

# Optimization of Wood Fiber Orientation Angle in Outer Layers of Variable Stiffness Plywood Plate

J. Sliseris, and K. Rocens

**Abstract**—The new optimization method for fiber orientation angle optimization of symmetrical multilayer plates like plywood is proposed. Optimization method consists of seeking for minimal compliance by choosing appropriate fiber orientation angle in outer layers of flexural plate. The discrete values of fiber orientation angles are used in method. Optimization results of simply supported plate and multispans plate with uniformly distributed load are provided. Results show that stiffness could be increased up to 20% by changing wood fiber orientation angle in one or two outer layers.

**Keywords**—Minimal compliance, flexural plate, plywood, discrete fiber angle optimization.

## I. INTRODUCTION

LAMINATED plates and shells with variable stiffness have been intensively investigated during past two decades.

These kinds of structures are becoming more popular due to ability to achieve increased strength-to-mass and stiffness-to-mass ratios by tailoring material properties. The fiber steering machines are becoming more popular in manufacturing variable stiffness glass or carbon fiber plates.

An optimal variable stiffness plate could be obtained by optimization of fiber orientation angle [1], [2], [3] or thickness optimization [4], [5]. A lamina with the variable stiffness and curved fibers provide a great flexibility to achieve needed natural frequencies, mode shapes [6], vibration amplitudes [7] and buckling load [8]. It is necessary to design constant thickness plates in many cases. Optimal properties of constant thickness plate or shell are obtained by using Genetic Algorithm [9], [10] or Ant Colony algorithm [11], [12], [13] in cases of complicated objective function or many design variables. It is necessary to take into account inter-laminar stress of variable stiffness lamina [14] in some cases.

Problem of optimal fiber orientation angle of multilayer lamina is successfully solved by the using of topology optimization approach [15], [16], discrete material optimization method [17], [18], [19], Ant colony algorithm [20] or Genetic algorithm [21]. Optimizations of structural elements are done by taking into account uncertainty and nonlinear effects [22], [23].

Flexural plates, like plywood, with variable stiffness are not investigated enough now. The optimization method for this type of structure should be specially created. Therefore there is proposed a new optimization method for plywood lamina

optimization and provided some typical results.

## II. OPTIMIZATION METHOD

The lamination parameters that define stacking sequence of lamina by 12 parameters are usually used in optimization because relationship between stiffness and lamination parameters is convex. The lamination parameters are related to each other, therefore the problem with feasible region always appears in optimization procedure. As well as an extra procedure for stacking sequence rendering from lamination parameters is necessary. To simplify this optimization technique and make it more applicable to the flexural plates with symmetrical layup we propose a new method. This method is based on structural compliance minimization:

$$\min_x c(x) = U^T(x)K(x)U(x) \quad (1)$$

where  $x$ - fiber orientation angle,  $c(x)$ - compliance function,  $U(x)$ - displacement vector,  $K(x)$ - stiffness matrix.

$$x = \{x_1, x_2, \dots, x_n\} \quad (2)$$

This method directly optimize fiber orientation angle of only outer layers of symmetrical lamina. The outer layers play the most significant role in stiffness of flexural plate.

The proposed method is based on algorithm that is shown in Fig. 1. The algorithm consists of three loops. The first loop runs until the convergence criteria are satisfied. The second loop goes through all finite elements from  $I$  to  $Ne$  (number of finite elements). The third loop goes through all discrete values of fiber orientation angles from  $I$  to  $n$ .

The fiber orientation angles are changed by special procedure  $R(x)=x_j$ . This procedure changes orientation angle to  $x_j$  in region with center in  $i$ -th finite element and radius  $R_{inf}$ .

The finite element analysis is done inside all loops. The value of compliance function  $C(i,j)$  (index  $i$  indicate  $i$ -th discrete angle and index  $j$  indicate  $j$ -th finite element) is calculated by using the results of finite element analysis.

There is a special procedure that updates values of fiber orientation angles  $x$  inside the first loop. The updated value of  $x$  is obtained in each finite element according to minimal compliance.

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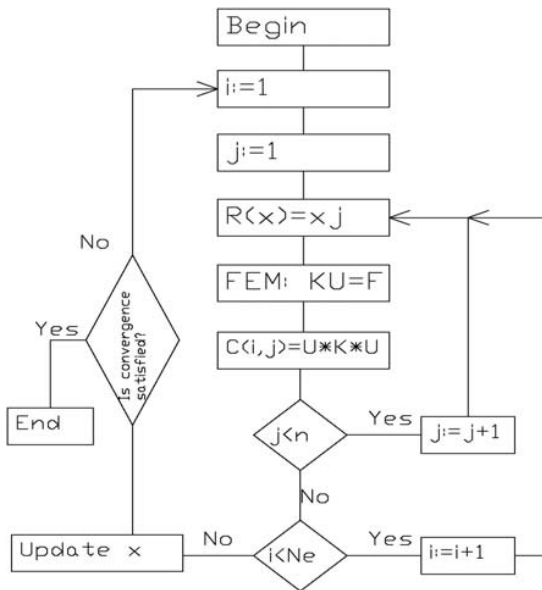


Fig. 1 The optimization algorithm

III. RESULTS

The proposed optimization method of fiber orientation angle is used to optimize rectangular one span simply supported plate and rectangular multispan plate. The simply supported plate is supported on all edges. The boundary conditions of multispan plate will be described further. In all cases, the plate is subjected to uniformly distributed load with intensity 1KPa. The plate is made of birch plywood with 19 layer symmetrical structure. Thickness of all layers is the same. The total thickness of plate is 26mm.

The optimization algorithm is implemented in MATLAB 7.10.0. The finite element analysis was done by using rectangular 4 node elements, which are based on Kirchhoff-Love linear theory of thin plates. Plywood layers are assumed orthotropic linear elastic materials.

Totally eighteen discrete values of fiber orientation angles were used in optimization:

0/10/20/30/40/50/60/70/80/90/100/110/120/130/140/150/160/170. The angle is between x-axis (horizontal axis) and fiber central axis. In all cases the influence radius was constant  $R_{inf}=0.2(m)$ .

A. Simply Supported Plate

Three different ratios of plate width and length were used in analysis (a/b=1;1.5;2). The initial fiber orientation direction of outer layer is along shorter span (see Fig. 2). The plots of optimal fiber distribution of outer layer are shown in Figs. 3, 4, 5. Due to symmetry of structure only one quarter part of plate is shown in figures.

The initial layer orientation angles are [90,0,90,0,90,0,90,0,90,0,90,0,90,0,90,0,90,0,90]. During the optimization, the outer layers are optimized:

[x,0,90,0,90,0,90,0,90,0,90,0,90,0,90,0,x].

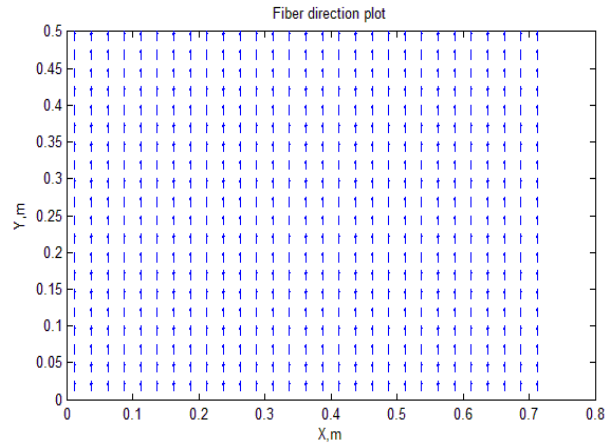


Fig. 2 The plot of fiber directions in each finite element of non-optimized plate with dimensions 1.0(m)x1.5(m)

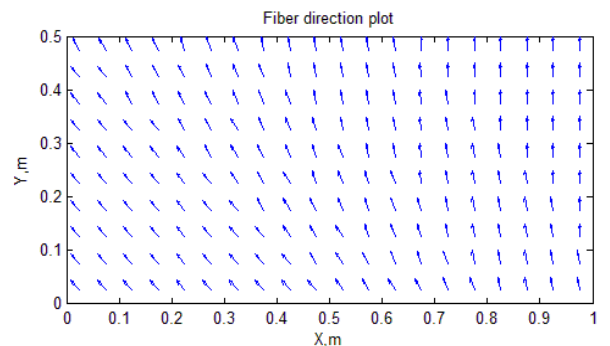


Fig. 3 The plot of fiber directions in each finite element of quarter of optimized plate with dimensions 1.0(m)x2.0(m)

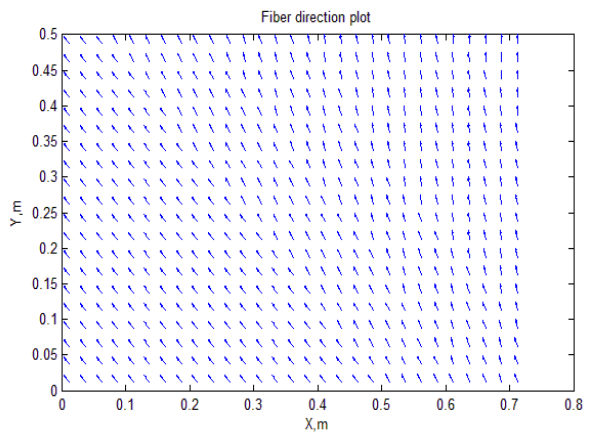


Fig. 4 The plot of fiber directions in each finite element of quarter of optimized plate with dimensions 1.0(m)x1.5(m)

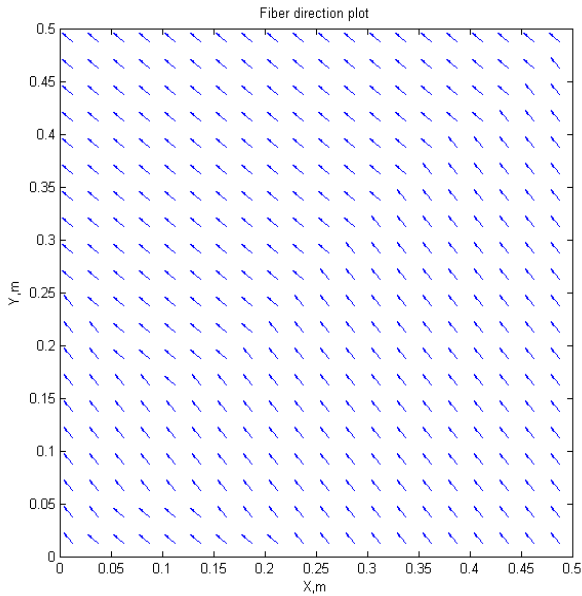


Fig. 5 The plot of fiber directions in each finite element of quarter of optimized plate with dimensions 1.0(m)x1.0(m)

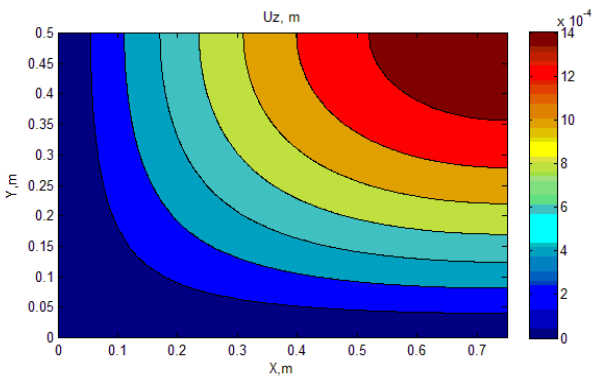


Fig. 6 Transversal displacements (m) of non-optimized plate

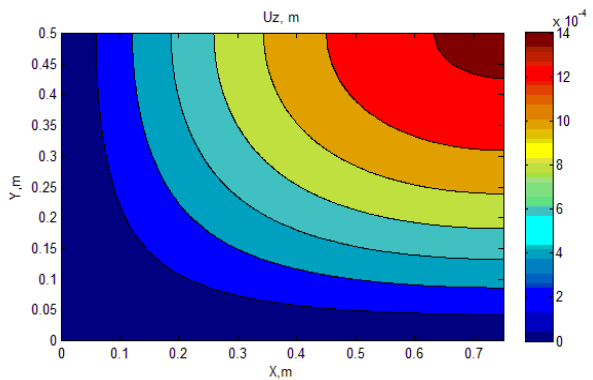


Fig. 7 Transversal displacements (m) of optimized plate

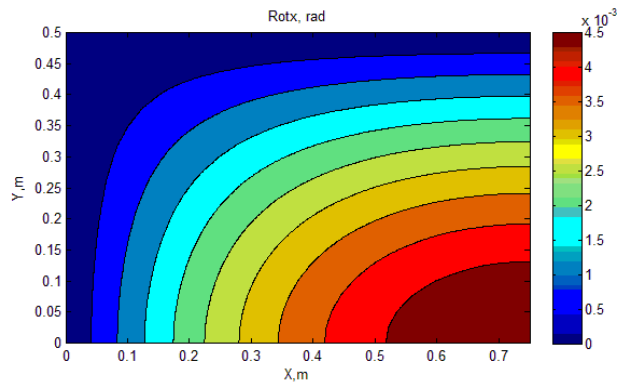


Fig. 8 Rotation around x-axis of non-optimized plate

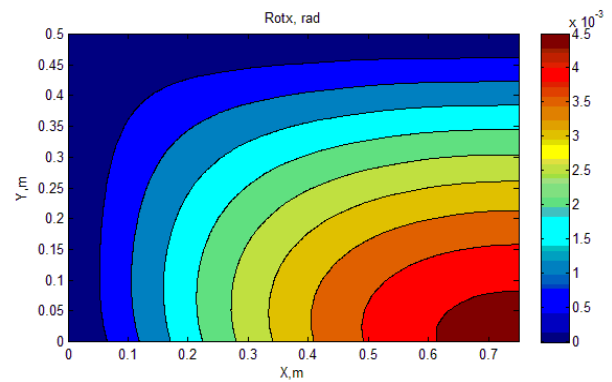


Fig. 9 Rotation around x-axis of optimized plate

Displacements and stress resultants- bending moments were compared for optimized and non-optimized plates. Transversal displacements of non-optimized and optimized plates (width/length=1.5) are shown in Figs. 6 and 7. The rotational displacements are shown in Figs. 8 and 9.

The summary of displacements and bending moments for optimized and non-optimized plates is shown in Table I. We can see that greater relative reduction of displacements could be achieved for plates with ratio of plate width to length that is closer to one. For example, maximal transversal displacement of optimized plate when  $a/b=1$  is reduced about 13% comparing to non-optimized plate. In case, when  $a/b=2$ , this difference is only 5 %.

The main stress resultants are smaller for optimized plate.

TABLE I  
DISPLACEMENTS AND BENDING MOMENTS OF OPTIMIZED AND NON-OPTIMIZED PLATE

	Non-Optimized			Optimized		
	a/b=2	a/b=1.5	a/b=1	a/b=2	a/b=1.5	a/b=1
$U_z$ , m	0.00143	0.00155	0.00133	0.00136	0.00144	0.00116
$Rot_x$ , rad	0.00456	0.00499	0.00434	0.00438	0.00473	0.00393
$Rot_y$ , rad	0.00282	0.00369	0.00439	0.00263	0.00339	0.00392
$M_x$ , KN*m/1m	0.141	0.185	0.241	0.135	0.167	0.243
$M_y$ , KN*m/1m	0.237	0.278	0.292	0.226	0.246	0.210
$M_{xy}$ , KN*m/1m	0.0239	0.0318	0.0383	0.0273	0.0354	0.0392

\* a/b- ratio of plate width and length. x-y Cartesian coordinate system, where x- horizontal direction, y- vertical direction

**B. Multispan Plate**

The multispan plate with two different boundary conditions is analyzed. In the first case, the three span plate with 0.5(m), 1.1(m) and 0.5 (m) spans in both directions are optimized. The fiber direction plots of non-optimized and optimized plates are shown in Figs. 10, 11. As well as transversal displacement plots are shown in Figs. 12, 13. Lines in bold indicate places where displacements are restricted  $U_z = 0$ . Due to symmetry, one quarter of plate is shown in plots.

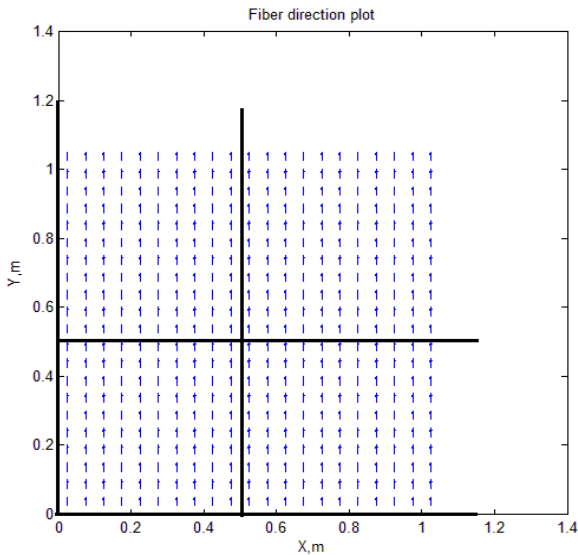


Fig. 10 The plot of fiber directions in each finite element of non-optimized three span plate (shown quarter of plate), bold lines indicate vertical supports

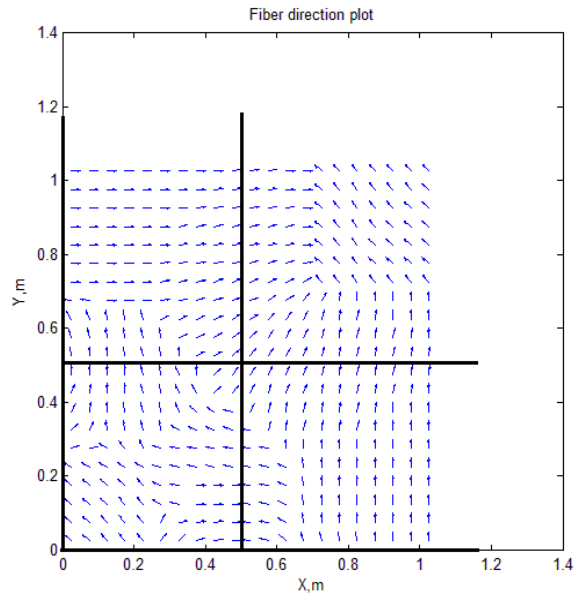


Fig. 11 The plot of fiber directions in each finite element of optimized three span plate (shown quarter of plate), bold lines indicate vertical supports

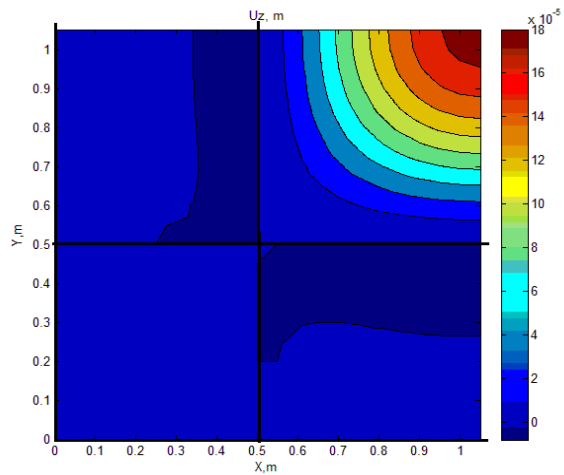


Fig. 12 The plot of transversal displacements (m) of non-optimized plate

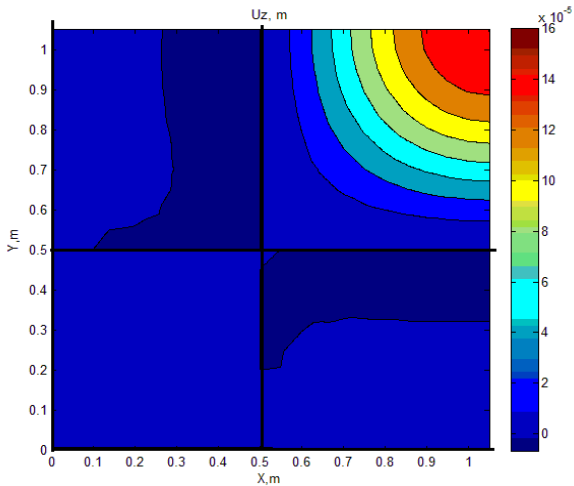


Fig. 13 The plot of transverse displacements (m) of optimized plate

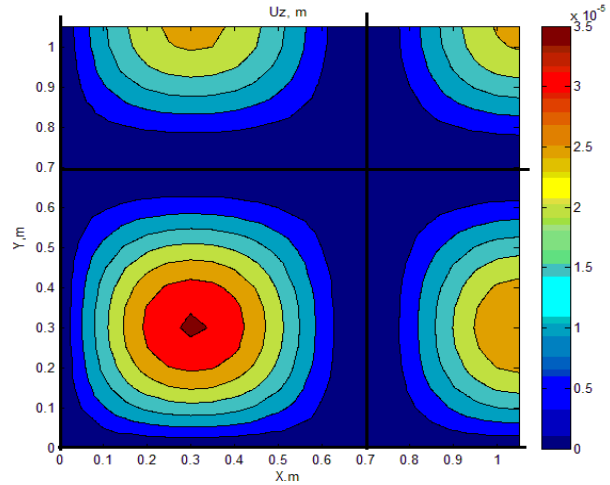


Fig. 15 The plot of transverse displacements of non-optimized plate

The plate with three equal spans in both directions – 0.7 (m) is analyzed as well. The plot of optimal fiber directions is shown in Fig. 14, but transverse displacement plots for non-optimized and optimized plate are shown in Figs. 15, 16.

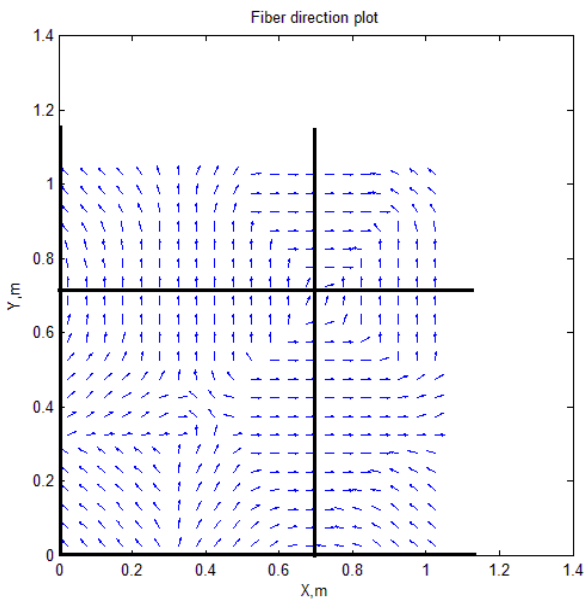


Fig. 14 The plot of fiber directions in each finite element of optimized three span plate (shown quarter of plate), bold lines indicate vertical supports

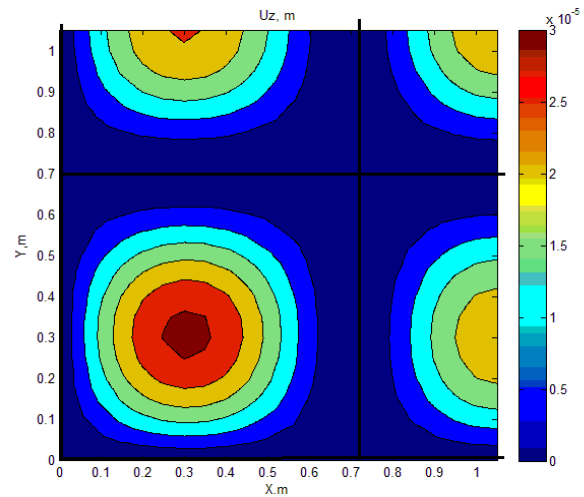


Fig. 16 The plot of transverse displacements of optimized plate

The possibility to increase stiffness (decrease displacements) by optimizing more than one outer layer was analyzed. It was done for plate with three equal spans in both directions- 0.7 (m).

The summary of results of this numerical experiment is given in table II. We can see that it is optimally to optimize only one or two outer layers, because when we optimize three layers, the increase of stiffness is not significant.

The maximal deflection could be reduced about 17% comparing to non-optimized plate, if two outer layers are optimized (for both layers fiber directions are the same). The increase of stiffness for other boundary conditions could be even bigger.

TABLE II  
DISPLACEMENTS AND BENDING MOMENTS OF NON-OPTIMIZED (FIRST NUMBER) AND OPTIMIZED (SECOND NUMBER) PLATE DEPENDING ON NUMBER OF OPTIMIZED OUTER LAYERS

	1 layer optimization	2 layer optimization	3 layer optimization
$U_z$ , mm	0.0036/0.0031	0.0036/0.0029	0.0036/0.0029
$Rot_x$ , rad	0.000193/0.000174	0.000193/0.000166	0.000193/0.000167
$Rot_y$ , rad	0.000195/0.000174	0.000203/0.000167	0.000203/0.000165
$M_x$ , KN*m/1m	0.0194/0.025	0.0125/0.0245	0.0125/0.0274
$M_y$ , KN*m/1m	0.0238/0.0223	0.0306/0.0268	0.0306/0.0265
$M_{xy}$ , KN*m/1m	0.00276/0.00285	0.00283/0.00418	0.00283/0.00586

\*Spans in both directions are 0.7m/0.7m/0.7m. Plate is subjected to uniformly distributed load

#### IV. CONCLUSION

A new optimization method of fiber orientation angle for symmetrically laminated flexural plates is proposed. The method was tested by optimizing single span and multispans plywood plates. A new kind of variable stiffness plywood and other material like glass or carbon fiber multilayer composite plates with increased stiffness could be obtained by using proposed optimization method. The results show that increase of stiffness could be more than 20%, depending on boundary conditions.

It is necessary to make further numerical and experimental investigations for this kind of plates with variable stiffness. Especially the places where connects regions with different fiber orientation angles should be investigated very detailed.

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