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Design of the Mathematical Model of the Respiratory System Using Electro-acoustic Analogy

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Abstract**—**The article deals with development, design and implementation of a mathematical model of the human respiratory system. The model is designed in order to simulate distribution of important intrapulmonary parameters along the bronchial tree such as pressure amplitude, tidal volume and effect of regional mechanical lung properties upon the efficiency of various ventilatory techniques. Therefore exact agreement of the model structure with the lung anatomical structure is required. The model is based on the lung morphology and electro-acoustic analogy is used to design the model.

Keywords**—**Model of the respiratory system, total lung impedance, intrapulmonary parameters.

I. INTRODUCTION

ARTIFICIAL lung ventilation is the most efficient method for treatment of acute respiratory failure. Despite the fact that artificial lung ventilation has been examined properly and new protective ventilatory modes have been introduced, there are still strong adverse effects of artificial ventilation upon patient's respiratory system. A quite new ventilatory strategy is called high frequency ventilation (HFV). HFV can be characterized by increased ventilatory frequency (up to 40 Hz) allowing a significant decrease in pressure amplitude and delivered tidal volume. Usage of the small pressure amplitudes in the airways and breathing with very low tidal volumes prevent the lungs from overdistension, barotrauma and volutrauma. These properties represent the most significant difference between HFV and conventional artificial lung ventilation (CV) and they identify unconventional ventilatory strategies.

Different effects of artificial ventilation can be observed when conventional ventilation or high frequency ventilation is used. Unpredictable differences mainly in oxygenation that have not been explained yet can be observed in clinical practice. Many parameters can influence the oxygenation, but their effect is mostly impossible to study directly in the human body. Therefore, derivation of a mathematical model of the respiratory system exactly corresponding with the reality can be the only possibility how to study influence of mechanical lung properties through the bronchial tree, distribution of tidal volume among generations of alveoli, etc. A unique modelling approach has been chosen in this study based on the respiratory system modelling according to its exact anatomical structure. Simulations using the model are carried out to describe unequal effects of both the ventilation modes upon various parameters characterizing intrapulmonary conditions.

II. METHODS

An electro-acoustic analogy [1] was used to develop the mathematical model of the respiratory system respecting its exact anatomical structure.

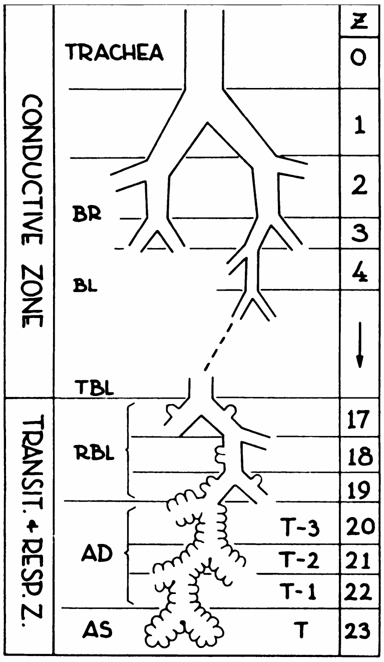


Fig. 1 Morphological model of the respiratory system. Reprinted from [3]

Anatomical structure of the respiratory system has a complex structure. The structure is difficult to be described mathematically because it has a lot of irregularities. Bronchial tree begins with trachea and divides with irregular dichotomy. It means that airways in the same generation of the bronchial tree have different geometrical proportions. The bronchial tree branches with each new generation of the airways. The parent airway divides into the two daughter airways generally, but it branches into the three daughter branches rarely. Therefore, a few morphological models have been introduced to describe the anatomical structure of the respiratory system mathematically [2, 3, 4]. These models neglect the irregularities in the bronchial tree. The airways in the same generation of the bronchial tree have uniform geometrical dimensions in the model. Morphological models are designed to represent the respiratory system including its anatomical structure.

The Weibel's model is used in this study to design the model of the respiratory system. The scheme of the model can be seen in Fig. 1. The geometrical dimensions of the elementary airways can be found in literature for adult and neonatal lung [2, 3, 4].

The respiratory system is considered as an acoustic system. All individual airways can be represented by short acoustic wave-guides. The wave-guides can be described by acoustic elements that are computed from its geometrical proportions using common acoustic principles.

Acoustic inertance m_a, acoustic compliance c_a and acoustic resistance r_a can be computed according to the functions (1), (2) a (3) [1]:

$$c_a = \frac{V}{\rho_0 {c_0}^2}, \tag{1}$$

$$m_a = \frac{\rho_0 l}{S}, \tag{2}$$

$$r_a = \frac{8\mu l}{\pi {R_t}^4}, \tag{3}$$

where ρ_0 stands for air density, l stands for length, S stands for cross-sectional area, V stands for volume, c_0 stands for propagation velocity, μ stands for air viscosity and R_t stands for diameter of the tube.

The electrical elements correspond to acoustic elements by mean of electro-acoustic analogy. The principle of electro-acoustic analogy is shown in Fig. 2.

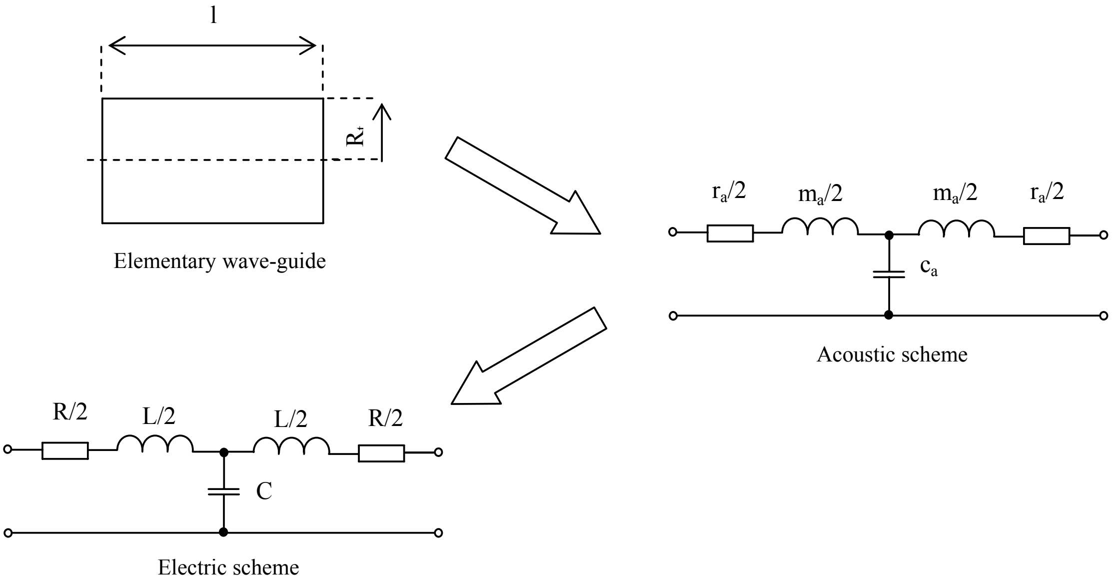
Electro-acoustic analogy can be used for rigid systems only. Therefore, airways are considered as rigid wave-guides and its electric parameters are computed by the mean of electro-acoustic analogy. It is supposed that main part of lung compliance is represented by alveolar space. The compliance of the airways is neglected.

Alveoli are compliant and electro-acoustic analogy cannot be used to model alveolar space. Electric capacity that represents alveoli is derived from the total lung compliance and distribution of alveoli along the bronchial tree.

The complete electric scheme of the respiratory system is shown in Fig. 3.

The model has a similar structure as a respiratory system and branches regularly.

Ventilatory frequency of 0.25 Hz is considered for conventional ventilation and 5 Hz for high-frequency ventilation. A special method has been developed so that such a complicated model could be used for simulations of the real situations. Distribution of tidal volume VT and pressure amplitude among generations of bronchial tree, total lung impedance (TLI) and other variables are studied under various conditions by modelling. The influence of respiratory mechanics upon the TLI was studied for frequencies that correspond with the ventilatory frequencies used during CV and HFV.



Elementary wave-guide

Acoustic scheme

Electric scheme

Fig. 2 The use of common acoustic principles and electro-acoustic analogy

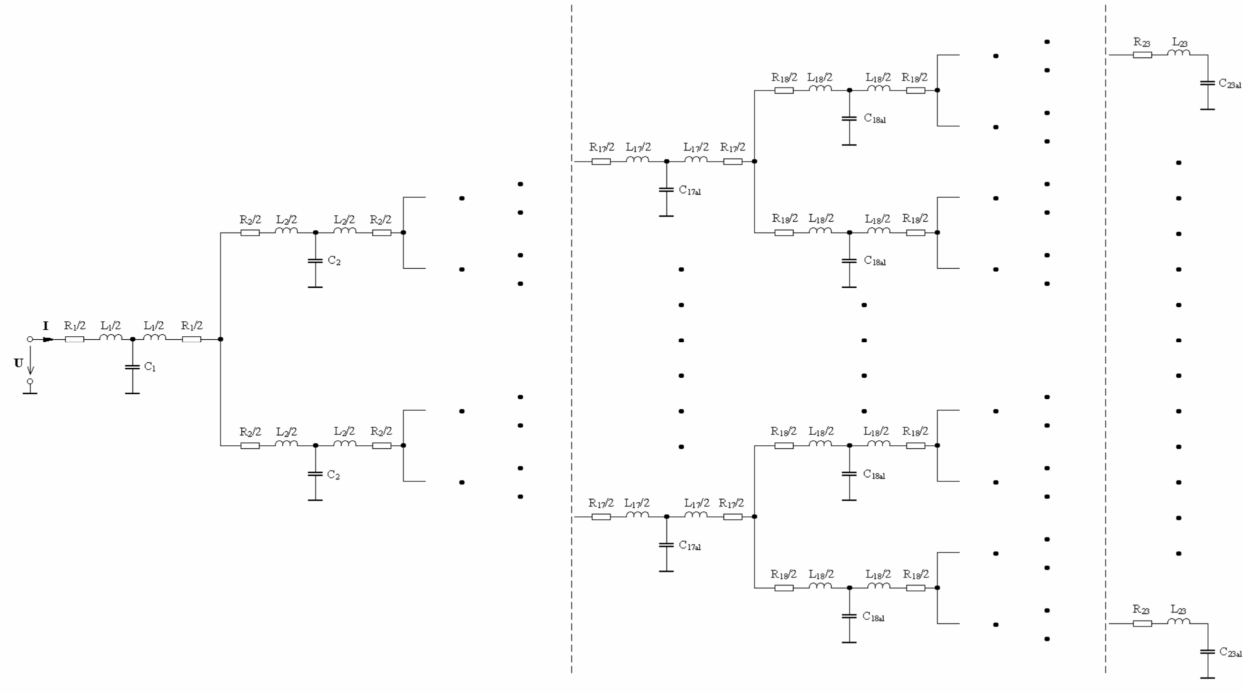


Fig. 3 Mathematical model of the respiratory system

III. RESULTS

The model can be designed for both adult and neonatal respiratory system. The simulations presented in the article were conducted for the adult man lung.

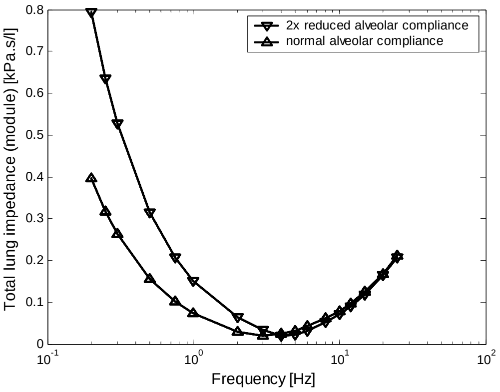


Fig. 4 Dependence of TLI upon frequency for normal and reduced alveolar compliance

The effect of the decreased alveolar compliance can be seen in Fig. 4. Changes of alveolar compliance have a significant effect on TLI during CV while TLI changes during HFV are not essential (due to the effect of airway inertances). There is only small shift in resonant frequency.

Contribution of airway resistance changes is significant mainly during HFV (Fig. 5) whereas the TLI is almost without change during CV. TLI is an essential variable for the pressure controlled ventilation modes. Results of simulations describe and explain some clinical experiences.

The results of conducted simulations suggest that HFV can be more efficient method of artificial lung ventilation during decreased alveolar compliance, whereas CV seems to be more efficient during increased airway resistrance.

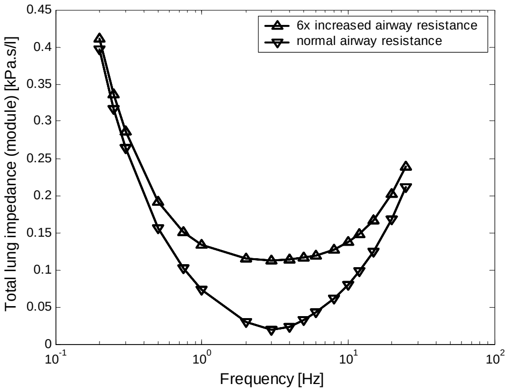


Fig. 5 Dependence of TLI upon frequency for normal and increased airway resistance

The effect of ventilatory frequency (CV, HFV) upon the pressure inside lung structure is shown in Fig. 6. Nearly 95 % of input pressure is present inside the lung structure if using CV. On the other hand only about 5 % of the input pressure amplitude is transferred into the distal parts of the respiratory system during HFV.

It suggests that HFV is protective ventilatory strategy contrary to CV. It means that the risk of barotrauma is minimized by small pressure amplitudes present in the distal parts of the respiratory system.

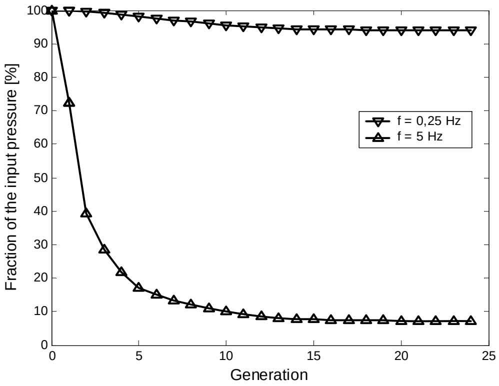


Fig. 6 The effect of frequency upon pressure inside the lung structure

IV. CONCLUSION

The model of the respiratory system that is designed according to anatomical structure of the respiratory system has been developed. It is possible to use the model to simulate the CV and HFV usage. Model allows study effects of different ventilatory regimens upon the intrapulmonary parameters. Also influence of the changes of mechanical properties upon the efficiency of ALV can be studied. The model allows study the intrapulmonary conditions during different mechanical parameters of the respiratory system. The mechanical properties of the lung tissue are changed during ARDS [5]. The change is dependent upon the origin of ARDS [6]. It is possible to use this model to observe the efficiency of the elementary ventilatory strategies during different mechanical properties of the respiratory system and choose the optimal ventilation therapy for different types of ARDS.

ACKNOWLEDGMENT

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