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Adaptive MPC using a Recursive Learning Technique

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***Abstract*—**A model predictive controller based on recursive learning is proposed. In this SISO adaptive controller, a model is automatically updated using simple recursive equations. The identified models are then stored in the memory to be re-used in the future. The decision for model update is taken based on a new control performance index. The new controller allows the use of simple linear model predictive controllers in the control of nonlinear time varying processes.

***Keywords*—**control, model predictive control, dynamic matrix control, online model identification

I. Introduction

IN many industrial plants, distributed control systems (DCS), mainly composed of PID controllers, have been used to control the process. The continuous need to increase productivity, improve efficiency, and the challenges caused by process disturbances, process nonlinearity, variance in raw material quality, have motivated the use of advanced process control (APC). Among different APC schemes, model predictive control (MPC), has received the most attention especially in refining, petrochemical and chemical industries [1]. Linear model predictive control (LMPC) has been successfully applied to many industrial processes. However the characteristics of many industrial processes are nonlinear and time varying, making LMPC not efficient [2]. In such conditions, MPC based on nonlinear models (NMPC) should be considered. Although NMPC has been successfully implemented in many cases [1], the designer is always challenged with the difficulty of obtaining accurate nonlinear models for complicated processes as well as the large computation effort required for nonlinear optimization algorithms. Another alternative is to use adaptive MPC where linear models are continuously estimated for each operating region. This can be achieved using online model identification using neural networks [3-5], fuzzy systems [6, 7], or based on actual measurements database [8,9]. The development of such scheme was restricted by the high computation time required, limiting its application to systems with large sampling time [10]. Another practical approach is to identify different linear models for different operating regions and switch between different models as the process moves between regions. This work presents a new adaptive MPC technique that uses a simple recursive learning method to update the process model when required. The decision of model update is taken based on a new control performance index.

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The model identified is stored in the memory to be re-used for the same operating region in the future. The number of the linear models is adaptively selected based on process non-linearity. The paper is organized as follows: Notation used in the paper is presented in section II. Section III is dedicated for a quick overview on the theory behind a famous type of MPC which is the dynamic matrix control (DMC). The new adaptive controller is described in section IV. The simulation results and the conclusion are presented in sections V and VI respectively.

II. Notation

Bold lower case letters are used for vectors while bold upper case letters are used for matrices. The hat accent is used to indicate that the variable is an estimated one. All notation used in this paper are given in Table I

TABLE I
Notation

Symbol	Description
h	Impulse response coefficients
y	Plant measured output
e	Control error
J	Optimization cost function
u	Controller output
I	Identity matrix
k	Process gain
θ	Process dead time
τ	Process time constant
d	Unmeasured disturbances
λ	Move suppression factor
p	Prediction horizon
m	Control horizon
FSR	Finite Step Response
LUT	Look Up Table

III. Background

A. Model Predictive Control

MPC refers to a family of controllers that uses a discrete form of the process model to predict future values of a process variable based on past values of controller output. The main idea behind MPC type controllers is illustrated in Fig. 1 for a SISO system [12]. At sampling time k, a set of m future manipulated variable moves (control horizon) are selected, so that the predicted response over a finite horizon p (prediction horizon) has certain desirable characteristics. This is achieved by minimizing an objective function based on the deviation of the future controlled variables from a desired trajectory over the prediction horizon p and the control energy over the control horizon m.

The MPC optimization is performed for a sequence of hypothetical future control moves over the control horizon and only the first move is implemented [13]. The problem is solved again at time k + 1 with the measured output y (k + 1) as the new starting point. Model uncertainty and unmeasured process disturbances are handled by calculating an additive disturbance as the difference between the process measurement and the model prediction at the current time step.

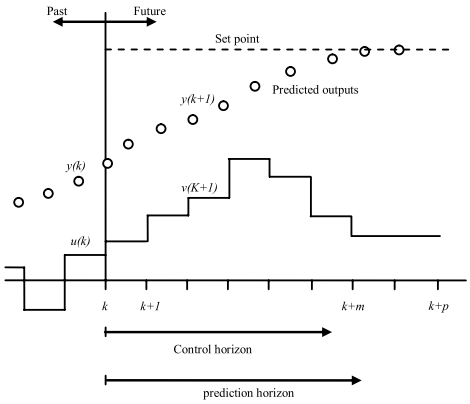

Fig. 1 Illustration for the concept of model predictive control

Although this work focus only on SISO controllers, MPC algorithm can be generally extended to control MIMO processes, subject to numerous disturbances and dynamically varying constraints. Based on the model used, different MPC algorithms are described in literatures [11]

B. Dynamic Matrix Control:

Dynamic matrix control DMC is a widely used algorithm developed by Cutler and Ramaker in the seventies. DMC uses a linear finite step response (FIR) model to predict the future process variable, $\hat{y}(n+j)$, over sampling future samples reference to the current time n:

$$\hat{y}(n+j) = y_{o+} \sum_{i=1}^{j} h_i \Delta u(n+j-i) + d(n+j) \qquad (1)$$

Where $y_o$is the initial plant output, Δu_i is the change in the controller output at the i th sample, h_i is the i th unit step response coefficient of the process, the term $d(n+j)$ combines the unmeasured disturbances and the inaccuracies due to plant-model mismatch.

The goal of the controller is to compute a series of moves such that:

$$e(n+j) = R(n+j) - \hat{y}(n+j) = 0 \quad j = 1,2,\dots,P \qquad (2)$$

where $R(n)$ is the reference trajectory or target set point, is the prediction horizon, and $e(n)$ is the control error.

Using (1), it can be shown that:

$$\begin{bmatrix} e(n+1) \\ e(n+2) \\ \vdots \\ e(n+P) \end{bmatrix}_{P\times 1} = \begin{bmatrix} h_1 & 0 & 0 & \dots & 0 \\ h_2 & h_1 & 0 & & 0 \\ h_3 & h_2 & h_1 & \ddots & \vdots \\ h_4 & h_3 & h_2 & & \\ \vdots & \vdots & \vdots & & \\ h_p & h_{p-1} & h_{p-2} & \cdot & h_{p-m+1} \end{bmatrix}_{P\times M} \times \begin{bmatrix} \Delta u(n) \\ \Delta u(n+1) \\ \vdots \\ \Delta u(n+M-1) \end{bmatrix}_{M\times 1} \qquad (3)$$

Or in compact matrix notation:

$$e = H.\Delta u \qquad (4)$$

where e is the vector of predicted errors over the next P samples, H is the dynamic matrix, and Δu is the vector of controller output moves to be determined.

Eq. (4) can be solved by minimizing a quadratic cost function of the form:

$$\mathrm{Min}_{\Delta u}\, J = [e - H\Delta u]^T[e - H\Delta u] + \Delta u^T \lambda \Delta u \qquad (5)$$

where is the move suppression factor used to avoid excessive control action. In the unconstrained case, this minimization problem has a closed form solution, which represents the DMC control law:

$$\Delta u = (H^T H + \lambda I)^{-1} H^T e \qquad (6)$$

C. Multi-Model MPC

Several linear empirical model can be used together to control a nonlinear plant using DMC. One technique uses a gain and time scheduling technique for updating the process model as the process move from one region to another [14]. An extension of this method is to use multiple models to update the controller model. Linear models that describes the system at various operating regions are developed using actual plant measurements or through step tests. It has been shown in [15] that linear models can be combined in order to obtain an approximation of the nonlinear process that approaches its true behavior. This scheme can be implemented practically by using several controllers in parallel, each designed and tuned for a specific operating region and switch between controllers depending on the operating region. To avoid bumping at switchover, the controllers' moves are weighted based on the prediction error calculated for each [16]. The resulting weights are obtained using recursive identification to minimize the error of the final controller output. Another technique uses fuzzy logic to combine controllers' outputs [17]. Generally the success of this scheme depends on the accuracy of the individual models used, the number of linear models required to approximate plant nonlinearity and the technique used to switch from on model to another. Multi-model MPC can also be implemented using online recursive formulations that use actual plant measurements to develop "local" linear models for the MPC [18]. The success of this scheme depends on the presence of enough excitations in the data used, the level of noise, and the convergence of the modeling formulations used.

The local linear model can be approximated using a first order plus dead time (FOPDT) model that has the form

$$\tau\frac{dy(t)}{dt} + y(t) = k.u(t-\theta) \quad (7)$$

Or,

$$\frac{y(s)}{u(s)} = \frac{ke^{-\theta s}}{1+\tau s} \quad (8)$$

Where k is the process gain, τ is the time constant and θ is the process dead time. Although a FOPDT model approximation does not capture all the features of higher order process, it often reasonably describes the process gain, overall time constant and effective dead time of such processes [19].

In this work, approximated FOPDT models are identified for the common operating regions. At each change in the set point, a performance index is used to evaluate the controller performance. If the performance is below an accepted level, a local FOPDT model is identified by comparing the actual plant measurements with the model output. The three model parameters together with the target set point are then stored in a look-up table in the memory. For each future change in the set point the stored look-up table and linear interpolation is used to estimate the local model for this new operating region. The final number of linear model used is adaptively selected depending on degree of process nonlinearity.

IV. Adaptive MPC Using Recursive Learning

A. Recursive Model Identification

Now assume that the controller output is applied to the model as well as the plant as illustrated in Fig.2

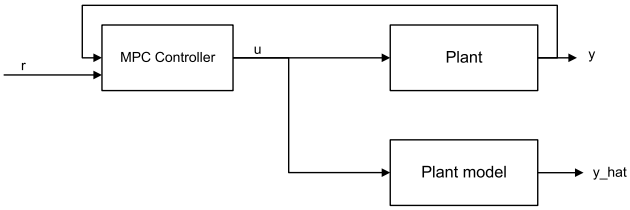

Fig. 2 Using plant model in model identification

Fig. 3, 4 and 5 show a plot for both y and $\hat{y}$ in case of gain mismatch, time constant mismatch, and in case of mismatch in all FOPDT parameters. Fig.6 shows the first derivatives $y'$and $\hat{y}$' corresponding to the case in Fig.5

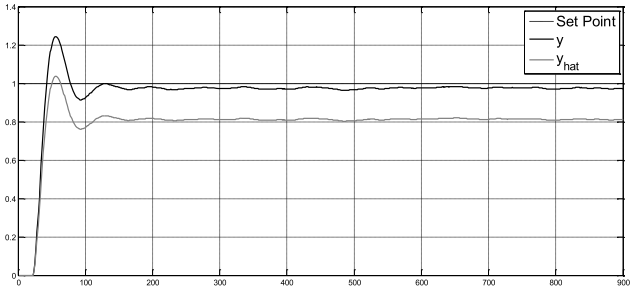

Fig. 3 Outputs in case of process gain mismatch

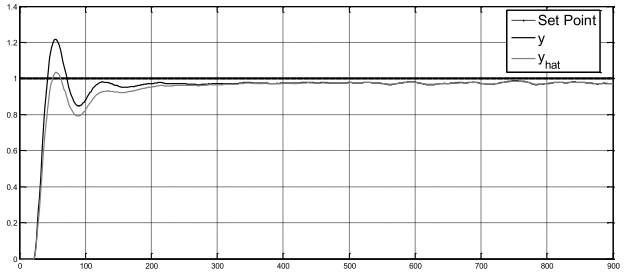

Fig. 4 Outputs in case of process time constant mismatch

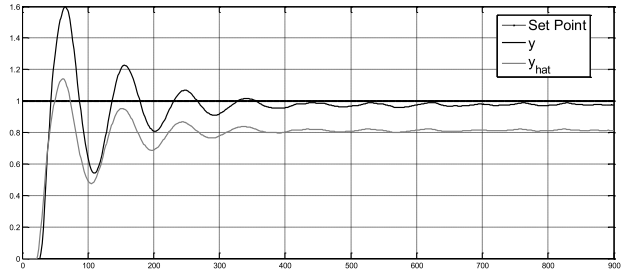

Fig. 5 Outputs in case of process gain, time constant and dead time mismatch

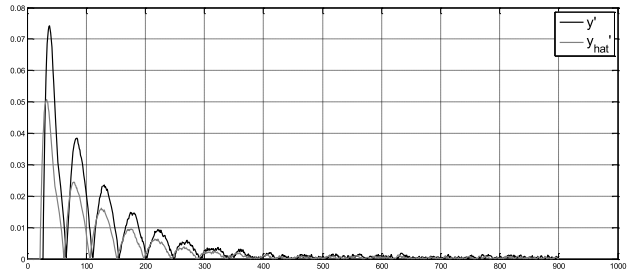

Fig. 6 Outputs' first derivative in case of process gain, time constant and dead time mismatch

If the new set point is applied at t_1, and the process is stabilized at t_2, FOPDT model parameters can be estimated by comparing $y, \hat{y}, y'$and $\hat{y}'$ using the following recursive equations:

$$k_{new} = k_{old}.\frac{y(t_2)-y(t_1)}{\hat{y}(t_2)-\hat{y}(t_1)} \quad (9)$$

$$\tau_{new} = \tau_{old}.\frac{k_{new}}{k_{old}}.\frac{\hat{y'}(t)\big|_{max}}{y'(t)|_{max}}\Bigg|_{t_1<t<t_2} \quad (10)$$

The dead time θ can be calculated by estimating the time shift between y and $\hat{y}$. Fig.7 shows the improvement in control performance after one learning pass using Eq.(9) and (10).

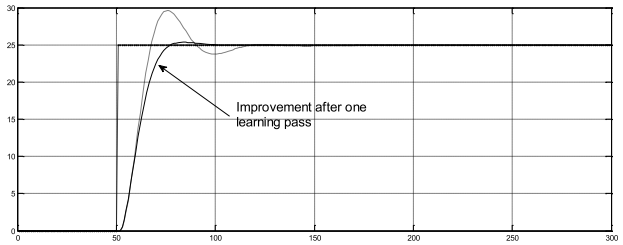

Fig. 7 Example 1 for control improvement after one learning pass

Fig.8 shows another example for more severe mismatch and quick change in set point.

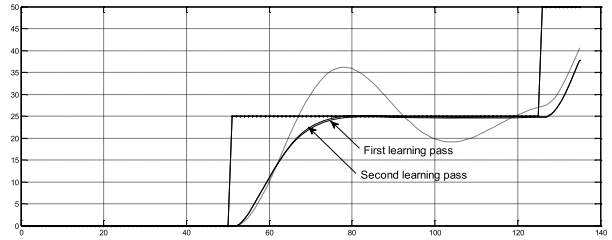

Fig. 8 Example 2 for control improvement after two learning passes

B. Adaptive Multi-Model MPC:

For the sake of this work, a new performance index is introduced and is defined as:

$$I = \frac{\max(y'(t))}{\max(\hat{y}'(t))} \cdot \frac{\max(y(t))}{R}\Big|_{t_1<t<t_2} \quad (11)$$

where R is the final controller set point. The index *I* is close to 1 for well performing control. The steps for implementing the new adaptive controller is summarized in the flow chart of fig.9

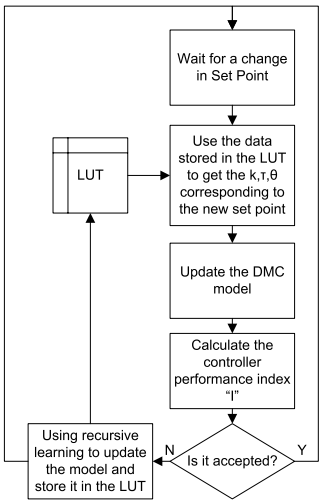

Fig. 9 Flow chart for the proposed adaptive controller

V. SIMULATION RESULTS

A. Gravity Drained Tank

The performance of the new controller is first demonstrated through the level control of gravity drained tanks. This is highly nonlinear process where the gain may vary by more than 500%. A schematic for system is shown in fig. 10

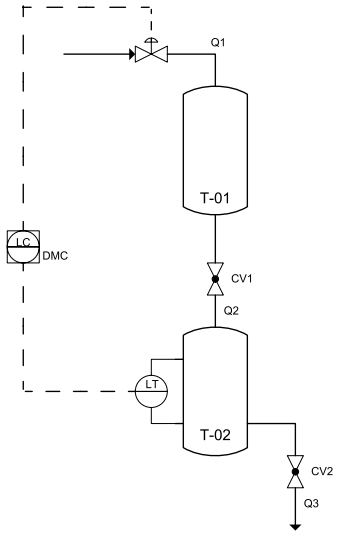

Fig. 10 Schematic for gravity drained tanks

The system consists of two non-interacting tanks stacked one above the other. Each of the two tanks has a diameter of 1m and a height of 2.75 m. The level transmitter is of differential pressure type and its tap points are installed at heights 0.25m and 2.5m reference to tank base, offering a total measurement span of 2.25m. Two globe valves are installed at the outlet of each tank with valve constants Cv1 equals to 0.5 and Cv2 equals to 1. The outlet of T-02 is at 0.25m height and the outlet valve is installed at the same level of tank base. The controlled variable is the level of the lower tank, and the manipulated variable is the control valve position at the inlet of the first tank. Using mass balance, the dynamic model of the system can be modeled using the two following differential equations:

$$A_1 \cdot \frac{dh_1}{dt} = Q_1 - Cv1\sqrt{h_1 + x_1} \quad (12)$$

$$A_2 \cdot \frac{dh_2}{dt} = Cv_1\sqrt{h_1 + x_1 + x_2} - Cv_2\sqrt{h_2} \quad (13)$$

Where $h1$,$h2$, $A1$ and $A2$ are the height and cross-section area of tanks T-01 and T-02 respectively, x_1 and x_2 are the distance between T-01 and valve 1 and between valve 1 and T-02. Plant measurements around h2=5% were used to construct the FSR model used in the DMC controller. Fig.11 shows the performance of a single model DMC controller for this non-linear process compared to the performance of the adaptive recursive learning DMC after one learning pass. It is clear that as the level set point increases, overshoot increases due to process gain increase. RL-DMC can successfully capture the change in the model diameter and thus avoid overshooting.

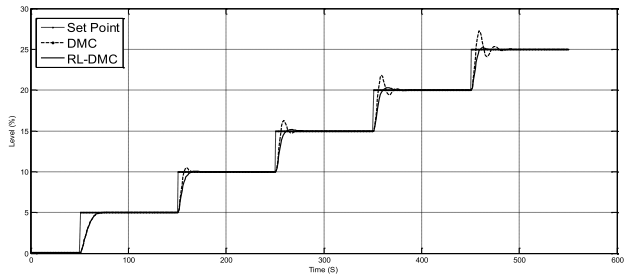

Fig. 11 Control performance of RL-DMC compared to DMC

TABLE II
IDENTIFIED MODELS

	Initial Model			Model after one pass		
	K	τ	θ	K	τ	Θ
5	0.23	3.4	0	0.23	3.6	0
10	0.23	3.4	0	0.54	3.2	1
15	0.23	3.4	0	0.7	3.7	2
20	0.23	3.4	0	0.83	4.1	3
25	0.23	3.4	0	0.95	4.4	3

To better illustrate the idea, consider a change in set point between 10% and 15% up and down as shown in Fig.12. At each change in the set point the RL-DMC identify a more accurate model leading to better control. This test represents the way the plant operator or the control engineer should use the consecutive set point changes to allow adaptation to varying operating conditions.

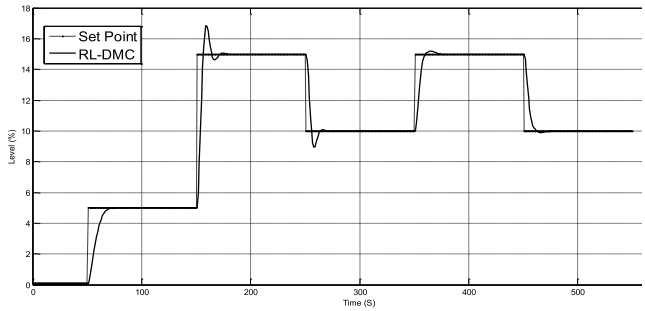

Fig. 12 using successive set point changes to improve the control performance

B. Nonlinear Mixing Process

This section demonstrates the performance of the new controller on a nonlinear mixing process. This process consists of an open tank with an agitator as shown in Fig. 13.

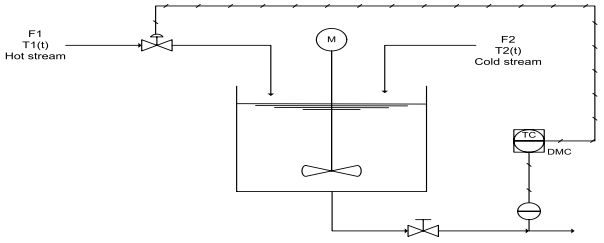

Fig. 13 Nonlinear Mixing Process

A hot water stream $F_1(t)$ is manipulated to mix with a cold water stream $F_2(t)$ to obtain an output flow F(t) at a desired temperature T'(t). All flow measurements are expressed in m3 /s. The temperature transmitter is located at a distance L from the mixing tank bottom. The volume of the tank varies freely without overflowing. The tank level can be estimated through mass balance as:

$$A.\frac{dh}{dt} = F_1 + F_2 - F \tag{14}$$

The output flow F(t) can be modeled as a function of the liquid level and the manual valve used in bottom of tank:

$$F(t) = C_v\sqrt{h(t)} \tag{15}$$

Where Cv represents the valve constant. The outlet temperature is obtained through mass balance as follows:

$$F_1(t).CP.T_1(t) + F_2(t).CP.T_2(t) - F(t).CP.T(t) = A.CV.h(t).\frac{dT}{dt} \tag{16}$$

where CP and CV are the heat capacity of the liquid at pressure constant and volume constant respectively. T1(t) is the hot water stream temperature, T2(t) is the cold water stream temperature. T(t) is the temperature just in the bottom of the tank. Because the temperature transmitter is located at a distance L from the tank bottom, there is a delay time between T(t) and the temperature registered by the sensor/transmitter T'(t). That delay time to(t) can be calculated as:

$$t_o = \frac{LA_t\rho}{F(t)} \tag{17}$$

where At is the pipe cross section an L is the distance between the tank bottom and the temperature transmitter position. The temperature transmitter can be modeled as:

$$\tau_T.\frac{dc(t)}{dt} + c(t) = K_T.T'(t) \tag{18}$$

Where τ_T and K_T are the transmitter time constant and gain respectively. Finally the control valve can be modeled as:

$$\tau_v.\frac{dF_1(t)}{dt} + F_1(t) = K_v m(t) \tag{19}$$

Where τ_v and K_v are the control valve time constant and gain respectively. Matlab is used to solve the 4 differential equations (14,16,18 and 19) using ode45 command. The final steady state model parameters used in the simulation are shown in table III:

TABLE III
MODEL STEADY STATE PARAMETERS

Parameter	Value	Unit
F1	0.475	m3/s
F2	1.1	m3/s
F	1.56	m3/s

T1	80	C
T2	15	C
T	35	C
P	1000	Kg/m3
A	10	m2
CV	1	Kcal/C.Kg
CP	1	Kcal/C.Kg
Cv	0.6	
L	3	m
At	0.005	M
Tv	0.5	S
Kv	0.016	
Kt	1	
τt	0.5	S
H	6.8	m

Fig. 14 and 15 compare the performance of DMC with RL-DMC after one learning pass and 3 learning passes respectively. Note that the DMC model was identified around an outlet temperature of 35C using a FOPDT model with k=0.54, τ=22.9,and θ=14. As the set point decrease, the process nonlinearity causes overshoots as indicated by the dashed line. In contrast, the RL-DMC successfully identified the real model for each operating set point leading to a smoother control.

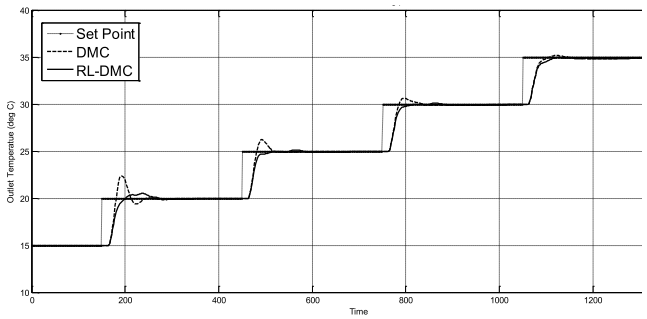

Fig. 14 simulation results for one learning pass

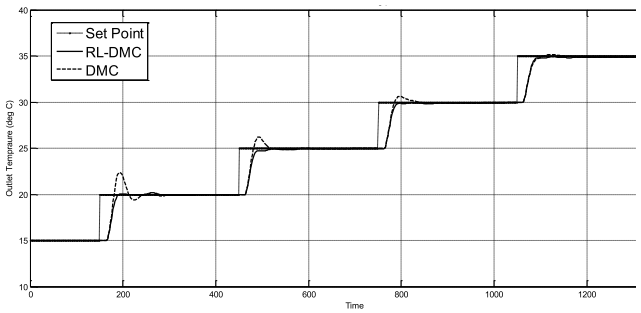

Fig. 15 Simulation results for 3 learning passes

The final identified models by the RL-DMC are shown in table IV

TABLE IV
IDENTIFIED LOCAL FOPDT MODELS

Set Point for outlet temperature (C)	Identified Gain	Identified Time constant
20	0.95	25.2
25	0.84	26.4
30	0.71	27.3
35	0.57	28.5

VI. CONCLUSION

This paper presents a new approach in the implementation of SISO multiple models MPC controller for non-linear plants. In this approach, approximate model(s) collected form physical equations, dynamic simulations packages can be used directly to the plant. Simple learning technique is used to identify different models for different operating regions based on controller performance. The simplicity of the calculations avoids the high computation capacity normally required for online identification. Adaptation can usually be accomplished in one or two step changes. This approach can considerably save the large cost spent on online identification packages and on expert process control engineers.

REFERENCES

[1] Paul Serban Agachi, Zoltan K. Nagy, Mircea Vasile Cristea, and Arpad Imre-Lucaci, "Model Based Control - Case Studies in Process Engineering", WILEY-VCH, 2006, pp. 26-32.
[2] Kalra L, Georgakis C. Effects of process nonlinearity on the performance of linear model predictive controllers for the environmentally safe operation of a fluid catalytic cracking unit. Ind Eng Chem Res 1994;33:3063–9.
[3] Yingyi Jin, Chengli Su, "Adaptive Model Predictive Control Using Diagonal Recurrent Neural Network", IEEE Fourth International Conference on Natural Computation, 2008.
[4] Chengli Su, Yun Wu, "Adaptive Neural Network Predictive Control Based on PSO Algorithm", IEEE Chinese Control and Decision Conference (CCDC 2009), 2009.
[5] Vincent A. Akpan, George D. Hassapis, "Nonlinear model identification and adaptive model predictive control using neural networks", ISA transactions, 2011.
[6] XUE Meisheng, TAO Chenggang, ZHUGE Jinjun, "A Fuzzy Compensation Mechanism in FFRLS-based Adaptive MPC Strategy", Proceedings of the 29th Chinese Control Conference , 2010.
[7] Edinzo J. Iglesias Sanchez, "Using fuzzy logic to enhance control performance of sliding mode control and dynamic matrix control", PhD thesis, 2006.
[8] N.A. Wahab, R. Katebi, J. Balderud, M.F. Rahmat, Data-driven adaptive model-based predictive control with application in wastewater systems, IET Control Theory and Applications, 2010
[9] Yasuki Kansha, Min-Sen Chiu, Adaptive generalized predictive control based on JITL technique, Journal of Process Control, 2009
[10] Dones I, Manenti F, Preisig HA, Buzzi-Ferraris G. Nonlinear model predictive control: a self-adaptive approach. Ind Eng Chem Res 2010;49(10):4782–91.
[11] J.A. Rossiter, "Model Based Predictive Control A Practical Approach", CRC Press, 2003
[12] R. Sivakumar, K. Suresh Manic, V. Nerthiga, R. Akila, K. Balu, "Application of Fuzzy Model Predictive Control in Multivariable Control of Distillation Column" International Journal of Chemical Engineering and Applications, Vol. 1, No. 1, June 2010
[13] Piotr Tatjewski and Maciej Lawryn Czuk (2006), "Soft computing in model–based predictive control", International Journal of Applied Maths and Computer Science, Vol. 16, No. 1, pp. 7–26, 2006

[14] Chow, C., Kuznetsoc, A. G., & Clarke, D. W., Successive one step-ahead predictions in a multiple model predictive control, International Journal of Systems Science, 1998
[15] Banerjee, A., Arkun, Y., Ogunnaike, B., & Pearson, R., Estimation of nonlinear systems using linear multiple models. AICHE Journal, 1997
[16] Yu, C., Roy, R. J., Kaufman, H., & Bequette, B. W., Multiple model adaptive predictive control of mean arterial pressure and cardiac output. IEEE Transactions on Biomedical Engineering,1992
[17] Hou Guolian, Zhang Jinfang, Liu Junjun, Zhang Jianhua , Multiple-model Predictive Control Based on Fuzzy Adaptive Weights and its Application to Main-Steam Temperature in Power Plant, 5th IEEE Conference on Industrial Electronics and Applications, 2010
[18] Chikkula, Y., & Lee, J. H., Robust adaptive predictive control of nonlinear processes using nonlinear moving average system models, Industrial & Engineering Chemical Research, 2000.
[19] Cohen, G. H., & Coon, G. A. , Theoretical considerations of retarded control, Transactions of the ASME, 1953
[20] Danielle Dougherty, Doug Cooper, A practical multiple model adaptive strategy for single-loop MPC, Control Engineering Practice,2003