Sustainable Design of Impinging Premixed Slot Jets

T.T. Wong, C.W. Leung and M.C. Wong

Abstract—Cooktop burners are widely used nowadays. In cooktop burner design, nozzle efficiency and greenhouse gas(GHG) emissions mainly depend on heat transfer from the premixed flame to the impinging surface. This is a complicated issue depending on the individual and combined effects of various input combustion variables. Optimal operating conditions for sustainable burner design were rarely addressed, especially in the case of multiple slot-jet burners. Through evaluating the optimal combination of combustion conditions for a premixed slot-jet array, this paper develops a practical approach for the sustainable design of gas cooktop burners. Efficiency, CO and NOx emissions in respect of an array of slot jets using premixed flames were analysed. Response surface experimental design were applied to three controllable factors of the combustion process, viz. Reynolds number, equivalence ratio and jet-to-vessel distance. Desirability Function Approach(DFA) is the analytic technique used for the simultaneous optimization of the efficiency and emission responses.

Keywords—optimization, premixed slot jets

I. INTRODUCTION

GAS cooktop burners are widely used in households for cooking and water heating in metropolitan cities like Hong Kong. In a recent estimation, more than three millions cooktop burners are used in Hong Kong. There are two major concerns in designing the gas cooktop burners: energy utilization and indoor air pollution, the present study attempts to address these two issues.

Because of the large number of gas cooktop burners involved in our daily life, even a slight improvement in their thermal performance would result in a significant impact on the total energy consumption and the environment. Recently the multislot jet design for cooktop burners is becoming popular, however, research on evaluating the effects of jet array configuration on gas cooktop thermal performance and gaseous emissions have rarely been reported. Literature on round jets shows that some of the responses are related to air/fuel mixture properties, e.g. fuel type, equivalence ratio and Reynolds number of the air/fuel mixture[1]. Others are related to configuration of the burner system: shape and size of the

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jets, jet-to-jet spacing, orientation of the jet, and the jet-to-plate distance[2].

Moreover, some of these parameters are interrelated, for example, equivalence ratio of the air/fuel supply is highly affected by the system configuration[3]. Hence an investigation on the thermal performance of an array of slot jets, with the aim of (i)determining the combined effects of selected design parameters and (ii)developing a sustainable design approach for gas cooktop burners, was carried out.

A row of three rectangular jets is chosen for the study because it provides the most basic configurations commonly found in domestic and commercial cooktop gas-fired burners. The configuration of a central jet with two side jets is similar to the situation of any jet in a jet array - flame-flame coupling(interaction) for the centre jet will enhance the thermal efficiency and flame stability but the flame at end slots will have larger heat loss and shorter residence time, leading to incomplete combustion. The combustion efficiency and GHG emissions of a jet array depend on several geometrical and flow parameters. In this study the jet-array geometrical parameter chosen is the jet-to-vessel distance(H) and the flow parameters chosen are Reynolds number(Re) and equivalence ratio(φ) of the air/fuel jets. The aspect ratio(ratio of width to length) of the slot jets is 1. Based on previous work on circular jets[4], the central composite design(CCD) was used to investigate the effects of the three input factors on each of the three response variables.

According to Padre[5], it is difficult to optimize simultaneously responses in complex process by single-response method and engineering judgment is primarily used to resolve such complicated problems. An engineer's judgment often increases the degree of uncertainty during decision making process, making it most critical to the quality of cooktop design. The approach adopted by Taguchi practitioners to tackle multiple response optimization problems by employing engineering knowledge together with their experience introduces some degree of uncertainty and, therefore, the validity and robustness of results cannot be guaranteed using this approach alone.

The use of fuzzy attribute decision-making assisted in taking into account multiple decision goals and constraints simultaneously. However, the methodology requires rather complicated computation and involves difficulty in implementation of expert's knowledge into the formulae. Antony[6] proposed the application of multivariate statistical methods for determining the optimal condition in industrial experiments with multiple responses. Hsu[7] presented an integrated optimization approach based on neural networks, exponential desirability functions and Tabu search to optimize a fused biconic taper process for a

Taiwanese fiber-optic passive component manufacturer. Sardinas et al.[8] proposed a multi-objective optimization of the drilling process of a laminate composite material. A micro-genetic algorithm was implemented to carry out the optimization process. Singh and Kumar [9] carried out multi-response optimization for turning EN-24 steel using TiC coated carbide inserts through Taguchi's approach and utility concept. This paper, based on three combustion variables, is aimed to determine a preliminary set of operating parameters for designing a burner with the best overall response.

Face-centered Central composite design(CCD) was applied to the three controllable factors concerned – each factor at three levels. Single-response and multi-response optimization were carried out through desirability function approach. Single-response optimization means determining the values of controllable factors that gave the most desirable value of a particular response, e.g. highest value for thermal efficiency and lowest value for CO and NOx emissions. On the other hand, multi-response optimization means determining the values of controllable factors that gave the desirable value of all responses in combination (thermal efficiency, CO and NOx emissions in the present study). These responses can be given equal weightage or the weightage of responses can be varied according to industry requirements. The desirability concept has been used for multi-response optimization owing to its better readability, acceptability and visualization as compared to other multi-characteristic optimization techniques like utility concept, principal component analysis, etc.

II. DESIRABILITY FUNCTION

Derringer and Suich[10] suggest a multiple response method called desirability. The method makes use of an objective function, di, called the desirability function and transforms a measured response to a "0 to 1" scale. Regardless of the shape, a response of 0 represents a completely undesirable response and 1 represents the most desirable response. Creating these desirability functions requires some prior knowledge from the analyst since the shape can be extremely flexible. In order to simultaneously optimize several responses, each of these di's are combined using the geometric mean to create the overall desirability (D)

$$D = (d_1 \times d_2 \times d_3 \times \ldots \times d_n)^{1/n} \qquad (1)$$

where n is the number of responses in the measure. If any of the responses falls outside the desirability range, the overall function becomes zero. Using the product of the desirability functions ensures that if any single desirability is 0 (undesirable), the overall desirability is 0. Thus, the simultaneous optimization of several responses has been reduced to optimizing a single response: the overall desirability, D.

For simultaneous optimization, each response must have a low and high value assigned to each goal. In most design software the "Goal" field for responses can be one of following choices: "none", "maximum", "minimum", or "in

range". Factors will always be included in the optimization at their design range by default, or as a maximum, minimum of target goal.

III. EXPERIMENTAL DESIGN

A three-factor, three level, central composite design(CCD) was employed to investigate the relationships of the design parameters of cooktop burner. The actual CCD parameter values are shown in Table I.

TABLE I
FACE-CENTERED CENTRAL COMPOSITE DESIGN

CCD(Actual values)			
Exp.No.	Eq	Re	H/d_e
1	1.00	1500.00	4.00
2	1.20	1200.00	2.00
3	1.20	1800.00	2.00
4	1.20	1200.00	6.00
5	1.20	1800.00	6.00
6	1.50	995.46	4.00
7	1.50	2004.54	4.00
8	1.50	1500.00	0.64
9	1.50	1500.00	7.36
10	1.50	1500.00	4.00
11	1.50	1500.00	4.00
12	1.50	1500.00	4.00
13	1.50	1500.00	4.00
14	1.50	1500.00	4.00
15	1.50	1500.00	4.00
16	1.80	1200.00	2.00
17	1.80	1800.00	2.00
18	1.80	1200.00	6.00
19	1.80	1800.00	6.00
20	2.00	1500.00	4.00

A tailor-made household cooktop burner with a ring of 128 mm diameter with three rectangular jets (jet area = 100 mm^2 and equivalent diameter, d_e = 11.3mm) were used in the experiment. The jet-to-jet spacing is 45.2mm (corresponding to $4d_e$). The aspect ratio of the jets was 1:3 and each jet was parallel to each other as shown in Fig.1. Essentially fuel-rich flames corresponding to equivalence ratios between 1 and 2 (corresponding to coded φ values of -1.68 and +1.68) were used. The range of Reynolds number considered varies from 995 to 2005 (corresponding to coded values of -1.68 and +1.68). The jet-to-vessel distance varies from 0.64 to 7.36 (corresponding to coded H/d_e values varying from -1.68 to +1.68).

The experimental setup comprises a heat generation system and a heat absorption system, as shown schematically in Fig. 2. In the heat generation system, the burner is made of brass. Liquefied petroleum gas (LPG) was used in the experiment. The composition of the LPG was 70% butane (C_4H_{10}) and 30% propane (C_3H_8) by vol., with a low heating value of 117 MJ/m^3.

Metered compressed air and LPG were mixed in a stainless steel premixed chamber and then delivered to the burner via a 2 500 mm long high-pressure pipe with an

inner diameter 10 mm. The air/LPG mixture was then supplied to the burner and ignited at the rim.

$-0.02*Eq*Re +3.15*Eq*(H/d) – 0.0005*Re*(H/d)$
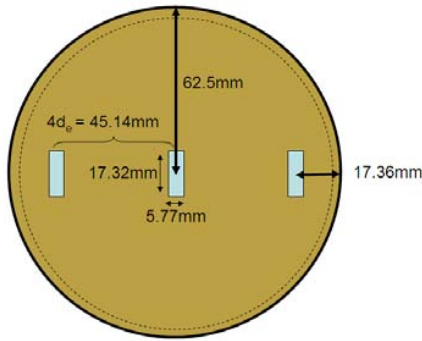$+4.46*Eq^2 –0.10*(H/d)^2$

$$(3)$$

Fig. 1 Array of three parallel slot jets

For the heat absorption system, a loading vessel with an outer diameter 250 mm was fixed on the metallic frame. Four standard K-type thermocouples spaced uniformly were used to provide average readings for the water temperature. These thermocouples were installed at the mid-point of the water depth and distributed in the loading vessel approximately 90 degrees apart. Heat loss due to the mixing process was not considered. In view of the insignificant CO_2 emissions generated, only CO and NOx emissions were studied, and they were absorbed by a sampling ring located vertically and 15 mm away from the outer surface of the loading vessel. Their intensity was measured by a Gas Analyzer (Anapol EU-2000).

In this study the thermal efficiency of burner is defined as the percentage of the thermal input transferred to the water in the loading vessel. It was determined by measuring the elapsed time for a standard 4 kg load of water to be heated from 30℃ to 80℃ and the corresponding consumption of LPG. Mathematically, the thermal efficiency is defined as:

$$\eta = \frac{M \times C_p \times \Delta T}{Q \times H_v} \times 100 \ \% \qquad (2)$$

where, M (kg) is the load mass of water, C_p(kJ/kg℃) is the specific heat of water, Q(m^3) is the LPG consumption, ΔT(℃) is the temperature rise in degree Celsius and H_v(kJ/m^3) denotes the heating value of LPG.

IV. EXPERIMENTAL RESULTS

Based on the experimental results of the jet array configuration shown, analyses of variance(ANOVA) were used to generate the regression models. Stepwise method was used to remove the insignificant terms. Presence of second-order terms in each of the models shown in Eq.(3), (4) and (5) indicate that second-order models would be required to estimate the burner responses.

A. Burner Efficiency(η) Model

$\eta = 36.64 + 1.24*Eq + 0.03*Re + 2.03*(H/d)$

The Adjusted R-Squared coefficient, i.e. Adjusted Coefficient of Determination for this multiple regression model was found to be 0.9717.

B.CO and NOx Emission Models

The CO and NOx emission models were found to be:

CO = 2227.90071+107.45* Eq – 3.86*Re
–219.37*H/D+3.72*Eq*Re–2125.87(H/d)*Eq*(H/d)
$–0.37216*Re*(H/d)+2049.97720*Eq^2+0.00017*Re^2$
$+378.52*(H/d)^2$

$$(4)$$

NO_x = 100.69 -17.05*Eq +4.79*Re + 22.41*(H/d)
+22.41*Eq*Re+0.98*Eq*(H/d)– 18.83*Re*(H/d)
$–9.41*Eq^2 – 1.06*Re^2-12.26*(H/d)^2$

$$(5)$$

Adjusted R-Squared coefficients in the case of CO and NO_x emissions are 0.8293 and 0.8526 respectively. For illustration purpose, the ANOVA result for the NOx response surface quadratic model is shown in Table II. The Model F-value of 13.21 implies the model is significant. There is only a 0.02% chance that a "Model F-Value" of this magnitude could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case Eq, H/d, Re*H/d, Eq^2, $(H/d)^2$ are significant NOx model terms. Values greater than 0.1000 indicate the model terms are not significant. Similarly the significance of the efficiency and CO models are reflected by the model F-values of 17.56 and 11.30 respectively(not shown). Furthermore, H/d, Eq*H/d and $(H/d)^2$ are signficant efficiency model terms, whereas Eq, H/d, Eq*H/d and $(H/d)^2$ are significant CO model terms.

"Adeq Precision" measures the signal to noise ratio. It compares the range of the predicted values at the design points to the average prediction error. Ratios greater than 4 indicate adequate model discrimination. The indicated ratio of 14.357 in the case of NOx indicates an adequate signal. This model can be used to navigate the design space. "Adeq Precision" for the slot-jet efficiency and CO quadratic models were found to be 11.691 and 20.702 respectively. Hence both models can be used to navigate the design space. As the adjusted R-squared for the efficiency, CO and NOx emission responses are 0.9717, 0.8293 and 0.8526 respectively, it can be implied that these measures of the amount of variation around the mean explained by the model, adjusted for the number of terms in the model, are acceptable.

V. SIMULTANEOUS OPTIMIZATION OF RESPONSES

For a practical approach to substainable burner design, the simultaneous optimization technique derived by Derrringer and Suich[10] is proposed. Their approach makes use of desirability functions. The procedure is to

first convert each response y_i into an individual desirability function d_i that varies over the range $0 \leq d_i \leq 1$, where if the response y_i is at its goal or target, then $d_i=1$, and if the response is outside an acceptable region, $d_i=0$. Then the design variables are chosen to maximize the overall desirability

$$D=(d_1*d_2*d_3)^{1/3} \qquad (6)$$

Using a commercially available statistical software, the optimization results are shown in Table III. Tt can be seen that the highest overall desirability is 0.795. Note that it results in on-target Eq and Re, and acceptable H/d. The corresponding responses for efficiency, CO and NOx are 64%, 128ppm and 68ppm respectively.

VI. CONCLUSION

Based on the ANOVA results, it may be concluded that in modeling cooktop burner thermal performance and CO and NOx emissions, the Reynolds number, the equivalence ratio and the jet-to-vessel distance, their squares, and most combined effects would need to be considered.

It has been proved through this study that by the adoption of non-linear regression modeling, ANOVA and the Desirability Function Approach, one can optimize various burner design parameters simultaneously for the purpose of achieving a substainable cooktop design.

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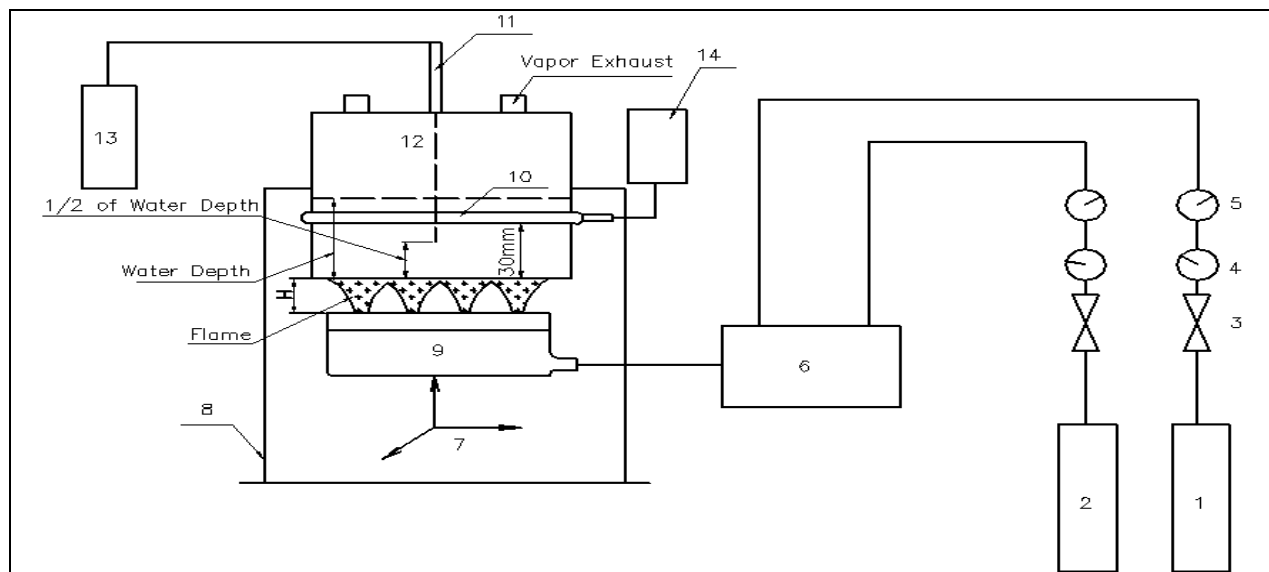
1. Compressed air supply   2. Fuel tank   3. Shut-off valve   4. Pressure gauge   5. Flow meter
6. Premixed Chamber   7. 3-D Positioner   8. Metallic frame   9. Burner   10. Sampling tube
11. Thermocouple   12. Loading vessel   13. Thermometer   14. Pollutant analyzer

Fig. 2 Experimental set-up

TABLE II
ANOVA FOR THE NO_x RESPONSE SURFACE QUADRATIC MODEL

Source	Sum of Square	df	Mean Square	F Value	p-value Prob>F	Implication
Model	17202.41	9	1911.38	13.21	0.0002	significant
A-Eq	3970.10	1	3970.10	27.44	0.0004	
B-Re	313.09	1	313.09	2.16	0.1720	
C-H/D	6858.74	1	6858.74	47.41	< 0.0001	
AB	61.36	1	61.36	0.42	0.5296	
AC	7.67	1	7.67	0.053	0.8225	
BC	2835.06	1	2835.06	19.60	0.0013	
A^2	1276.67	1	1276.67	8.82	0.0140	
B^2	16.19	1	16.19	0.11	0.7449	
C^2	2167.21	1	2167.21	14.98	0.0031	
Std Dev.	12.03				R-Squared	0.9224
Mean	85.16				Adj R-Squared	0.8526
C.V. %	14.12				Adeq Precision	14.357

TABLE III
SIMULTANEOUS OPTIMIZATION OF EFFICIENCY, CO, AND NO_x EMISSIONS

Constraints

Name Goal	Lower Limit	Upper Limit	Lower Weight	Upper Weight	Importance
Eq is in range	1.2	1.8	1	1	3
Re is in range	1200	1800	1	1	3
H is in range	2de	6de	1	1	3
Efficiency(%) maximize	40.768	66.2164	1	1	3
CO(ppm) minimize	67.2433	186.273	1	1	3
NOx(ppm) minimize	35.3529	102.502	1	1	3

Solutions

Number	Eq	Re	H/D	Efficiency(%)	CO(ppm)	NOx(ppm)	Desirability	
1	1.80	1200.00	3.25de	63.7083	127.88	68.0081	0.795	**Selected**