1.

Five BSWI43 beam elements with only 49 degrees of freedom (DOFs) are used and the frequency results are similar to those of 200 traditional beam elements with 402 DOFs, as shown in Table 2. The number of DOFs needed for the wavelet-based elements is only 1/8 of that for the traditional beam element. This shows the wavelet-based element has better performance in solving eigenvalue problems.

For the cases investigated (Table 1), the first four frequencies as functions of α_1 and α_1 with $\beta_1 = 0.2$ and $\beta_2 = 0.4$ can be seen in Fig. 4 (a), (b), (c) and (d).

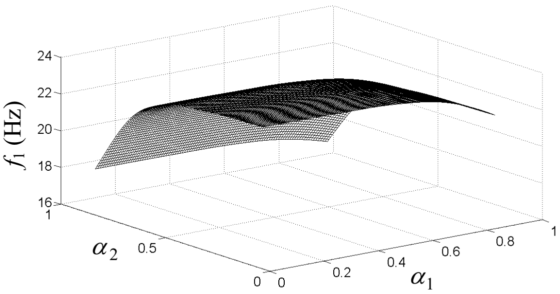
TABLE I
CRACK CASES

case	α_2	α_1	β_2	β_1
1	0.3	0.5	0.4	0.2
2	0.3	0.3	0.3	0.3
3	0.4	0.3	0.7	0.1
4	0.2	0.5	0.6	0.2
5	0.5	0.4	0.5	0.3
6	0.4	0.5	0.5	0.3
7	0.2	0.3	0.3	0.1
8	0.4	0.5	0.8	0.7
9	0.6	0.3	0.6	0.2
10	0.3	0.6	0.2	0.1

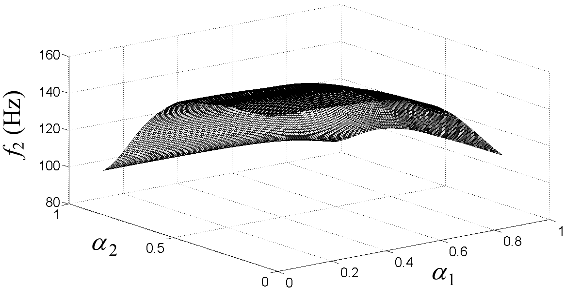
Note: α_2, α_1, β_2 and β_1 denote the crack depths and locations

TABLE II
THE COMPARISION BETWEEN THE WAVELET-BASED ELEMENT AND TRADITIONAL BEAM ELEMENT

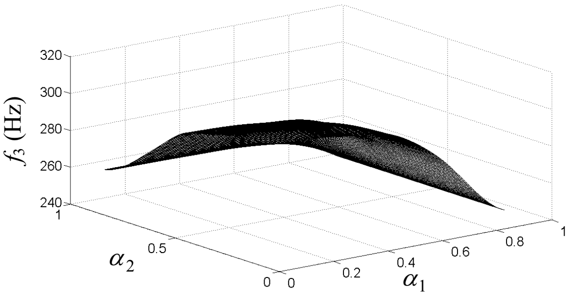
case	Wavelet-based element				Traditional element			
	f_1	f_2	f_3	f_4	f_1	f_2	f_3	f_4
1	23.2	153.5	294.5	588.9	23.2	153.5	294.3	589.1
2	26.0	162.3	315.4	620.2	26.1	162.2	315.4	620.2
3	22.8	155.1	302.8	598.7	22.8	155.0	302.8	598.9
4	23.2	154.8	296.9	589.3	23.1	154.8	297.1	589.5
5	22.7	148.3	292.8	600.2	22.6	148.3	292.9	600.4
6	22.9	149.0	291.5	592.1	23.0	148.9	291.6	592.1
7	23.3	157.5	301.7	610.4	23.4	157.7	301.6	610.5
8	21.9	152.5	301.1	583.0	22.0	152.5	301.1	583.0
9	21.8	144.0	298.4	590.0	21.7	144.2	298.3	590.2
10	23.3	155.6	296.6	583.1	23.2	155.8	296.5	583.0



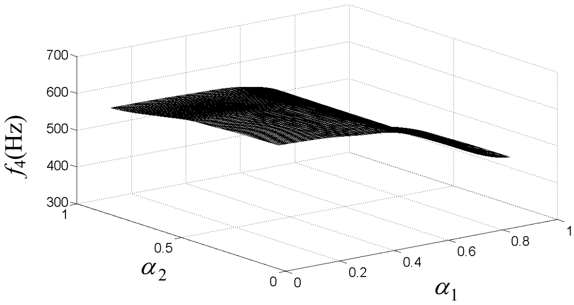
(a) The first frequency



(b) The second frequency



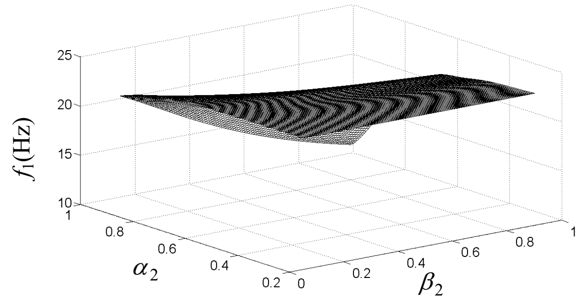
(c) The third frequency



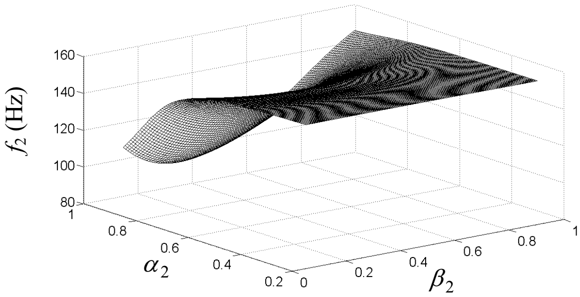
(d) The fourth frequency

Fig. 4 The first four frequencies as functions of the cracks' α_1 and α_1 with $\beta_1 = 0.2$ and $\beta_2 = 0.4$

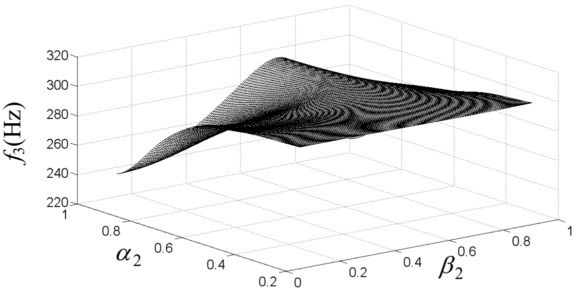
Fig. 5 shows the first four frequencies as functions of the second crack's α_2 and β_2 with α_1 and β_1 fixed at 0.4 and 0.1 respectively. When one crack is kept constant, the relationships between the first four frequencies and α_2 and β_2 are shown in Fig. 5 (a), (b), (c) and (d).It is observed from Figs. 4 and 5 that the first four frequencies are different for different each crack cases. Therefore, we can detect the two cracks based on such differences between different crack cases. However, the relationships between the frequencies and the corresponding crack locations and depths are very complex. Therefore, we need use RMS or other optimization methods to detect multiple cracks in rotor systems. In the simulation analysis, the measured four frequencies for crack detection are replaced by the first four simulated frequencies computed using traditional beam element as shown in Table 2. To simulate frequency measurement errors, we add some random noise whose amplitude is bounded by [-1,1] to each simulation frequency. Table III shows the predicted crack locations and depths are 100 % accurate (compared to Table 1). It should be pointed out that if there exist large measured errors introduced by, e.g., measuring systems, structural boundary conditions, and material inner damping, the prediction may not be 100 % accurate. However, we can select the inimum root-mean-square (RMS) values to determine the crack parameters. The results in Table 3 can also help to determine the actual number of cracks. For example in case 2, $\beta_1 = \beta_2 = 0.3$ indicates that there is only one crack.



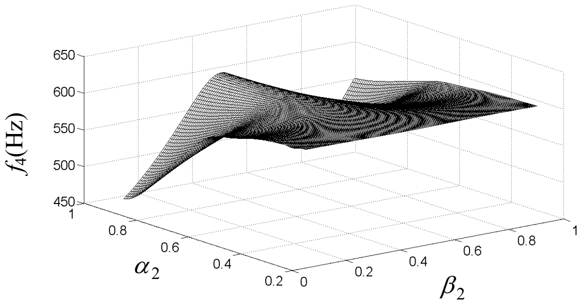
(a) The first frequency



(b) The second frequency



(c) The third frequency



(d) The fourth frequency

Fig.5 The first four frequencies as functions of the second crack's α_2 and β_2 with $\alpha_1 = 0.4$ and $\beta_1 = 0.1$

The above example clearly demonstrates that the proposed method yield results that are comparable to these obtained via the traditional beam element with substantially fewer elements. The computational time for the forward problem can thus be reduced considerably. The inverse problem can also be solved to determine the number of cracks, their locations and severity based on the minimum RMS values.

TABLE III
THE PREDICTED RESULTS

case	α_2^*	α_1^*	β_2^*	β_1^*	RMS
1	0.3	0.5	0.4	0.2	0.8377
2	0.3	0.3	0.3	0.3	1.1295
3	0.4	0.3	0.7	0.1	1.0597
4	0.2	0.5	0.6	0.2	0.5715
5	0.5	0.4	0.5	0.3	1.0953
6	0.4	0.5	0.5	0.3	0.9158
7	0.2	0.3	0.3	0.1	0.9991
8	0.4	0.5	0.8	0.7	0.8710
9	0.6	0.3	0.6	0.2	1.2199
10	0.3	0.6	0.2	0.1	0.8163

Note: α_2^*, α_1^*, β_2^* and β_1^* denote the predicted crack depths and locations

V. CONCLUSION

A new methodology based on BSWI element for the detection of the locations and sizes of multiple cracks has been developed. The BSWI element presented in this paper is a useful tool with high computational efficiency in structural crack identification. Our numerical analysis indicates that the proposed method can be used to accurately detect locations as well as sizes of multiple cracks.

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