Some Solid Transportation Models With Crisp and Rough Costs

Pradip Kundu, Samarjit Kar, Manoranjan Maiti

Abstract—In this paper, some practical solid transportation models are formulated considering per trip capacity of each type of conveyances with crisp and rough unit transportation costs. This is applicable for the system in which full vehicles, e.g. trucks, rail coaches are to be booked for transportation of products so that transportation cost is determined on the full of the conveyances. The models with unit transportation costs as rough variables are transformed into deterministic forms using rough chance constrained programming with the help of trust measure. Numerical examples are provided to illustrate the proposed models in crisp environment as well as with unit transportation costs as rough variables.

Keywords—Solid transportation problem, Rough set, Rough variable, Trust measure.

I. INTRODUCTION

THE solid transportation problem (STP), first stated by Schell [21] is an extension of classical transportation problem (TP). The STP is a problem of transporting goods from some sources to some destinations through some conveyances (modes of transportation), i.e., the STP deals with three types of constraints, namely source, destination and conveyance capacity constraints. Haley [3] described a solution procedure of a solid transportation problem, which is an extension of the Modi method. Till now, STP [5,12-15,29] is modeled taking total supply capacity of all the conveyances and it is assumed that this total capacity is available for utilization for all source to destination routs whatever be the amount of product allocated in the routs for transportation. But in many practical situations this may not always happen. Practically most of time full vehicles, e.g., trucks, rail coaches are to be booked and the availability of each type of conveyance at each source may not be the same and vehicles available at one source may not be utilized at another source due to long distance between them or some other problems. Also fulfillment of capacity of a vehicle effects the optimal transportation policy. These practical situations motivated us to formulate some useful solid transportation models.

The available data in a transportation system, such as supplies, demands, conveyance capacities are not always crisp but are uncertain in nature due to insufficient information, lack of evidence, fluctuating financial market, etc. Many researchers studied STP in various uncertain environments. Jiménez and

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Verdegay [5] studied two types of uncertain STP, one with interval numbers and other with fuzzy numbers. Yang and Liu [29] presented expected value model, chance-constrained programming model and dependent-chance programming for a fixed charge solid transportation problem in fuzzy environment. Ojha et al. [14] investigated an entropy based STP with general fuzzy costs and time. Yang and Feng [28] investigated expected value goal programming, chance-constrained goal programming and dependent-chance goal programming for a bicriteria STP with fixed charge under Stochastic environment. Nagarjan and Jeyaraman [13] formulated and solved chance-constrained programming model for a multi-objective STP with the parameters as stochastic intervals.

Though transportation problems in various types of uncertain environments such as fuzzy, random are studied by many researchers, there are few research papers about TP in rough uncertain environment. Since rough set theory is proposed by Pawlak [16], it is developed by many researchers [11,17,19,20,32] in theoretical aspect and applied into many practical fields such as data envelopment analysis (DEA) [22,26], data mining [6], multi-criteria decision analysis [1,2,18], medical diagnosis [4,24], neural network [7], signal processing [27], etc. Liu [8] proposed the concept rough variable which is a measurable function from rough space to the set of real numbers. Liu [10] discussed some inequalities of rough variables and convergence concept of sequence of rough variables. Liu [8,9] studied some rough programming models with rough variables as parameters. Xu and Yao [25] studied a two-person zero-sum matrix games with payoffs as rough variables. Tao and Xu [23] developed a rough multi-objective programming for dealing with multi-objective solid transportation problem assuming that the feasible region is not fixed but flexible due to imprecise parameters. Youness [31] introduced a rough programming problem considering the decision set as a rough set. Xu et al. [26] proposed a rough DEA model to solve a supply chain performance evaluation problem with rough parameters. Xiao and Lai [27] considered power-aware VLIW instruction scheduling problem with power consumption parameters as rough variables. But at the best of our knowledge none studied STPs with any of the parameters such as cost coefficients, supplies, demands, etc as rough variables. In this paper we formulate and solve STP with vehicle capacity taking unit transportation costs and unit additional costs as rough variables.

The rest of the paper is organized as follows: In Section 2, we provide some definitions and properties of rough set and rough variable. Section 3 presents description of our proposed solid transportation problems and corresponding

model formulations. Section 4 formulates the problems with unit transportation and additional costs (penalty) as rough variables. Rough chance constrained programming models are formulated for the problems with the help of trust measure and corresponding deterministic forms are obtained. Numerical examples are provided in the Section 5 to illustrate the proposed models in crisp environment and as well as with rough unit transportation and additional costs. Finally the paper is concluded in Section 6.

II. PRELIMINARIES

Here we introduced some basic idea of approximation of a subset of a certain universe by means of lower and upper approximation and rough set theory. Suppose U is a non-empty finite set of objects called the universe and A is a non-empty finite set of attributes, then the pair $S = (U, A)$ is called information system. For any $B \subseteq A$ there is associated an equivalence relation $I(B)$ defined as $I(B) = \{(x, y) \in U \times U \mid \forall a \in B, a(x) = a(y)\}$, where $a(x)$ denotes the value of attribute a for element x. $I(B)$ is called the B-indiscernibility relation. The equivalence classes of the B-indiscernibility relation are denoted by $[x]_B$.
For an information system $S = (U, A)$ and $B \subseteq A$, $X \subseteq U$ can be approximated using only the information contained in B by constructing the B-lower and B-upper approximations [17] of X, denoted $\underline{B}X$ and $\overline{B}X$ respectively, where

$$\underline{B}X = \{x \mid [x]_B \subseteq X\} \; and \; \overline{B}X = \{x \mid [x]_B \cap X \neq \phi\}.$$

Clearly, lower approximation $\underline{B}X$ is the definable (exact) set contained in X so that the objects in $\underline{B}X$ can be with certainty classified as members of X on the basis of knowledge in B, while the objects in $\overline{B}X$ can be only classified as possible members of X on the basis of knowledge in B. The B-boundary region of X is defined as

$$BN_B = \overline{B}X - \underline{B}X$$

and thus consists of those objects that we cannot decisively classify into X on the basis of knowledge in B. The boundary region of a crisp (exact) set is an empty set as the lower and upper approximation of crisp set are equal. A set is said to be rough if the boundary region is non-empty, i.e., if $BN_B \neq \phi$ then X is referred to as rough with respect to B.

A. Rough variable

The concept of of rough variable is introduced by Liu[8]. The following definitions are based on Liu[8,9].
Definition 1.2.1. Let Λ be a nonempty set, $\mathcal{A}$ be a σ-algebra of subsets of Λ, Δ be an element in $\mathcal{A}$, and π be a nonnegative, real-valued, additive set function on $\mathcal{A}$. Then $(\Lambda, \Delta, \mathcal{A}, \pi)$ is called a rough space.
Definition 1.2.2. A rough variable ξ on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$ is a measurable function from Λ to the set of real numbers $\Re$ such that for every Borel set B of $\Re$, we have $\{\lambda \in \Lambda \mid \xi(\lambda) \in B\} \in \mathcal{A}$.
Then the lower and upper approximations of the rough variable ξ are defined as follows:

$$\underline{\xi} = \{\xi(\lambda) \mid \lambda \in \Delta\} \; and \; \overline{\xi} = \{\xi(\lambda) \mid \lambda \in \Lambda\}.$$

Definition 1.2.3. Let ξ be a rough vector on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$, and $f_j : \Re^n \to \Re$ be continuous functions, $j = 1, 2, ..., m$. Then the upper trust of the rough event characterized by $f_j(\xi) \leq 0; j = 1, 2, ..., m$ is defined by

$$T\bar{r}\{f_j(\xi) \leq 0_{,j=1,2,...,m}\} = \frac{\pi\{\lambda \in \Lambda | f_j(\xi(\lambda)) \leq 0, \; j = 1, 2, ..., m\}}{\pi(\Lambda)},$$

and the lower trust of the rough event characterized by $f_j(\xi) \leq 0; j = 1, 2, ..., m$ is defined by

$$T\underline{r}\{f_j(\xi) \leq 0_{,j=1,2,...,m}\} = \frac{\pi\{\lambda \in \Delta | f_j(\xi(\lambda)) \leq 0, \; j = 1, 2, ..., m\}}{\pi(\Delta)}.$$

If $\pi(\Delta) = 0$, then $T\bar{r}\{f_j(\xi) \leq 0_{,j=1,2,...,m}\} \equiv T\underline{r}\{f_j(\xi) \leq 0_{,j=1,2,...,m}\}$.
The trust of the rough event is defined as the average value of the lower and upper trusts, i.e.,
$Tr\{f_j(\xi) \leq 0_{,j=1,2,...,m}\} = \frac{1}{2}(T\bar{r}\{f_j(\xi) \leq 0_{,j=1,2,...,m}\} + T\underline{r}\{f_j(\xi) \leq 0_{,j=1,2,...,m}\})$.
Definition 1.2.4. Let ξ be a rough variable on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$ and $\alpha \in (0, 1]$, then

$$\xi_{sup}(\alpha) = \sup\{r | Tr\{\xi \geq r\} \geq \alpha\}$$

is called α-optimistic value to ξ; and

$$\xi_{inf}(\alpha) = \inf\{r | Tr\{\xi \leq r\} \geq \alpha\}$$

is called α-pessimistic value to ξ.
Definition 1.2.5. Let ξ be a rough variable on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$. The expected value of ξ is defined by

$$E[\xi] = \int_0^\infty Tr\{\xi \geq r\}dr - \int_{-\infty}^0 Tr\{\xi \leq r\}dr.$$

Example 1.2.1. Consider that $\xi = ([a, b], [c, d])$ be a rough variable with $c \leq a < b \leq d$, where $[a, b]$ is the lower approximation and $[c, d]$ is the upper approximation. This means the elements in $[a, b]$ are certainly members of the variable and that of $[c, d]$ are possible members of the variable. Here $\Delta = \{\lambda | a \leq \lambda \leq b\}$ and $\Lambda = \{\lambda | c \leq \lambda \leq d\}$, $\xi(x) = x$ for all $x \in \Lambda$, $\mathcal{A}$ is the Borel algebra on Λ and π is the Lebesgue measure.
As an practical example consider the possible transportation cost of unit product to be transported from a source i to certain destination j through a conveyance k. But as transportation cost depends upon fuel price, labor charges, tax charges, etc. and each of which are fluctuate time to time, so it is not always possible to determine its exact value. Suppose four experts give the possible unit transportation cost for $i - j$ route via conveyance k, determined in a certain time period as intervals [3,5], [4,5], [3.5,6] and [4,6] respectively. Denotes c_{ijk} as- 'the possible value of the unit transportation cost according to the all experts'. Then c_{ijk} is not exact and can be approximated by means of lower and upper approximation. It is clear that [4,5] is the lower approximation of c_{ijk} as it is the greatest definable (exact) set that c_{ijk} contain, i.e. every member of [4,5] is certainly a value of c_{ijk}. Here [3,6] is the upper approximation. So c_{ijk} can be represented as the rough variable ([4,5],[3,6]).
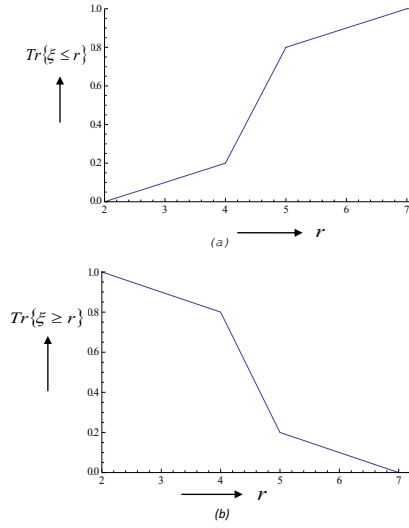
Fig. 1. The trust of the rough event characterized by (a) $\xi \leq r$ and (b) $\xi \geq r$.

For a given value r and $\xi = ([a, b], [c, d])$,

$$
Tr\{\xi \leq r\} = \begin{cases} 0, & \text{if } r \leq c; \\ \frac{r-c}{2(d-c)}, & \text{if } c \leq r \leq a; \\ \frac{1}{2}\left(\frac{r-a}{b-a} + \frac{r-c}{d-c}\right), & \text{if } a \leq r \leq b; \\ \frac{1}{2}\left(\frac{r-c}{d-c} + 1\right), & \text{if } b \leq r \leq d; \\ 1, & \text{if } r \geq d. \end{cases}
$$

$$
Tr\{\xi \geq r\} = \begin{cases} 0, & \text{if } r \geq d; \\ \frac{d-r}{2(d-c)}, & \text{if } b \leq r \leq d; \\ \frac{1}{2}\left(\frac{d-r}{d-c} + \frac{b-r}{b-a}\right), & \text{if } a \leq r \leq b; \\ \frac{1}{2}\left(\frac{d-r}{d-c} + 1\right), & \text{if } c \leq r \leq a; \\ 1, & \text{if } r \leq c. \end{cases}
$$

For rough variable $\xi = ([4, 5], [2, 7])$, $Tr\{\xi \leq r\}$ and $Tr\{\xi \geq r\}$ are depicted in Fig. 1.

α-optimistic value to $\xi = ([a, b], [c, d])$ is

$$
\xi_{sup}(\alpha) = \begin{cases} (1-2\alpha)d + 2\alpha c, & \text{if } \alpha \leq \frac{d-b}{2(d-c)}; \\ 2(1-\alpha)d + (2\alpha-1)c, & \text{if } \alpha \geq \frac{2d-a-c}{2(d-c)}; \\ \frac{d(b-a)+b(d-c)-2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & \text{otherwise.} \end{cases}
$$

α-pessimistic value to ξ is

$$
\xi_{inf}(\alpha) = \begin{cases} (1-2\alpha)c + 2\alpha d, & \text{if } \alpha \leq \frac{a-c}{2(d-c)}; \\ 2(1-\alpha)c + (2\alpha-1)d, & \text{if } \alpha \geq \frac{b+d-2c}{2(d-c)}; \\ \frac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & \text{otherwise.} \end{cases}
$$

The expected value of ξ is $E(\xi) = \frac{1}{4}(a + b + c + d)$.

III. DESCRIPTION OF THE PROBLEM AND MODEL FORMULATIONS

In traditional STP total transportation capacity of conveyances is taken and the problem is solved assuming that this total capacity can be utilized for all routes whatever the allocation of products is in the routes. But in many real transportation systems, full vehicles (e.g. trucks for road transportation, coaches for rail transportation, etc.) are to be booked and number of vehicles required are according to amount of product to be transported through a particular route. The difficulty in this case arises when the amount of allocated product is not sufficient to fill up the capacity of the vehicle, because then extra cost is incurred despite the unit transportation cost due to not fulfilling the vehicle capacity. Here we formulate some solid transportation models with vehicle capacity to deal with such situations.

Suppose q_k be the capacity of singe vehicle of k-th type conveyance. Let z_{ijk} be the frequency (number of required vehicles) of conveyance k for transporting goods from source i to destination j via conveyance k and x_{ijk} (decision variable) be the corresponding amount of goods. Then z_{ijk} is a decision variable which takes only positive integer or zero. Also we have

$$x_{ijk} \leq z_{ijk} \cdot q_k.$$

Now in such vehicle transportation system obviously unit transportation cost depends upon the utilizes of the capacity of the vehicle. That is for a particular route $i - j - k$ if the unit transportation cost c_{ijk} is according to full utilization of the vehicle capacity q_k then an extra cost (penalty) will be added if the capacity q_k is not fully utilized. Determination of additional cost for deficit amount depends upon the relevant transportation authority. Two cases may arise, either authority do not want to compromise for deficit amount and so direct cost c_{ijk} is also represent the additional cost for unit deficit amount, or they agree to compromise and fixed an additional cost for unit deficit amount. For calculating additional cost first deficit amount of goods is to be calculated for each route. This can be done by two ways - calculating deficit amount for $i - j - k$ route directly as $(z_{ijk} \cdot q_k - x_{ijk})$ or by calculating the empty ratio [30] of each vehicle of k-th type conveyance for transporting goods from source i to destination j as

$$
d_{ijk} = \begin{cases} 0, & \text{if } \frac{x_{ijk}}{q_k} = \left[\frac{x_{ijk}}{q_k}\right]; \\ 1 - \left(\frac{x_{ijk}}{q_k} - \left[\frac{x_{ijk}}{q_k}\right]\right), & \text{otherwise.} \end{cases}
$$

Then the amount of deficit amount for $i - j - k$ route is given by $q_k \cdot d_{ijk}$. Now if u_{ijk} represents additional cost for unit amount of deficit from source i to destination j via conveyance k, then additional cost for this route is given by

$$\epsilon_{ijk} = u_{ijk}(z_{ijk} \cdot q_k - x_{ijk}) \text{ or } \epsilon_{ijk} = u_{ijk} \cdot q_k \cdot d_{ijk}.$$

The total additional (penalty) cost for the problem is

$$C(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \epsilon_{ijk}.$$

So the STP model becomes

$$Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (c_{ijk}\ x_{ijk} + \epsilon_{ijk})$$

$$s.t. \ \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \ \leq \ a_i, \ \ i=1,2,...,m,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ \geq \ b_j, \ \ j=1,2,...,n, \qquad (1)$$

$$x_{ijk} \leq z_{ijk} \cdot q_k, \ i=1,2,...,m; j=1,2,...,n; k=1,2,...,K,$$

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j, \ x_{ijk} \geq 0, \ z_{ijk} \in Z^+, \forall \ i,j,k.$$

In the above model it is assumed that there are sufficient number of vehicles of each type of conveyance available to transport the required amount of goods (i.e., there is no restriction on number of available vehicles of each type of conveyances). If number of vehicles of conveyances limited to certain number, suppose Q_k for k-th type conveyance then an another constraint

$$\sum_{i=1}^{m}\sum_{j=1}^{n} z_{ijk} \leq Q_k, \ k=1,2,...,K$$

is added to the model (1), then the above model becomes

$$Min \ Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} (c_{ijk} \ x_{ijk} + \epsilon_{ijk})$$

$$s.t. \ \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \ \leq \ a_i, \ \ i=1,2,...,m,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ \geq \ b_j, \ \ j=1,2,...,n, \qquad (2)$$

$$x_{ijk} \leq z_{ijk} \cdot q_k, \ i=1,2,...,m; j=1,2,...,n; k=1,2,...,K,$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} z_{ijk} \leq Q_k, \ k=1,2,...,K,$$

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j, \ x_{ijk} \geq 0, \ z_{ijk} \in Z^+, \forall \ i,j,k.$$

This limitation of number of vehicles can effect the optimal transportation policy. For example unavailability of sufficient number of vehicles of certain type of conveyance may force to use another type of conveyance which costs higher than the previous.

In the above two models it is assumed that total available vehicles can be utilized in each source as they required. But in reality in each source, the availability of different vehicles may not be the same and the vehicles available at one sources may not be utilized for another source due to long distance between them. So there may be a situation arise that in a certain source there are more than sufficient number of particular vehicles available to transport product to destinations but at the same time in an another source there are less number of that vehicles available than the requirement. As a result it may happen that vehicle having less transportation cost leaving from certain source to destination without being fully loaded, while vehicle having comparably high transportation cost leaving with fully loaded. So it is realistic to develops a solid transportation
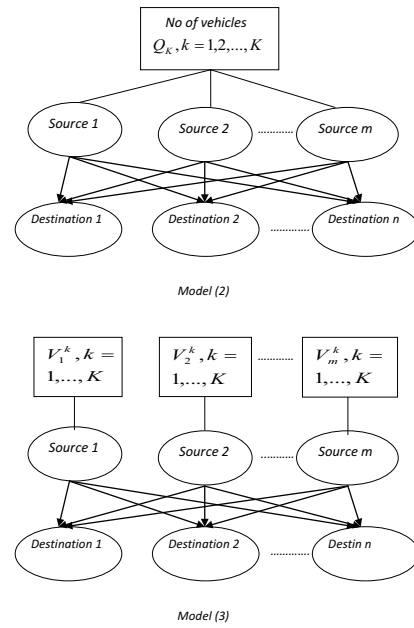

Fig. 2.   The hierarchical structures of the models (2) and (3).

model with source-wise vehicle availability. Suppose at source i, the number of available vehicles of k-th type conveyance is V_i^k and vehicles at each source can not be shared to other sources. Then the constraints

$$\sum_{j=1}^{n} z_{ijk} \leq V_i^k, \ i=1,2,...,m; k=1,2,...,K$$

is added to the model (1) and so the model becomes

$$Min \ Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} (c_{ijk} \ x_{ijk} + \epsilon_{ijk})$$

$$s.t. \ \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \ \leq \ a_i, \ \ i=1,2,...,m,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ \geq \ b_j, \ \ j=1,2,...,n, \qquad (3)$$

$$x_{ijk} \leq z_{ijk} \cdot q_k, \ i=1,2,...,m; j=1,2,...,n; k=1,2,...,K,$$

$$\sum_{j=1}^{n} z_{ijk} \leq V_i^k, \ i=1,2,...,m; k=1,2,...,K,$$

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j, \ x_{ijk} \geq 0, \ z_{ijk} \in Z^+, \forall \ i,j,k.$$

The hierarchical structures of the models (2) and (3) are shown in the Fig. 2.

IV. PROBLEM WITH UNIT TRANSPORTATION AND ADDITIONAL COSTS (PENALTY) AS ROUGH VARIABLES

Consider the unit transportation costs c_{ijk} and as well as unit additional costs u_{ijk} for the model (1) are rough variables represented by $c_{ijk} = ([c_{ijk}^2, c_{ijk}^3], [c_{ijk}^1, c_{ijk}^4])$, $c_{ijk}^1 \leq c_{ijk}^2 < c_{ijk}^3 \leq c_{ijk}^4$ and $u_{ijk} = ([u_{ijk}^2, u_{ijk}^3], [u_{ijk}^1, u_{ijk}^4])$, $u_{ijk}^1 \leq u_{ijk}^2 < u_{ijk}^3 \leq u_{ijk}^4$. Then the objective function of the model (1), $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (c_{ijk} \; x_{ijk} + \epsilon_{ijk})$, $\epsilon_{ijk} = u_{ijk}(z_{ijk} \cdot q_k - x_{ijk})$ becomes a rough variable defined as $Z = ([Z^2, Z^3], [Z^1, Z^4])$, where

$$Z^r = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (c_{ijk}^r \; x_{ijk} + \epsilon_{ijk}^r), \; r = 1, 2, 3, 4,$$

$\epsilon_{ijk}^r = u_{ijk}^r(z_{ijk} \cdot q_k - x_{ijk}), \; r = 1, 2, 3, 4.$

Rough chance-constrained programming: We formulate rough chance-constrained programming (CCP) for the model (1) with rough costs. Since the problem is a minimization problem, we minimize the smallest objective $\bar{Z}$ satisfying $Tr\{Z \leq \bar{Z}\} \geq \alpha$, where $\alpha \in (0, 1]$ is a specified trust (confidence) level, i.e., we minimize the α-pessimistic value $Z_{inf}(\alpha)$ of Z. This implies that the optimum objective value will below the $\bar{Z}$ with a trust level at least α. So the rough CCP becomes

$$Min \; (Min \; \bar{Z}) \tag{4}$$

$$\begin{aligned} s.t. \; Tr\{Z \leq \bar{Z}\} &\geq \alpha, \\ \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} &\leq a_i, \; i = 1, 2, ..., m, \end{aligned} \tag{5}$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \geq b_j, \; j = 1, 2, ..., n, \tag{6}$$

$$x_{ijk} \leq z_{ijk} \cdot q_k, \; i = 1, 2, ..., m; j = 1, 2, ..., n; k = 1, 2, ..., K, \tag{7}$$

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j, \; x_{ijk} \geq 0, \; z_{ijk} \in Z^+, \forall \; i, j, k. \tag{8}$$

From the definition of α-pessimistic value, the above CCP equivalently becomes

$$Min \; Z'$$

$$s.t. \; the \; constraints \; (5) - (8), \tag{9}$$

where

$$Z' = \begin{cases} (1 - 2\alpha)Z^1 + 2\alpha Z^4, & if \; \alpha \leq \frac{Z^2 - Z^1}{2(Z^4 - Z^1)}; \\ 2(1 - \alpha)Z^1 + (2\alpha - 1)Z^4, & if \; \alpha \geq \frac{Z^3 + Z^4 - 2Z^1}{2(Z^4 - Z^1)}; \\ \frac{Z^1(Z^3 - Z^2) + Z^2(Z^4 - Z^1) + 2\alpha(Z^3 - Z^2)(Z^4 - Z^1)}{(Z^3 - Z^2) + (Z^4 - Z^1)}, \\ \hspace{4cm} otherwise. \end{cases}$$

Now we also formulate another rough CCP for the model (1) with rough costs, to minimize the greatest objective $\underline{Z}$ satisfying $Tr\{Z \geq \underline{Z}\} \geq \alpha$, where $\alpha \in (0, 1]$ is a specified trust (confidence) level, i.e., we minimize the α-optimistic value $Z_{sup}(\alpha)$ of Z. This implies that the optimum objective value will above the $\underline{Z}$ with a trust level at least α. So the rough CCP becomes

$$Min \; (Max \; \underline{Z})$$

$$s.t. \; Tr\{Z \geq \underline{Z}\} \geq \alpha, \tag{10}$$

$$and \; the \; constraints \; (5) - (8).$$

From the definition of α-optimistic value, the above CCP equivalently becomes

$$Min \; Z''$$

$$s.t. \; the \; constraints \; (5) - (8), \tag{11}$$

where

$$Z'' = \begin{cases} (1 - 2\alpha)Z^4 + 2\alpha Z^1, & if \; \alpha \leq \frac{Z^4 - Z^3}{2(Z^4 - Z^1)}; \\ 2(1 - \alpha)Z^4 + (2\alpha - 1)Z^1, & if \; \alpha \geq \frac{2Z^4 - Z^2 - Z^1}{2(Z^4 - Z^1)}; \\ \frac{Z^4(Z^3 - Z^2) + Z^3(Z^4 - Z^1) - 2\alpha(Z^3 - Z^2)(Z^4 - Z^1)}{(Z^3 - Z^2) + (Z^4 - Z^1)}, & otherwise. \end{cases}$$

Since for $0.5 < \alpha \leq 1$, $Z_{inf}(\alpha) \geq Z_{sup}(\alpha)$, so solving the problems (9) and (11) with trust level α $(0.5 < \alpha \leq 1)$ we conclude that optimum objective value lie within the range $[Z'', Z']$ with the trust level at least α.

In case of models (2) and (3) with unit transportation and additional costs as rough variables, rough CCP can be developed same way as above.

V. NUMERICAL EXAMPLE

A. Problems with unit transportation and additional costs as crisp numbers

Consider a problem with three sources $(i = 1, 2, 3)$, three destinations $(j = 1, 2, 3)$, two types of conveyances $(k = 1, 2)$. The unit transportation costs are given in Table I. The availabilities at each source, demands of each destination and capacity of single vehicle of each type of conveyance are given in Table II. For convenience suppose additional costs for unit deficit amount is $u_{ijk} = 0.8 \cdot c_{ijk}$.

Now if there are sufficient number of vehicles of each type

TABLE I
UNIT TRANSPORTATION COSTS c_{ijk}

$i \backslash j$	1	2	3	1	2	3
1	8	11	12	12	9	13
2	8	10	7	11	8	10
3	9	14	9	12	10	9
k		1			2	

TABLE II
AVAILABILITIES, DEMANDS AND VEHICLE CAPACITY.

$a_1 = 25.6, a_2 = 16.8, a_3 = 32.4, b_1 = 14.8, b_2 = 26.8,$ $b_3 = 23.8, q_1 = 2.48, q_2 = 3.78.$

conveyance available as required (i.e., there is no restriction on number of available vehicles of each type of conveyances), then for the above problem solving the model (1) we have the following solution (Table III) So total number of required vehicles of conveyance $k = 1$ is 10 and that of conveyance $k = 2$ is 11.

Now as we say earlier, it may happen that number of vehicles of certain type of conveyance is so limited that it is not sufficient to fulfill its requirement for a transportation system.

TABLE III
OPTIMUM RESULTS FOR MODEL (1)

$x_{111} = 14.8$, $x_{121} = 2.44$, $x_{221} = 1.68$, $x_{331} = 4.96$, $x_{122} = 7.56$, $x_{222} = 15.12$, $x_{332} = 18.84$, Min $Z = 572.936$, $z_{111} = 6$, $z_{121} = 1$, $z_{221} = 1$, $z_{331} = 2$, $z_{122} = 2$, $z_{222} = 4$, $z_{332} = 5$.

Suppose in the above example the number of available vehicles of conveyance $k = 1$ is 14 and that of conveyance $k = 2$ is 10, i.e., $Q_1 = 14$ and $Q_2 = 10$. Then with the same data as given in Tables I and II, solving the model (2) we have the following solution (Table IV). It should be mentioned that

TABLE IV
OPTIMUM RESULTS FOR MODEL (2)

$x_{111} = 14.8$, $x_{121} = 2.48$, $x_{221} = 1.64$, $x_{231} = 3.82$, $x_{331} = 4.86$, $x_{122} = 7.56$, $x_{222} = 11.34$, $x_{322} = 3.78$, $x_{332} = 15.12$, Min $Z = 579.536$, $z_{111} = 6$, $z_{121} = 1$, $z_{221} = 1$, $z_{231} = 2$, $z_{331} = 2$, $z_{122} = 2$, $z_{222} = 3$, $z_{322} = 1$, $z_{332} = 4$.

here in case of model (2), if number of available vehicles of each type of conveyance at each source are greater or equal to as required in model (1), i.e., if $Q_1 \geq 10$ and $Q_2 \geq 11$ then model (2) gives the same result as model (1).

Now to demonstrate model (3), consider the same data as given in Tables I and II and suppose availability of vehicles of each type conveyances at each sources are $V_1^1 = 5$, $V_1^2 = 3$, $V_2^1 = 4$, $V_2^2 = 6$, $V_3^1 = 4$, $V_3^2 = 5$. Then solving the model (3) we have the solution as presented in Table V.

TABLE V
OPTIMUM RESULTS FOR MODEL (3)

$x_{111} = 9.92$, $x_{121} = 1.64$, $x_{221} = 2.48$, $x_{231} = 2.48$, $x_{311} = 4.88$, $x_{331} = 2.48$, $x_{122} = 11.34$, $x_{222} = 11.34$, $x_{332} = 18.9$, Min $Z = 576.54$, $z_{111} = 4$, $z_{121} = 1$, $z_{221} = 1$, $z_{231} = 1$, $z_{311} = 2$, $z_{331} = 1$, $z_{122} = 3$, $z_{222} = 3$, $z_{332} = 5$.

B. Problems with unit transportation and additional costs as rough variables

Consider the model (1) with three sources ($i = 1, 2, 3$), three destinations ($j = 1, 2, 3$), two types of conveyances ($k = 1, 2$). The unit transportation costs are rough variables as given in Tables VI and VII. The availabilities at each sources, demands of each destinations and capacity of single vehicle of each type of conveyances are same as in Table II. For convenience suppose additional costs for unit deficit amount is $u_{ijk} = 0.8 \cdot c_{ijk}$. Now constructing rough CCP as (4)-(8) with trust level $\alpha = 0.9$, we have corresponding deterministic form using (9) as follows:

$$Min \ Z'$$

TABLE VI
UNIT TRANSPORTATION COSTS c_{ij1}

$i \setminus j$	1	2	3
1	([7,9],[6,10])	([10,11],[8,12])	([11,13],[10,12])
2	([6,8],[5,9])	([9,10],[7,11])	([5,7],[4,8])
3	([8,10],[7,11])	([13,15],[12,16])	([8,10],[7,11])

TABLE VII
UNIT TRANSPORTATION COSTS c_{ij2}

$i \setminus j$	1	2	3
1	([10,12],[9,13])	([8,10],[7,11])	([12,14],[11,15])
2	([11,12],[9,13])	([6,8],[5,9])	([9,10],[7,11])
3	([11,12],[10,13])	([10,11],[9,12])	([8,9],[7,11])

$$s.t. \sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk} \leq a_i, \quad i = 1, 2, 3,$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{ijk} \geq b_j, \quad j = 1, 2, 3, \tag{12}$$

$$x_{ijk} \leq z_{ijk} \cdot q_k, \ i = 1, 2, 3; j = 1, 2, 3; k = 1, 2,$$

$$x_{ijk} \geq 0, \ z_{ijk} \in Z^+, \forall \ i, j, k.$$

where,

$$Z' = \begin{cases} -0.8Z^1 + 1.8Z^4, & \text{if } 0.9 \leq \frac{Z^2 - Z^1}{2(Z^4 - Z^1)}; \\ 0.2Z^1 + 0.8Z^4, & \text{if } 0.9 \geq \frac{Z^3 + Z^4 - 2Z^1}{2(Z^4 - Z^1)}; \\ \frac{Z^1(Z^3 - Z^2) + Z^2(Z^4 - Z^1) + 1.8(Z^3 - Z^2)(Z^4 - Z^1)}{(Z^3 - Z^2) + (Z^4 - Z^1)}, & \text{otherwise.} \end{cases}$$

$$Z^r = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2} (c_{ijk}^r \ x_{ijk} + \epsilon_{ijk}^r), \ r = 1, 2, 3, 4,$$

$\epsilon_{ijk}^r = 0.8c_{ijk}^r(z_{ijk} \cdot q_k - x_{ijk}), \ r = 1, 2, 3, 4.$
Solving this problem we get the solution presented in Table VIII. From this solution we conclude that the objective value

TABLE VIII
OPTIMUM RESULTS FOR MODEL (1) WITH TRANSPORTATION
COSTS AS ROUGH VARIABLES

$x_{111} = 12.4$, $x_{121} = 4.87$, $x_{231} = 2.43$, $x_{311} = 2.4$, $x_{331} = 2.46$, $x_{122} = 7.56$, $x_{222} = 14.36$, $x_{332} = 18.9$, Min $Z' = 630.2688$,, $z_{111} = 5$, $z_{121} = 2$, $z_{231} = 1$, $z_{311} = 1$, $z_{331} = 1$, $z_{122} = 2$, $z_{222} = 4$, $z_{332} = 5$.

will less or equal to 630.2688 with trust level at least 0.9. We now construct rough CCP as (10) with trust level $\alpha = 0.9$ and then we have corresponding deterministic form using (11) as follows:

$$Min \ Z''$$

$$s.t. \sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk} \leq a_i, \quad i = 1, 2, 3,$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{ijk} \geq b_j, \quad j = 1, 2, 3, \tag{13}$$

$$x_{ijk} \leq z_{ijk} \cdot q_k, \ i = 1, 2, 3; j = 1, 2, 3; k = 1, 2,$$

$$x_{ijk} \geq 0, \; z_{ijk} \in Z^+, \forall \; i,j,k.$$

where,

$$Z'' = \begin{cases} -0.8Z^4 + 1.8Z^1, & \text{if } 0.9 \leq \frac{Z^4-Z^3}{2(Z^4-Z^1)}; \\ 0.2Z^4 + 0.8Z^1, & \text{if } 0.9 \geq \frac{2Z^4-Z^2-Z^1}{2(Z^4-Z^1)}; \\ \frac{Z^4(Z^3-Z^2)+Z^3(Z^4-Z^1)-1.8(Z^3-Z^2)(Z^4-Z^1)}{(Z^3-Z^2)+(Z^4-Z^1)}, & \text{otherwise.} \end{cases}$$

Solving this we get $MinZ'' = 471.427$. So the objective value will greater or equal to 471.427 with trust level at least 0.9. As we know for $0.5 < \alpha \leq 1$, $Z_{inf}(\alpha) \geq Z_{sup}(\alpha)$, here our results $(Z' > Z'')$ shows this truth. Finally we can conclude that the optimum objective value lie within the range $[471.427, 630.2688]$ with trust level at least 0.9. To validate these results let us find the optimum expected objective value $E(Z)$. For the rough objective function $Z = ([Z^2, Z^3], [Z^1, Z^4])$, we have $E(Z) = 1/4(Z^1 + Z^2 + Z^3 + Z^4)$. We find the expected objective value for this problem 547.358. So we see that the expected objective value lie within the range of objective value as obtained by rough programming.

VI. Conclusion

This paper presents some solid transportation models for the transportation system where full vehicles are used for transportation so that unit transportation costs are determined according to full utilization of the vehicle capacity. To deal with different situations like availability of each type of conveyances, whether the available vehicles at one source can be utilized at another source or not, different solid transportation models are formulated. STP with different types of uncertain variables such as fuzzy, random, fuzzy random are discussed by many researchers, but STP with rough variables is not discussed before. In this paper we only assume the unit transportation costs as rough variables, the STP with all the parameters, i.e., costs, availabilities, demands, conveyance capacities as rough variables may be taken as a future work.

References

[1] K. Dembczynski, S. Greco, R. Slowinski, Rough set approach to multiple criteria classification with imprecise evaluations and assignments, European Journal of Operational Research 198 (2009) 626-636.

[2] S. Greco, B. Matarazzo, R. Slowinski, Rough sets theory for multicriteria decision analysis, European Journal of Operational Research 129 (2001) 1-47.

[3] K.B.Haley, The sold transportation problem, Operations Research 10 (1962) 448-463.

[4] S. Hirano, S. Tsumoto, Rough representation of a region of interest in medical images, International Journal of Approximate Reasoning 40 (2005) 23-34.

[5] F.Jiménez, J.L.Verdegay, Uncertain solid transportation problems, Fuzzy Sets and Systems, 100 (1998) 45-57.

[6] T. Lin, Y. Yao, L. Zadeh, Data Mining, Rough Sets, and Granular Computing, Springer Verlag, 2002.

[7] P. Lingras, Rough Neural Networks, International Conference on Information Processing and Management of Uncertainty, Granada, Spain, 1996, 1445-1450.

[8] B. Liu, Theory and Practice of Uncertain Programming, Physical-Verlag, Heidelberg, 2002.

[9] B. Liu, Uncertainty Theory: An Introduction to its Axiomatic Foundation, Springer-Verlag, Berlin, 2004.

[10] B. Liu, Inequalities and Convergence Concepts of Fuzzy and Rough Variables, Fuzzy Optimization and Decision Making 2 (2003) 87-100.

[11] G. Liu, W. Zhu, The algebraic structures of generalized rough set theory, Information Sciences 178 (2008) 4105-4113.

[12] S. Liu, Fuzzy total transportation cost measures for fuzzy solid transportation problem, Applied Mathematics and Computation 174 (2006) 927-941.

[13] A. Nagarjan, K. Jeyaraman, Solution of chance constrained programming problem for multi-objective interval solid transportation problem under stochastic environment using fuzzy approach, International Journal of Computer Applications 10(9) (2010) 19-29.

[14] A.Ojha, B.Das, S.Mondal, M.Maiti, An entropy based solid transportation problem for general fuzzy costs and time with fuzzy equality, Mathematical and Computer Modeling, 50 (1-2) (2009) 166-178.

[15] A. Ojha, B. Das, S. Mondal, M. Maiti, A stochastic discounted multi-objective solid transportation problem for breakable items using analytical hierarchy process, Applied Mathematical Modelling 34 (2010) 2256-2271.

[16] Z. Pawlak, Rough sets, International Journal of Information and Computer Sciences 11(5) (1982) 341-356.

[17] Z. Pawlak, Rough Sets - Theoretical Aspects of Reasoning About Data, Kluwer Academatic Publishers, Boston, 1991.

[18] Z. Pawlak, R. Slowinski, Rough set approach to multi-attribute decision analysis (invited review), European Journal of Operational Research 72 (1994) 443-459.

[19] Z. Pawlak, A. Skowron, Rough sets: some extensions, Information Sciences 177 (2007) 28-40.

[20] L. Polkowski: Rough Sets, Mathematical Foundations, Advances in Soft Computing, Physica Verlag, A Springer-Verlag Company, 2002.

[21] E.D.Schell, Distribution of a product by several properties, Proceedings of 2nd Symposium in Linear Programming, DCS/comptroller, HQ US Air Force, Washington D C (1955) 615-642.

[22] M. Shafiee, N. Shams-e-alam, Supply Chain Performance Evaluation With Rough Data Envelopment Analysis, 2010 International Conference on Business and Economics Research, vol.1 (2011), IACSIT Press, Kuala Lumpur, Malaysia.

[23] Z. Tao, J. Xu, A class of rough multiple objective programming and its application to solid transportation problem, Information Sciences 188 (2012) 215235.

[24] S. Tsumoto, Medical differential diagnosis from the viewpoint of rough sets, Information Sciences (2007) 28-34.

[25] J. Xu and L. Yao, A Class of Two-Person Zero-Sum Matrix Games with Rough Payoffs, International Journal of Mathematics and Mathematical Sciences 2010, doi:10.1155/2010/404792.

[26] J. Xu, B. Li, D. Wu, Rough data envelopment analysis and its application to supply chain performance evaluation, International Journal of Production Economics 122 (2009) 628-638.

[27] Shu Xiao and Edmund M-K. Lai, A Rough programming approach to power-aware VLIW instruction scheduling for digital signal processors, ICASSP 2005.

[28] L. Yang, Y. Feng, A bicriteria solid transportation problem with fixed charge under stochastic environment, Applied Mathematical Modelling 31 (2007) 2668-2683.

[29] L. Yang, L. Liu, Fuzzy fixed charge solid transportation problem and algorithm, Applied Soft Computing 7 (2007) 879-889.

[30] L. Yang, Z. Gao, K. Li, Railway freight transportation planning with mixed uncertainty of randomness and fuzziness, Applied Soft Computing 11 (2011) 778-792.

[31] E. Youness, Characterizing solutions of rough programming problems, European Journal of Operational Research 168 (2006) 1019-1029.

[32] W. Zhu, Relationship between generalized rough sets based on binary relation and covering, Information Sciences 179 (2009) 210-225.

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