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A Novel Estimation Method for Integer Frequency Offset in Wireless OFDM Systems

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Abstract—Ren et al. presented an efficient carrier frequency offset (CFO) estimation method for orthogonal frequency division multiplexing (OFDM), which has an estimation range as large as the bandwidth of the OFDM signal and achieves high accuracy without any constraint on the structure of the training sequence. However, its detection probability of the integer frequency offset (IFO) rapidly varies according to the fractional frequency offset (FFO) change. In this paper, we first analyze the Ren's method and define two criteria suitable for detection of IFO. Then, we propose a novel method for the IFO estimation based on the maximum-likelihood (ML) principle and the detection criteria defined in this paper. The simulation results demonstrate that the proposed method outperforms the Ren's method in terms of the IFO detection probability irrespective of a value of the FFO.

Keywords—Orthogonal frequency division multiplexing, integer frequency offset, estimation, training symbol

I. Introduction

DUE to the fact that orthogonal frequency division multiplexing (OFDM) has many advantages including robustness to multipath fading, simple equalizer structure, and high transmission efficiency [1], OFDM has been widely adopted in communication systems such as digital audio broadcasting (DAB), digital video broadcasting-terrestrial (DVB-T), and the IEEE 802.11a wireless local area network (WLAN) [2]. However, OFDM is very sensitive to carrier frequency offset (CFO) caused by Doppler shift and/or the mismatch of the oscillators in the transmitter and receiver, which could destroy orthogonality among sub-carriers, bringing on inter-carrier interference (ICI). Thus, accurate estimation of CFO is one of the most important technical issues for reliable transmission in OFDM systems.

Many methods for CFO estimation have been proposed [3]-[6]. Moose presented the maximum-likelihood (ML) CFO estimation method based on two consecutive and identical training symbols [3]. The estimation range of the Moose's method is equal to half the sub-carrier spacing. Schmidl and Cox (SC) proposed the CFO estimation method using a training symbol with two identical halves [4], whose estimation range is equal to the sub-carrier spacing. Morelli and Mengali (MM) improved the SC method by using the best linear unbiased estimation (BLUE) principle [5]. The MM method uses only one training symbol composed of $L > 2$ identical parts and its estimation performance is quite close to the Cramer-Rao lower bound (CRLB). Laourin et al. proposed a new training symbol structure having phase difference and a CFO estimation method which offers a wide estimation range with the reduced computational load [6].

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In order to provide high accuracy in the CFO estimation, all of the methods mentioned above require a specific training symbol structure so that their application to the generalized training symbol is limited. Recently, an efficient three-step CFO estimation method of which the performance is independent from the structure of the training symbol has been developed by Ren et al. [7]. The Ren's method does not only provide the accurate estimation, but also has an estimation range as large as overall signal bandwidth without loss of accuracy. However, the Ren's method has a problem that the probability of detection of the integer frequency offset (IFO) varies according to the fractional frequency offset (FFO) change.

In this paper, we first analyze the Ren's method and define two detection criteria suitable for detection of IFO, and then propose a novel IFO estimation method based on the ML principle and the new detection criteria. The proposed method still maintains the advantages of Ren's method and provides a better IFO detection probability than Ren's method irrespective of a value of FFO. The rest of this paper is organized as follows. Section II describes the OFDM signal model. In Section III, we analyze the Ren's method and propose a novel IFO estimation method based on new detection criteria. Simulation results are presented in Section IV, and Section V concludes this paper with a brief summary.

II. Signal Model

The complex-valued OFDM samples $\{x(n)\}$ in time domain are generated by taking the inverse fast Fourier transform (IFFT), and can be expressed as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad n = 0, 1, \cdots, N-1, \quad (1)$$

where N is the size of the IFFT and X_k is a phase shift keying (PSK) or a quadrature amplitude modulation (QAM) symbol in the k-th sub-carrier. The useful part of the OFDM symbol has duration of T seconds, and the cyclic prefix (CP) whose length is longer than the channel impulse response is inserted in order to avoid the intersymbol inference (ISI).

At the receiver, the received signal $r(t)$ is sampled with period $T_s = T/N$. Assuming that timing synchronization is perfect, the n-th received OFDM sample $r(n)$ obtained every T_s seconds can be expressed as

$$r(n) = s(n)e^{j2\pi\varepsilon n/N} + w(n), \quad (2)$$

where ε represents the CFO normalized to the sub-carrier spacing $1/T$ and $w(n)$ is the complex-valued additive white Gaussian noise (AWGN) sample with zero mean and variance $\sigma_w^2 = \mathbf{E}\{|w(n)|^2\}$, where $\mathbf{E}\{\cdot\}$ and $|\cdot|$ denote the expectation and the absolute value operators, respectively. In (2), the signal component $s(n)$ is given by

$$s(n) = \sum_{k=0}^{L-1} h_k x(n-k), \tag{3}$$

where h_k denotes the k-th channel filter tap and L represents the channel memory. In this paper, we assume that the channel is static over one OFDM block period. The signal to noise ratio (SNR) ρ is defined as $\rho \triangleq \sigma_s^2/\sigma_w^2$ with $\sigma_s^2 \triangleq \mathbf{E}\{|s(n)|^2\}$.

III. Estimation Method

A. The Conventional Method

In the conventional method, the estimation performance can be independent from the structure of the training sequence by employing the envelope equalized processing (EEP) factor f_n defined as [7]

$$f(n) = \frac{x(n)^*}{\|x(n)\|^2}, \tag{4}$$

where * and $\|\cdot\|$ denote the complex conjugate and Euclidean norm, respectively. Then, the received OFDM sample equalized by the EEP factor is represented as

$$\begin{aligned} y(n) &= r(n)f(n) \\ &= h_0 x(n) e^{j2\pi\varepsilon n/N} f(n) \\ &+ \left\{ \sum_{k=1}^{L-1} h_k x(n-k) e^{j2\pi\varepsilon n/N} + w(n) \right\} f(n) \\ &= h_0 e^{j2\pi\varepsilon n/N} + w'(n), \end{aligned} \tag{5}$$

where $w'(n) = \{\sum_{k=1}^{L-1} h_k x(n-k) e^{j2\pi\varepsilon n/N} + w(n)\} f(n)$.

The CFO ε normalized to the sub-carrier spacing can be expressed as

$$\varepsilon = \varepsilon_I + \varepsilon_F, \tag{6}$$

where ε_I and ε_F are true values of IFO and FFO, respectively. The conventional method [7] estimates the CFO in three steps. First, IFO is estimated from a periodogram of the received OFDM symbol.

$$\hat{\varepsilon}_I = \arg\max_{f_k} \{ I(f_k) + I(f_k+1) \}, \tag{7}$$

where $f_k \in \{-\frac{N}{2}, -\frac{N}{2}+1, \cdots, \frac{N}{2}-1\}$ and $I(f_k)$ is the periodogram of signal $y(n)$ defined as

$$I(f_k) = \left| \sum_{i=0}^{N-1} y(i) e^{-j2\pi f_k i/N} \right|^2. \tag{8}$$

Next, the estimate of FFO is computed by using the value $\hat{\varepsilon}_I$ obtained in the first step.

$$\hat{\varepsilon}_F = \frac{\sqrt{I(\hat{\varepsilon}_I+1)}}{\sqrt{I(\hat{\varepsilon}_I)} + \sqrt{I(\hat{\varepsilon}_I+1)}}. \tag{9}$$

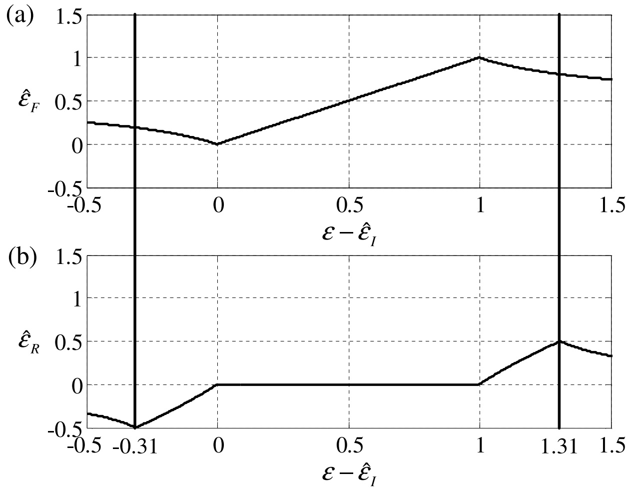

Fig. 1. The frequency estimates $\hat{\varepsilon}_F$ and $\hat{\varepsilon}_R$ versus $\varepsilon - \hat{\varepsilon}_I$ for the conventional method.

In order to improve the accuracy of the estimation, the residual frequency error between the true CFO ε and the sum of two estimates, $\hat{\varepsilon}_I$ and $\hat{\varepsilon}_F$, is estimated in the third step.

$$\hat{\varepsilon}_R = \frac{1}{2} \frac{\sqrt{I(\hat{\varepsilon}_I + \hat{\varepsilon}_F + \frac{1}{2})} - \sqrt{I(\hat{\varepsilon}_I + \hat{\varepsilon}_F - \frac{1}{2})}}{\sqrt{I(\hat{\varepsilon}_I + \hat{\varepsilon}_F + \frac{1}{2})} + \sqrt{I(\hat{\varepsilon}_I + \hat{\varepsilon}_F - \frac{1}{2})}}. \tag{10}$$

Finally, the estimate of the CFO is obtained by

$$\hat{\varepsilon} = \hat{\varepsilon}_I + \hat{\varepsilon}_F + \hat{\varepsilon}_R. \tag{11}$$

B. The Novel IFO Estimation Method

For convenience in derivation, we assume that $w'(n)$ in (5) is zero. Then, the periodogram of the signal in (8) is computed as

$$\begin{aligned} I(f_k) &= \left| \sum_{i=0}^{N-1} h_0 e^{j2\pi(\varepsilon - f_k)i/N} \right|^2 \\ &= \|h_0\|^2 \frac{\sin^2\{\pi(f_k - \varepsilon)\}}{\sin^2\{\pi(f_k - \varepsilon)/N\}}. \end{aligned} \tag{12}$$

Substituting (12) into (9) and (10), we can rewrite $\hat{\varepsilon}_F$ and $\hat{\varepsilon}_R$ as

$$\hat{\varepsilon}_F = \frac{Z(\hat{\varepsilon}_I)}{Z(\hat{\varepsilon}_I) + Z(\hat{\varepsilon}_I + 1)} \tag{13}$$

and

$$\hat{\varepsilon}_R = \frac{Z(\hat{\varepsilon}_I + \hat{\varepsilon}_F - \frac{1}{2}) - Z(\hat{\varepsilon}_I + \hat{\varepsilon}_F + \frac{1}{2})}{Z(\hat{\varepsilon}_I + \hat{\varepsilon}_F - \frac{1}{2}) + Z(\hat{\varepsilon}_I + \hat{\varepsilon}_F + \frac{1}{2})}, \tag{14}$$

respectively, where $Z(x) = |\sin \pi(\varepsilon - x)/N|$.

Figs. 1(a) and 1(b) show the frequency estimates $\hat{\varepsilon}_F$ and $\hat{\varepsilon}_R$ versus $\varepsilon - \hat{\varepsilon}_I$ for the conventional method. From the figures, we can observe that $\hat{\varepsilon}_F$ is correctly estimated when $\varepsilon - \hat{\varepsilon}_I$ takes a value within the range of $0 \le \varepsilon - \hat{\varepsilon}_I < 1$ where $\hat{\varepsilon}_R$ has a zero value. It is also found that when the value of $\varepsilon - \hat{\varepsilon}_I$ is obtained within the range of $-0.31 < \varepsilon - \hat{\varepsilon}_I < 0$ or $1 < \varepsilon - \hat{\varepsilon}_I < 1.31$ (we round off the numbers to three decimal places to obtain -0.31 and 1.31), the residual error can be

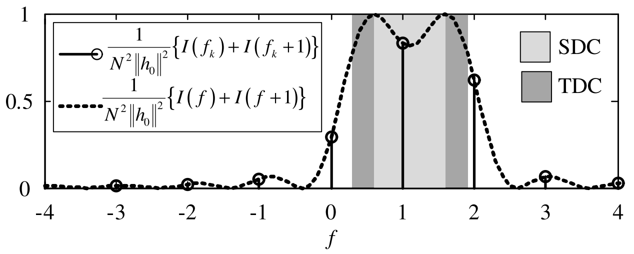

Fig. 2. The conventional IFO estimation metric normalized to $N^2\|h_0\|^2$ with SDC and TDC when $\varepsilon = 1.6$ ($f \in [-N/2, N/2]$).

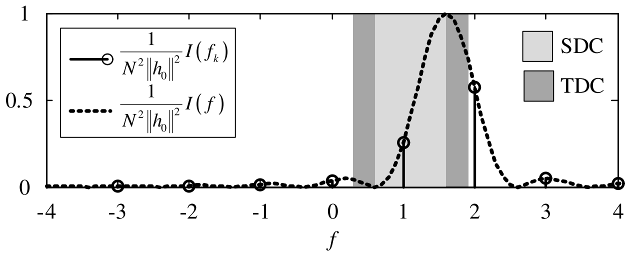

Fig. 3. The ML IFO estimation metric normalized to $N^2\|h_0\|^2$ with SDC and TDC when $\varepsilon = 1.6$ ($f \in [-N/2, N/2]$).

compensated even though $\hat{\varepsilon}_F$ is not correctly estimated. Based on these ranges, we define new detection criteria for the estimate $\hat{\varepsilon}_I$, which are named as *strict detection criterion* (SDC) and *tolerable detection criterion* (TDC), respectively.

Definition 1. SDC: the estimate $\hat{\varepsilon}_I$ takes a value within the range $(\varepsilon - 1, \varepsilon]$.

Definition 2. TDC: the estimate $\hat{\varepsilon}_I$ takes a value within the range $(\varepsilon - 1.31, \varepsilon - 1)$ or $(\varepsilon, \varepsilon + 0.31)$.

If the estimate $\hat{\varepsilon}_I$ obtained in the first stage meets SDC, both $\hat{\varepsilon}_F$ and $\hat{\varepsilon}_R$ can be estimated correctly as shown in Figs. 1(a) and 1(b). It means that we have two chances to obtain the correct CFO estimate. However, in the case that the estimate $\hat{\varepsilon}_I$ falls within the range satisfying TDC, we just have one chance for the correct estimation because $\hat{\varepsilon}_F$ gives an incorrect value as shown in Fig. 1(a) so that only $\hat{\varepsilon}_R$ can help to compensate the residual error in the estimate of CFO. Therefore, when $\hat{\varepsilon}_I$ is estimated within the range meeting the criterion SDC, we can obtain more accurate result than when $\hat{\varepsilon}_I$ takes a value within the range of TDC.

Fig. 2 shows the conventional IFO estimation metric in (7) normalized to $N^2\|h_0\|^2$ with two detection criteria, $0.6 < \hat{\varepsilon}_I \leq 1.6$ for SDC and $0.29 < \hat{\varepsilon}_I < 0.6$ or $1.6 < \hat{\varepsilon}_I < 1.91$ for TDC when $\varepsilon = 1.6$. As we can see from the figure, the conventional IFO metric has relatively large values outside the ranges of SDC and TDC, which can lead to an increase of the probability of false alarm, resulting in serious performance degradation. From the observations, it is found that the accuracy of the conventional estimation method depends on two factors, one is which detection criterion the estimate $\hat{\varepsilon}_I$ is satisfied with and the other is how large value IFO metric has outside the ranges of SDC and TDC.

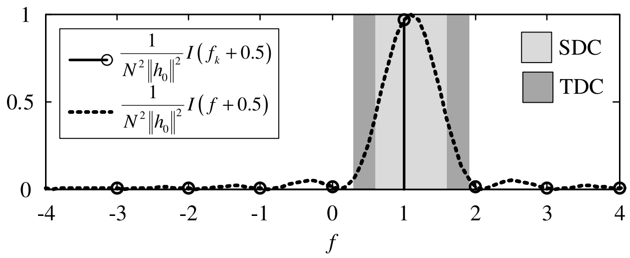

Fig. 4. The proposed IFO estimation metric normalized to $N^2\|h_0\|^2$ with SDC and TDC when $\varepsilon = 1.6$ ($f \in [-N/2, N/2]$).

The ML IFO estimation metric for $\hat{\varepsilon}_I$ is given by [8]

$$\hat{\varepsilon}_I = \arg\max_{f_k} I(f_k). \tag{15}$$

In Fig. 3, it is observed that the index of the periodogram $I(f_k)$ having maximum value ($f_k = 2$ in the figure) does not exist within the range $0.29 < \hat{\varepsilon}_I < 1.91$, which is sum of ranges of SDC and TDC. However, we can anticipate that the shifted version of (15) to the left by 0.5 will give the index of the periodogram corresponding to the maximum amplitude inside the range of SDC. Based on this observation, we propose a new IFO estimation method which is modified from the ML estimation metric.

$$\hat{\varepsilon}_I = \arg\max_{f_k} I(f_k + 0.5). \tag{16}$$

Fig. 4 shows the proposed IFO estimation metric with two detection criteria. In the figure, we can see that the proposed method has the estimate $\hat{\varepsilon}_I$ within the range satisfying SDC. It is also founded that the amplitude of the periodogram for trial values outside the range of TDC is smaller than those of both (7) and (15), which helps to decrease the probability of false alarm and increase the probability of detection.

IV. Simulation Results

In this section, we compare the performance of the proposed IFO estimation method with the conventional method. Simulation results have been obtained under following conditions. The symbols X_k are quadrature PSK (QPSK) modulated sequence and the size of the FFT N is 64. A CP of length 8 samples is used. We consider two channel models, AWGN and four-path Rayleigh fading channels. In Rayleigh fading channel model, the four channel filter taps with path delays of 0, 2, 4, and 6 samples are used. The amplitude A_l of the l-th path changes independently from the others according to a Rayleigh distribution with exponential power delay profile. The power ratio of the first and last tap is set to be 20 dB (i.e., $\mathbf{E}\{A_l^2\} = \exp(-0.8l)$). The Doppler bandwidth of 0.025 (corresponding to a mobile speed of 135 km/h) and a carrier frequency of 1 GHz are assumed. All simulation results are obtained with 2×10^4 iterations for AWGN channel and 3×10^4 iterations for Rayleigh fading channel.

Figs. 5 and 6 illustrate the IFO detection probabilities of the conventional and the proposed method as a function of ε_F over the AWGN channel when SNR is 0 dB and over the

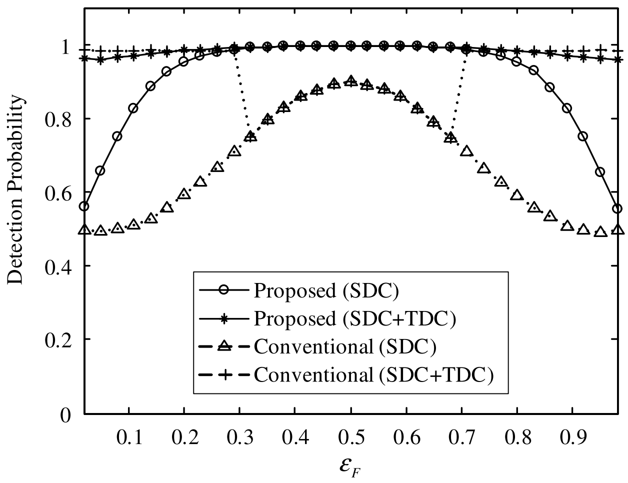

Fig. 5. IFO detection probabilities versus ε_F in AWGN channel when SNR is 0 dB.

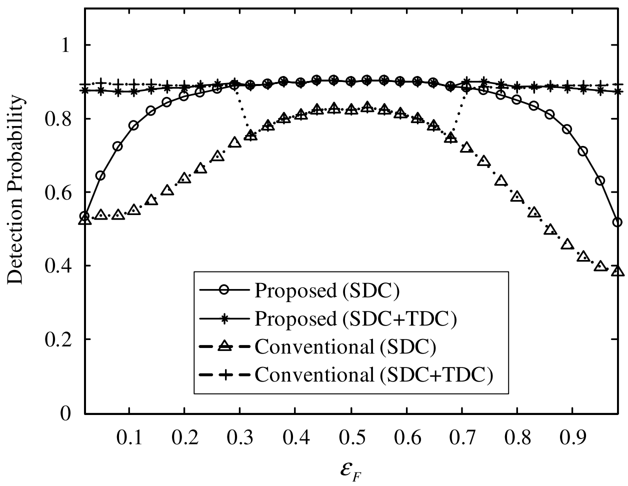

Fig. 6. IFO detection probabilities versus ε_F in fading channel when SNR is 5 dB.

Rayleigh fading channel when SNR is 5 dB, respectively. From the figures, we can see that the conventional method has a low detection probability over the range $\varepsilon_F \in (0.31, 0.69)$. This is because the conventional IFO estimation metric (7) has large values outside the ranges of SDC and TDC, which makes the conventional IFO estimation metric give an incorrect estimate. On the other hand, the proposed method still maintains a good detection probability over the range $\varepsilon_F \in (0.31, 0.69)$. It stems from the fact that the proposed method gives the estimate of IFO $\hat{\varepsilon}_I$ taking a value within the range satisfying the criterion SDC more than the conventional method, as well as its periodogram values corresponding to the trial values outside the range of TDC are smaller than those of the conventional method. Therefore, IFO detection probability of the proposed method is superior to that of the conventional method regardless of the value of ε_F.

V. Conclusion

In this paper, we have analyzed the conventional CFO estimation method proposed by Ren et al., and have found out that the IFO detection probability of the proposed method varies according to the FFO change. Then, we have defined two detection criteria, SDC and TDC, suitable for IFO detection. Based on these detection criteria and the ML principle, we have proposed the novel IFO estimation method which gives a better IFO detection probability compared with the conventional method irrespective of a value of FFO.

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