

Approximate Bounded Knowledge Extraction using Type-I Fuzzy Logic

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Abstract—Using neural network we try to model the unknown function f for given input-output data pairs. The connection strength of each neuron is updated through learning. Repeated simulations of crisp neural network produce different values of weight factors that are directly affected by the change of different parameters. We propose the idea that for each neuron in the network, we can obtain quasi-fuzzy weight sets (QFWS) using repeated simulation of the crisp neural network. Such type of fuzzy weight functions may be applied where we have multivariate crisp input that needs to be adjusted after iterative learning, like claim amount distribution analysis. As real data is subjected to noise and uncertainty, therefore, QFWS may be helpful in the simplification of such complex problems. Secondly, these QFWS provide good initial solution for training of fuzzy neural networks with reduced computational complexity.

Keywords—Crisp neural networks, fuzzy systems, extraction of logical rules, quasi-fuzzy numbers.

I. INTRODUCTION

FISSION of artificial neural networks and fuzzy inference systems have attracted the growing interest of researchers in various scientific and engineering areas due to the growing need of adaptive intelligent systems to solve the real world problems. A crisp or fuzzified neural network can be viewed as a mathematical model for brain-like systems. The learning process increases the sum of knowledge of the neural network by improving the configuration of weight factors. Fuzzy neural networks are generalization of crisp neural networks to process both numerical information from measuring instruments and linguistic information from human experts, see [2], [14], and [15]. Thus, fuzzy inference systems can be used to emulate human expert knowledge and experience. An overview of different fuzzy neural network architectures is discussed by [5], [7] and classified them as,

- 1) A fuzzy neural network may take crisp or fuzzy values as inputs and can return crisp or fuzzy output.

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- 2) another class of fuzzy neural networks is feedforward neural networks which are defined from conventional feedforward neural networks by substituting fuzzified neurons for crisp ones. These are named as regular fuzzy neural networks.

It is much more difficult to develop the learning algorithms for the fuzzy neural networks than for the crisp neural networks; this is because the inputs, connections weights and bias terms related to a regular fuzzy neural network are fuzzy sets, see [17], [22] and [24].

The paper is organized as follows. In section II, we made a short study of learning procedures in crisp neural networks. In section III, we present concepts of fuzzy logic and quasi-fuzzy sets. In section IV, simulation experiments using crisp neural network is performed repeatedly to achieve quasi-fuzzy sets. These sets provide the initial solution for type-I neuro-fuzzy networks as discussed by [9], [28] and [29]. To our knowledge, the concept of obtaining fuzzy weights through crisp neural network has not been investigated in the literature.

II. NEURAL NETWORKS

Using neural network we try to model the unknown function f for given input-output data pairs. The existing algorithms for these problems are regression modeling, neural networks, and wavelet theory. A neural network can be regarded as representation of a function determined by its weight factors and networks architecture [15]. The overall mapping is thus characterized by a composite function relating feedforward network inputs to output. That is

$$\mathbf{O} = \mathbf{f}_{\text{composite}}(\mathbf{x})$$

Using p-mapping layers in a p+1 layer feedforward net yield

$$\mathbf{O} = \mathbf{f}^{L_p}(\mathbf{f}^{L_{p-1}}(\dots(\mathbf{f}^{L_1}(\mathbf{x}))\dots))$$

Usually, we train a neural network with a training set, present inputs to the neural networks, and interpret the outputs according to the logical rules in the training set see [1], [3],[4] and [21]. The most commonly used technique to adjust weight parameters of a neural network is backpropagation method based on LMS learning defined as

$$J = E \left[\sum_k e_k^2(n) \right]$$

where k = number of output neurons.

$$w_{ji}^l(n+1) = w_{ji}^l(n) + \eta \delta_j^l(n) y_i^{l-1}(n),$$

η is the learning rate and $\delta_j^l(n)$ is the local change made at each neuron in the learning, see [15]

$$\delta_j^l(n) = \begin{cases} e^{-v_j^l(n)} \Phi'(v_j^l(n)) & \text{(for neuron } j \text{ in output layer } L) \\ \Phi'(v_j^l(n)) \sum_k \delta_k^{l+1}(n) w_{kj}^{l+1}(n) & \text{(for neuron } j \text{ in output layer } l) \end{cases}$$

But to deal with noisy and uncertain information, a crisp neural network has to use concepts of fuzzy inference systems [27].

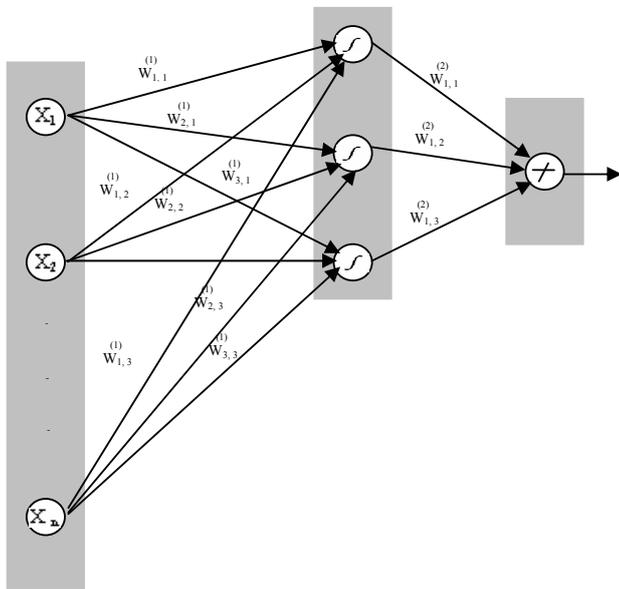


Fig. 1 Structure of a crisp artificial neural network

III. FUZZY LOGIC

Fuzzy logic was originally proposed by Prof. Lotfi A. Zadeh to quantitatively and effectively handle problems involving uncertainty; ambiguity and vagueness see [12] and [13]. The theory which is now well-established was specifically designed to mathematically represent uncertainty and vagueness and provide formalized tools for dealing with the imprecision that is intrinsic to many real world problems. The ability of fuzzy logic is inherently robust since it does not require precision and noise-free inputs. Fuzzy inference systems are the most reliable alternative if the mathematical model of the system to be controlled is unavailable [11],[18] and [26]. The fuzzy sets and fuzzy rules can be formulated in terms of linguistic variables. Methods of fuzzy logic are commonly used to model a complex system by a set of rules provided by the experts. But fuzzy rules can also be applied in

reverse problems: given the input-output behavior of a system, what are the rules which are governing the behavior.

We cite definitions of fuzzy set and membership function cross over points, alpha-cut sets and convexity of a fuzzy set see [10].

Definition 1: If X is a collection of objects denoted generically by x , then a fuzzy set A is defined as a set of ordered pairs,

$$A = \{(x, \mu_A(x)), | x \in X\}$$

Where $\mu_A(x)$ is called the membership function for the fuzzy set A . The membership function maps each element of X to a membership grade value between 0 and 1.

Definition 2: The α - cut or α - level set is a non-fuzzy set of a fuzzy set A denoted by A^α and defined as

$$A^\alpha = \{x | \mu_A(x) \geq \alpha\} \tag{1}$$

Thus every fuzzy set can be represented as a set of it's α - cuts as

$$A = \{A^{\alpha_1}, A^{\alpha_2}, \dots, A^{\alpha_m}\} \tag{2}$$

Definition 3: A fuzzy set is convex if and only if for any $x_1, x_2 \in X$ and any $\lambda \in [0,1]$

$$\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1) + \mu_A(x_2))$$

Alternatively, A is convex if all of its α - cut sets are convex.

Definition 4: A quasi-fuzzy number A is a fuzzy set of the real line with a normal, fuzzy convex and continuous membership function satisfying the following conditions,

$$\lim(t \rightarrow -\infty) A(t) = 0, \lim(t \rightarrow \infty) A(t) = 0 \tag{3}$$

Let A be a fuzzy number. Then A^γ is a closed convex subset of \mathbf{R} for all $\gamma \in [0,1]$ defined as

$$\begin{aligned} a_l(\gamma) &= \min(A^\gamma), a_r(\gamma) = \max(A^\gamma) \\ a_l : [0,1] &\rightarrow \mathbf{R}, a_r : [0,1] \rightarrow \mathbf{R} \end{aligned} \tag{4}$$

Then $A^\gamma = [a_l(\gamma), a_r(\gamma)]$. The support of A is the open interval $(a_l(\gamma), a_r(\gamma))$.

Definition 5: A triangular membership function is specified by three parameters (a_m, a_l, a_r) as follows:

$$trian(x; a_l, a_m, a_r) = \max\left(\min\left(\frac{x - a_l}{a_m - a_l}, \frac{a_r - x}{a_r - a_m}\right), 0\right) \quad (5)$$

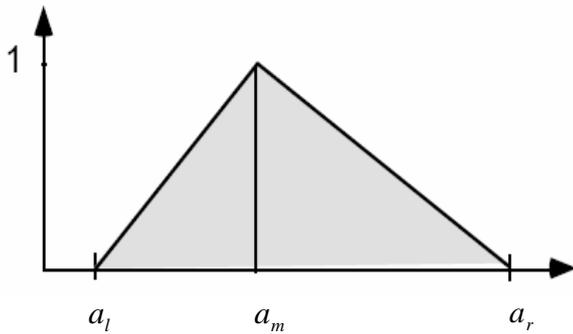


Fig. 2 Quasi triangular fuzzy set

In order to reduce computational expense, we use triangular fuzzy numbers $\tilde{a} = (a_m, a_l, a_r)_{trian}$ to define the fuzzy weight. These quasi-fuzzy weights sets follow fuzzy arithmetic, and thus can be used for fuzzy neural networks.

IV. EXPERIMENT

In this paper we demonstrate the learning and obtaining fuzzy membership functions of weight vectors to obtain quasi-fuzzy weight sets. The input/target pair presented to the network is $\{\underline{\mathbf{X}}, \underline{t}\}$ where $\underline{\mathbf{X}} = [x_1, x_2, x_3, x_4, x_5]$. A crisp neural network with 3 hidden and one output neuron is trained with performance function 1e-06 and repeated the simulation for first 100 successes.

Daily close share prices are considered from Karachi stock exchange for 200 trading days and are preprocessed. For each of the hidden neuron and output neuron, the simulated weight values may be plotted.

The QFWS of first input connected to all the three neuron are shown in figure 4. The triangular membership is constructed due to its reduced complexity [8] and [19] and [20]. For $w_{1,1}^1$ as shown in fig. 3, using (5) the parameters of triangular-mf will be,

$$a_l = \min(w_{1,1}^1) \quad a_r = \max(w_{1,1}^1) \quad a_m = (a_l + a_r) / 2 \quad (6)$$

Secondly, [15] defines that each hidden weight connection of neuron lies approximately in the interval

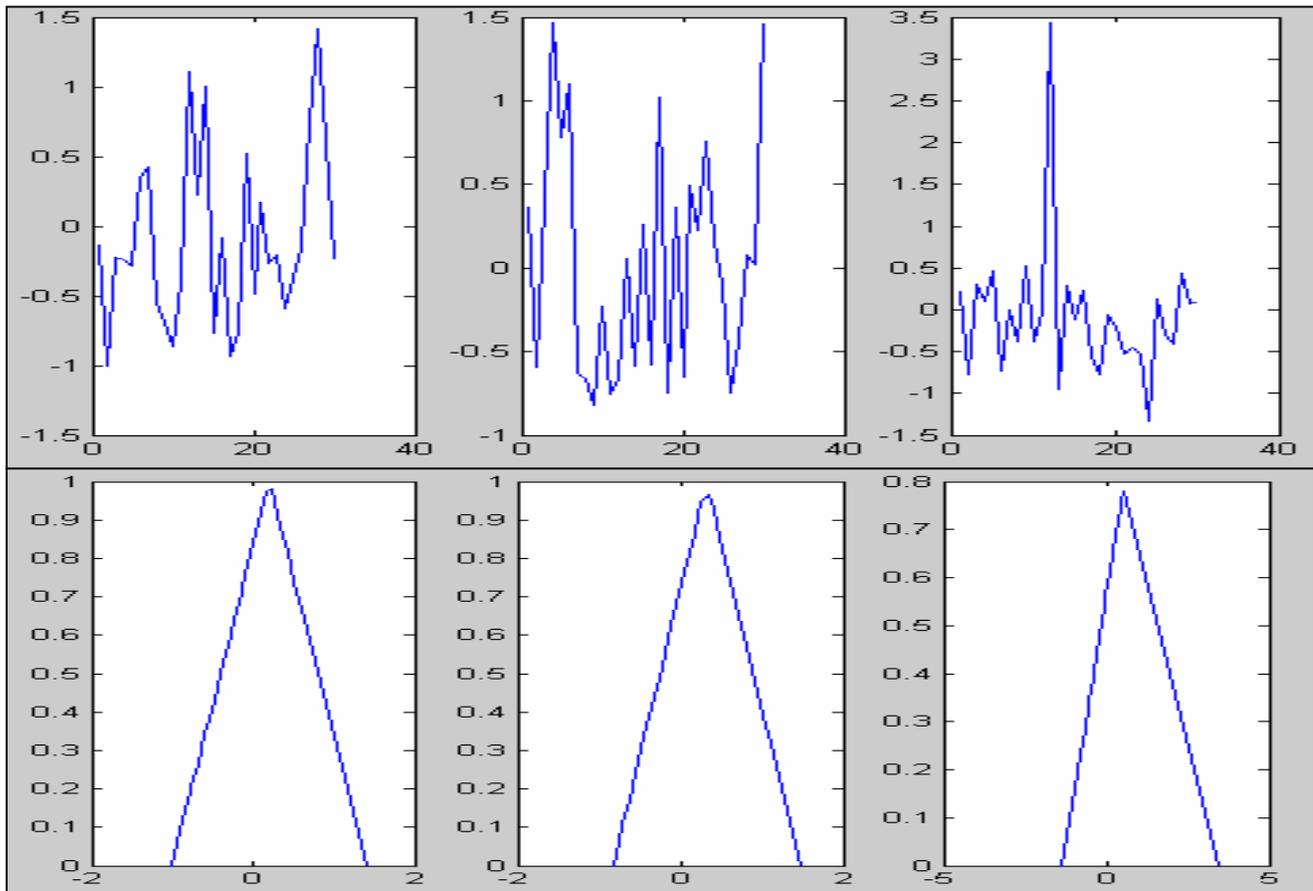


Fig. 3 (a) Input weight matrix for first input vector, (b) Triangular-mf for first weight matrix $\mathbf{w}^{i,1}$, $i=1,2,3$ (no. of neurons)

$$-\frac{1}{\sqrt{n_h}} < w_{ij} < \frac{1}{\sqrt{n_h}} \quad (7)$$

Our proposed interval based weight set in eq. (6) provides little large interval to search for weights of hidden part of a fuzzy neural network.

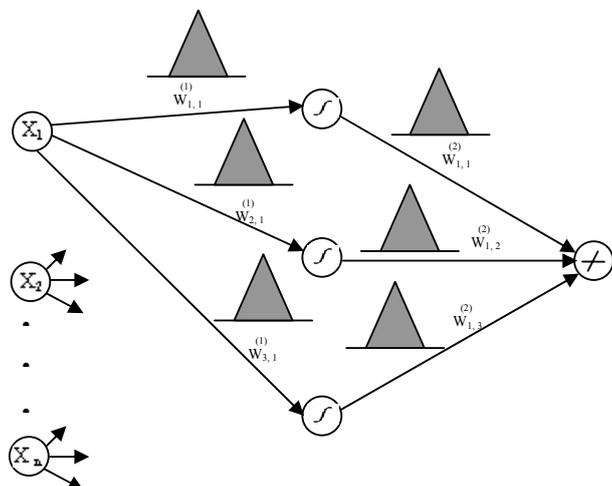


Fig. 4 Proposed quasi-fuzzy weight neural network

V. CONCLUSION

We described the architecture of QFWS based fuzzified neural networks and presented a general framework of learning algorithms of fuzzified neural networks. Learning in neuro-fuzzy learning with fuzzy weights requires initialization of an interval based fuzzy sets, which require higher computing than for crisp learning to deal with uncertainty, vagueness and linguistic behaviors of some real life situations see [6], [16], [23] and [25].

Further improved identification of suitable membership functions is possible by determining the underlying probability structure of synaptic connections of a crisp neural network. Thus based on this idea, we can form fuzzy inference systems with varying rules. This may provide new research directions to compare different QFWS based fuzzy neural networks.

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