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Support Vector Machines Approach for Detecting the Mean Shifts in Hotelling's T^2 Control Chart with Sensitizing Rules

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***Abstract*—**In many industries, control charts is one of the most frequently used tools for quality management. Hotelling's T^2 is used widely in multivariate control chart. However, it has little defect when detecting small or medium process shifts. The use of supplementary sensitizing rules can improve the performance of detection. This study applied sensitizing rules for Hotelling's T^2 control chart to improve the performance of detection. Support vector machines (SVM) classifier to identify the characteristic or group of characteristics that are responsible for the signal and to classify the magnitude of the mean shifts. The experimental results demonstrate that the support vector machines (SVM) classifier can effectively identify the characteristic or group of characteristics that caused the process mean shifts and the magnitude of the shifts.

***Keywords*—**Hotelling's T^2 control chart, Neural networks, Sensitizing rules, Support vector machines.

I. Introduction

STATISTICAL Process Control (SPC) is often used in quality management for modern industry. The main purpose of SPC supervise process is stable or out of control. Control chart has become some of the most frequently used tools for improving quality. However, there are many instances requiring the simultaneous control of two or more quality characteristics of the output of a production process in which the corresponding quality measurements are correlated. Traditional single variant control chart such as Shewhart, exponentially weighted moving average (EWMA), and cumulative sum (CUSUM) cannot solve the correlation problem. We will be interested in the monitoring of the vector of means of a multivariate quality measurement, such as Multivariate EWMA (MEWMA), Multivariate CUSUM (MCUSUM), Hotelling's T^2 control chart, etc. Hotelling's T^2 control chart is a widely tool to detect process mean variation. It has not significance effect when using Hotelling's T^2 control chart to monitor process means as mean shift range is medium and small. Aparisi et al. [1] shows that Hotelling's T^2 control chart with adding sensitizing rules to monitor process change can enhance the performance of control chart.

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When the mean of a process is shift, from Multivariate control chart we can only know process is out of control, but cannot know which quality characteristics is out-off-control. Reference [2] presented a method that incorporated principal component analysis (PCA) to transform the original p variables into lower dimensional principal components, which are linear combinations of the original variables. Reference [3] proposed used MYT method to identify particular subset that causes an out-of-control signal. However, the computation process is complex. Artificial neural networks (ANNs) are another method to solve this problem. Many researchers have investigated the application of artificial neural networks to SPC. Reference [4] successfully trained a back-propagation neural network to detect shifts in mean. Consequently, Reference [5] develops a fuzzy artificial neural network-based model to supplement the multivariate x^2 control chart. Subsequently, reference [6] develops an artificial neural network-based model to supplement the multivariate x^2 chart. The method not only identifies the characteristic or group of characteristics that cause the signal but also classifies the magnitude of the shifts.

Traditional SPC techniques of control chart are not applicable in many process industries where the data from the facilities are often autocorrelated. Reference [7] demonstrates that support vector machines (SVMs) can be extremely effective in minimizing both Type-I errors (probability that the method would wrongly declare the process to be out of control or generate a false alarm) and Type-II errors (probability that the method will be unable to detect a true shift or trend present in the process) in these autocorrelated processes.

This research used Hotelling's T^2 control chart with sensitizing rule to enhance the efficiency of multivariate control chart when monitoring the mean shift at small and medium range. Consequently, SVM is applied to classify quality characteristic of mean shift when process is out of control.

II. Methodology

The first procedure of the constructed model is to collect real-time online processes information. Consequently, four sensitizing rules of Hotelling's T^2 control chart are used to detect process whether it is out-of-control. Classifier SVM is used to identify which quality characteristics caused the mean shift as well as the magnitude of process variation.

The procedures of the proposed model can be divided into three steps. The first step is to collect real-time online process data. The second step is to detect Hotelling's T^2 control chart by adding a series of sensitizing rules. The third step is to use classifier (SVM) to recognize the quality characteristics causing out-of-control and magnitude of process mean shift.

There are four sensitizing rules of Hotelling's T^2 that applied to detect the mean shift of out-of-control, and the model therefore are named model 1, model 2, model 3 and model 4.

The classification results of SVM are corresponding to mean shift pattern. Consider *p* quality characteristics and every quality characteristic has *m* mean shift, there are totally m^p-1 shift combinations. The output range of SVM is 1 to m^p-1. When the output value of SVM is i, the type of shift for the qquality characteristic can be easily determined to be the i th shifted pattern. The mmagnitude of the mean shift also can be found.

The following section gives a detail description of these steps.

A. Hotelling's T^2 with Sensitizing Rule to Detect Control Chart

The most often used tool to detect the process mean variant is Hotelling's T^2 control chart. The large mean shift can be detected quickly. If the shift is median and small, the performance is worse. Reference [1] proposed Hotelling's T^2 control chart adding sensitizing rules to enhance the performance of detecting out-off-control process.

Suppose there are *p* quality characteristics in a process, Let $\bar{\mathbf{x}}$ be the sample mean of the *p* quality characteristics, i.e.,

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{bmatrix} \quad (1)$$

It can be shown that

$$n(\bar{x}-u)'\sum{}^{-1}(\bar{x}-u) \sim \chi_p^2 \quad (2)$$

where u is the mean and Σ is the covariance of the quality characteristics, and χ_p^2 here is the chi-squared distribution with *p* degrees of freedom.

If the Hotelling's T2 chart use the (1-α)th percentile, the upper control limit of the chart is expressed as $\chi_{p,\alpha}^2$

The following sensitizing rules are suggested for the Hotelling's x^2 control chart as shown in Fig.1.[1]

- Sensitizing rule 1: point above the control limit (CL).
- Sensitizing rule 2: two out of three consecutive points within the attention zone (zone A).
- Sensitizing s rule 3: eight consecutive points over the median (zone B).
- Sensitizing rule 4: seven consecutive rising points.

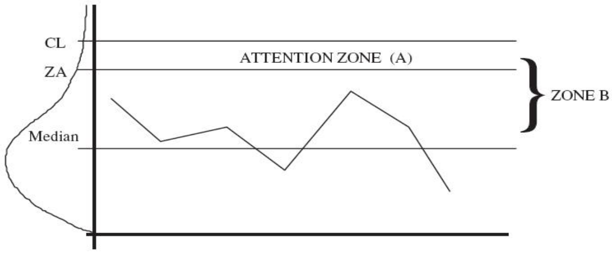

Fig. 1 Hotelling's x^2 control chart with supplementary sensitizing rules

where median line is defined as process is in control, $\chi_0^2 = n(\bar{x}-u)'\sum^{-1}(\overline{x-u})$ is the probability above 0.5.

Control limit (CL) and Alarm Zone (ZA) is determined by ARL. A widely used measure of the performance of a control chart is the average run length (ARL). The run length of the chart is the number of the first sample in which the chart gives the first signal of a potential out-of-control process. The mean of the run length distribution is commonly referred to as the ARL. The ARL is given by

$$ARL_d = \frac{1}{\alpha} \quad (3)$$

where $\alpha = p\left(T^2 \geq CL \middle| d\right)$. A value of 0.05 is used for α when d=0

The attention zone, zone A is delimited by the value called ZA, which complies with

$$p\left(T^2 \geq ZA \middle| d=0\right) = 0.03 \quad (4)$$

$$p\left(T^2 \geq \text{median} \middle| d=0\right) = 0.05 \quad (5)$$

B. SVM Classifier Applied to Justify the Process Mean Shifts and the Magnitude of the Shifts

In machine learning, support vector machines (SVMs) are supervised learning models with associated learning algorithms that analyze data and recognize patterns, used for classification and regression analysis. In its simplest linear form, an SVM is a hyperplane that separate a set of positive examples from a set of negative ones with maximum margin. In addition to performing linear classification, SVMs can efficiently perform non-linear classification.

SVMs are inherently two-class classifiers. Multiclass SVM aims to assign labels to instances by using support vector machines, where the labels are drawn from a finite set of several elements.The traditional way to do multiclass classification with SVMs is to use one of the methods discussed. In particular, the most common technique in practice one-against-one (OAO), one-against-all (OAA) and directed acyclic graph SVM (DAGSVM).That is selected by the most often used classifiers. The OAO is common often used to classify the mean shift for Hotelling T^2.

III. Numerical Experiment

A numerical experiment using simulated data was performed to demonstrate application of the proposed model. There are four out-off-controls for Hotelling T^2 with four sensitizing rules, and we construct a SVM (support vector machine) classifier corresponding four sensitizing rules named model 1, model 2, model 3 and model 4.

The details of formulation as follow:

A. Selecting Feature Vector

The creating process mean shift data refer to [6]. Mean shift pattern with adding sensitizing rule for Hotelling's T^2 control chart are generated. Reference [1] suggests 4 sensitizing rules. Sensitizing rule 1 detects large mean shift, sensitizing rule2, sensitizing rule 3 and sensitizing 4 detect medium and small mean shift. The samples in which the process is out of control are used as the training data and testing data of SVM classifier.

The selection of the feature vector in training crucially affects a neural network performance. The feature vector must be able to help in classifying shifts. Herein, each subgroup had five samples (n =5). The feature vector comprises raw observations X_{ij} and the corresponding mean vector

$$\overline{\mathbf{X}}_i = \frac{1}{n}\sum_{j=1}^{n}\mathbf{X}_{ij}$$

when sudden shifts occur at t_s, X_{ij} is expressed as follows.

$$\mathbf{X}(t) = \mu + \mathbf{Y}(t) + \mathbf{S}(t, t_s), \quad t \geq t_s \tag{6}$$

where

$\mathbf{X}(t)$: p quality characteristic measured at time t.

μ : process mean vector when the process is in control

$\mathbf{Y}(t)$: ~N(0,Σ)

$S(t,t_s):(k_1\sigma_1,k_2\sigma_2,...,k_p\sigma_p)$, where k_l is the magnitude of the process shifts in terms of σ_l, associated with the lth quality characteristic.

The study considered nine distinct types of shift associated with the lth quality characteristic; hence k_l has nine possible values, from -4 to +4 in increments of one. Hence simulation generates 75 input vectors for each type of shift. Two-thirds of these were used for training, and the rest of the data were for testing.

TABLE I
Relationship between Target Value and Type of Shift for the lth Quality Characteristic

Target value	Type of shift for the lth quality characteristic
-0.9	$-4\sigma_l$
-0.7	$-3\sigma_l$
-0.5	$-2\sigma_l$
-0.3	$-1\sigma_l$
0	0
0.3	$+1\sigma_l$
0.5	$+2\sigma_l$
0.7	$+3\sigma_l$
0.9	$+4\sigma_l$

Model 1 to detect mean large shift, hence k_l has nine possible values, and from -4 to + 4 therefore there are 48 kinds of mean shift combinations.

Model 2, model 3 and model 4 to detect medium and short mean shift, hence k_l has five possible values, from -2 to +2 in increments of one, therefore 16 kinds of mean shift combinations. Total there are 16 mean shift patterns. Table II shows the combinations.

TABLE II
16 Kinds of Mean Shift Combinations

16 combinations of shift pattern		
1 (0.3,0)	4 (0.5,0)	
2 (0.3,0.3)	5 (0.5,0.3)	13 (0,0.3)
3 (0.3,0.5)	6 (0.5,0.5)	14 (0,0.5)
7 (-0.3,0)	10 (-0.5,0)	
8 (-0.3,-0.3)	11 (-0.5,-0.3)	15 (0,-0.3)
9 (-0.3,-0.5)	12 (-0.5,-0.5)	16 (0,-0.5)

B. Data Formulation of Rule

The classifying module performs on-line classification. When the x^2 statistic value signals that a sample is out-of-control, the sample data are collected as input data to the trained network. As soon as the input data are passed through the trained network

Model 1, the sample point is one data point and the corresponding feature space is

$$\left(\mathbf{X}_{i1}, \mathbf{X}_{i2}, \ldots, \mathbf{X}_{ij}, \ldots, \mathbf{X}_{in}, \overline{\mathbf{X}}_i\right)$$

Model 2, the sample point is three data points and the corresponding feature space is

$$\begin{pmatrix} \mathbf{X}_{i1}, \ldots, \mathbf{X}_{in}, \overline{\mathbf{X}}_i, \mathbf{X}_{(i+1)1}, \ldots, \mathbf{X}_{(i+1)n}, \\ \overline{\mathbf{X}}_{(i+1)}, \mathbf{X}_{(i+2)1}, \ldots, \mathbf{X}_{(i+2)n}, \overline{\mathbf{X}}_{(i+2)} \end{pmatrix}$$

Model 3, the sample point is eight data points and the corresponding feature space is

$$\begin{pmatrix} \mathbf{X}_{i1}, \ldots, \mathbf{X}_{in}, \overline{\mathbf{X}}_i, \ldots, \mathbf{X}_{(i+j)1}, \ldots, \mathbf{X}_{(i+j)n}, \\ \overline{\mathbf{X}}_{(i+j)}, \ldots, \mathbf{X}_{(i+7)1}, \ldots, \mathbf{X}_{(i+7)n}, \overline{\mathbf{X}}_{(i+7)} \end{pmatrix}$$

Model 4, the sample point is seven data points and the corresponding feature space is

$$\begin{pmatrix} \mathbf{X}_{i1},\ldots,\mathbf{X}_{in},\overline{\mathbf{X}}_{i},\ldots,\mathbf{X}_{(i+j)1},\ldots,\mathbf{X}_{(i+j)n}, \\ \overline{\mathbf{X}}_{(i+j)},\ldots,\mathbf{X}_{(i+6)1},\ldots,\mathbf{X}_{(i+6)n},\overline{\mathbf{X}}_{(i+6)} \end{pmatrix}$$

C. Performance Evaluation

How to evaluate the correct rate of model are major concerns in this section. Meanwhile, in order to compare the performance of different methods, it is very important to have some performance index. The following metrics are used to evaluate the performance of the proposed methods. Table III shows the contingency table.

TABLE III
CONTINGENCY TABLE

Class C_i		Actual	
		YES	NO
Predicted	YES	TP_i	FP_i
	NO	FN_i	TN_i

where TP_i is the set of data correctly classified;
FP_i is the set of data wrongly accepted;
FN_i is the set of data wrongly rejected;
TN_i is the set of data correctly rejected;

1. Precision

$$\text{Precision} = \text{Pr}_i = \frac{|TP_i|}{|TP_i| + |FP_i|} \tag{7}$$

$$\text{Micro-average precision} = \hat{P}_r^U = \frac{\sum_{i=1}^{m}|TP_i|}{\sum_{i=1}^{m}(|TP_i| + |FP_i|)} \tag{8}$$

$$\text{Macro-average precision} = \hat{P}_r^M = \frac{\sum_{i=1}^{m}\text{Pr}_i}{m} \tag{9}$$

2. Recall

$$\text{Recall} = \text{Re}_i = \frac{|TP_i|}{|TP_i| + |FN_i|} \tag{10}$$

$$\text{Micro-average recall} = \hat{R}_e^U = \frac{\sum_{i=1}^{m}|TP_i|}{\sum_{i=1}^{m}(|TP_i| + |FN_i|)} \tag{11}$$

$$\text{Macro-average recall} = \hat{R}_e^M = \frac{\sum_{i=1}^{m}\text{Re}_i}{m} \tag{12}$$

3. F-Measure

$$F_\beta = \frac{(\beta^2+1)\times \text{Pr}\times \text{Re}}{(\beta^2 \times \text{Pr}) + \text{Re}} \tag{13}$$

$$\text{Micro-average F} = F_\beta = \frac{(\beta^2+1)\times \hat{P}_r^U \times \hat{R}_e^U}{(\beta^2 \times \hat{P}_r^U) + \hat{R}_e^U} \tag{14}$$

$$\text{Macro-average F} = F_\beta = \frac{(\beta^2+1)\times \hat{P}_r^M \times \hat{R}_e^M}{(\beta^2 \times \hat{P}_r^M) + \hat{R}_e^M} \tag{15}$$

$$F_\beta = \frac{(\beta^2+1)\times \text{Pr}\times \text{Re}}{(\beta^2 \times \text{Pr}) + \text{Re}} \tag{16}$$

By exercise above indices, one can evaluate the performance of the model.

IV. RESULTS

To elucidate the importance of sensitizing rules for Hotelling T^2 applied to SVM classification, four models were constructed to detect out-of-control samples with control chart. The characteristics of quality of cause out-of-control and the magnitude of mean shift were found. Then, the confusion matrix was used for the evaluation accuracy, precision, recall and F-measure. Accuracy Macro-average precision, Macro-average recall, Macro-average F are evaluated the model efficiency.

From Table IV, we can find, the accuracy of proposed model is up to 80% except for model 1. The classification accuracy of model 3 and model 4 is up to 98%.

The classification accuracy rate of model 1 is below obviously the 7 others three patterns; the reason can be input feature space is different. The input feature space of conducted model 1 is one data point; the input feature space of model 2 is three data points, eight data points for model 3 each time, seven data points for model 4. The information gain most is model 3, second is model 4, third is model 2, and the last is model1.

TABLE IV
THE OVERALL PERFORMANCE OF SVM MODEL

	Accuracy	Macro Average precision	Macro Average recall	Macro Average F
Model 1	58.75%	59.3%	58.83%	59.07%
Model 2	81.25%	83.46%	81.25%	82.34%
Model 3	99.75%	99.76%	99.75%	99.75%
Model 4	98.5%	99.01%	98.5%	98.75%

V. CONCLUSION

SVM is widely applied to many fields. This paper applied SVM to detect the mean shifts in Hotelling's T^2 control chart with sensitizing Rules. There are two findings in this paper. First, it demonstrates that the performance of Hotelling's T^2 with sensitizing rules is good at recognizing mean shifts quality characteristic as well as the individual pattern of mean shift in correlated manufacturing processes.

The classification accuracy for recognizing mean shifts manufacturing processes is superior. Some of cases, the feature vector input size will influence the classification accuracy. In general, the accuracy is better if the vector input size is bigger.

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