Constructing of Classifier for Face Recognition on the Basis of the Conjugation Indexes

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Abstract—In this work the opportunity of construction of the qualifiers for face-recognition systems based on conjugation criteria is investigated. The linkage between the bipartite conjugation, the conjugation with a subspace and the conjugation with the null-space is shown. The unified solving rule is investigated. It makes the decision on the rating of face to a class considering the linkage between conjugation values. The described recognition method can be successfully applied to the distributed systems of video control and video observation.

Keywords—Conjugation, Eigenfaces, Recognition.

I. INTRODUCTION

RECENTLY face recognition problem became one of the most popular ones [1]. The reasons of this fact are the practical requirements and the problem complexity, which makes it a good field for testing of new approaches and ideas.

At present there are two the most common methods of face recognition: the principal component analysis [2] and the method of comparing the sample and the pretender, that lies in calculating the coefficient of correlation between them. Usually the decision is taken per one realization. In this case correlation coefficient means conjugation of the vectors matched the comparing images.

In the papers [3],[4] the use of three types of conjugation indexes for separation of the learning sets into classes is considered. Ibidem the example, which illustrates efficiency of the approach for solving the problem of recognition of the graphic symbols on the images, is given.

In the face recognition systems the dimensions of the vectors that describe the faces and the relation between this dimension and number of learning samples can vary within wide limits. At the same time in the view of calculation it is practical to use different conjugation indices.

In this paper the relation between different conjugation indexes is considered more explicitly than in previous papers [3],[4]. The recommendations for using them for different

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dimensions of features vectors and learning sampling are given. The example of learning and classification of the faces from ORL base is given.

II. THE DEFINITION OF CONJUGATION INDICES AND THE RELATION BETWEEN THEM

In the papers [3],[4] three following types of conjugation indices are considered.

1. Bipartite conjugation index:

$$ r_{i,k}^2 = \frac{\left(\mathbf{x}_i^T \mathbf{x}_k \right)^2}{\left(\mathbf{x}_i^T \mathbf{x}_i \right) \left(\mathbf{x}_k^T \mathbf{x}_k \right)} , \tag{1} $$

where $\mathbf{x}_i$, $\mathbf{x}_k$ are $N \times 1$-vectors, their components are compared images definitions in features vectors.

2. Index of conjugation with a subspace stretched onto the vectors of features of objects belonging to the same class:

$$ R_{i,k} = \frac{\mathbf{x}_i^T \mathbf{X}_k \left[\mathbf{X}_k^T \mathbf{X}_k \right]^{-1} \mathbf{X}_k^T \mathbf{x}_i}{\mathbf{x}_i^T , \mathbf{x}_i} . \tag{2} $$

Here $\mathbf{x}_j$ is $N \times 1$-vector, describing an image produced for detection of belonging to some class (in this case it is k-th class), and $\mathbf{X}_k$ is $N \times M$-matrix, which consists of vectors of features of objects belonging to the k-th class.

3. Index of conjugation with the null-space of the same space:

$$ S_{i,k} = \frac{\mathbf{x}_i^T \mathbf{T}_{0,k} \mathbf{T}_{0,k}^T \mathbf{x}_i}{\mathbf{x}_i^T , \mathbf{x}_i} . \tag{3} $$

where $\mathbf{T}_{0,k}$ is matrix, which consists of N- M eigenvectors that conform to eigenvalues of $N \times M$-matrix $\mathbf{X}_k \mathbf{X}_k^T$, where $\mathbf{X}_k$ – $N \times M$-matrix, which consists of vectors of features of objects belonging to the k-th class.

It is easy to see that bipartite conjugation index (1) is a special case of index of conjugation with subspace of the vectors belonging to k-th class when this class consists of one vector $\mathbf{x}_k$, i.e. the dimension of $\mathbf{X}_k$ matrix is $N \times 1$.

There is also a link between classes $R_{i,k}$ and $S_{i,k}$:

$$ R_{i,k} = 1\text{-} \ S_{i,k}, \quad (S_{i,k} = 1\text{-} \ R_{i,k}) \tag{4} $$

Let us prove it using a following Lemma.

Lemma. Let $\mathbf{X}$ be $N \times M$-matrix: $N > M$ and $Rank(\mathbf{X}) = M$, and $\mathbf{x}$ is $N \times 1$-vector, then:

$$\mathbf{E}_N - \mathbf{X}_k \left[\mathbf{X}_k^T \mathbf{X}_k \right] = \mathbf{T}_0 \mathbf{T}_0^T \qquad (5)$$

where $\mathbf{E}_N$ - $N \times N$-unitary matrix, $\mathbf{T}_0$ is $N \times (N-M)$-matrix, which consists of eigenvectors that conform to eigenvalues of $N \times N$-matrix $\mathbf{XX}^T$.

Equation (5) can be proved using the properties $\mathbf{T}_0^T \mathbf{X} = 0$ $\left(\mathbf{X}^T \mathbf{T}_0 = 0 \right)$, $\mathbf{T}_0^T \mathbf{T}_0 = \mathbf{E}_{N-M}$ by multiplying the both parts by $\mathbf{T}_0^T$ at the left and by $\mathbf{T}_0$ at the right.

According to (2) we obtain:

$$1 - R_{i,k} = \frac{\mathbf{x}_i^T \mathbf{x}_i}{\mathbf{x}_i^T \mathbf{x}_i} - \frac{\mathbf{x}_i^T \mathbf{X}_k \left[\mathbf{X}_k^T \mathbf{X}_k \right]^{-1} \mathbf{X}_k^T \mathbf{x}_i}{\mathbf{x}_i^T \mathbf{x}_i} =$$
$$= \frac{\mathbf{x}_i^T \left[\mathbf{E}_N - \mathbf{X}_k \left[\mathbf{X}_k^T \mathbf{X}_k \right]^{-1} \mathbf{X}_k^T \right] \mathbf{x}_i}{\mathbf{x}_i^T \mathbf{x}_i} .$$

Subject to (5), from this expression follows: $1 - R_{i,k} = S_{i,k}$. It is evident that $R_{i,k} = 1 - S_{i,k}$.

Thus indices (1)-(3) are equal for prescribed i and k. To choose one of them for classifier constructing we follow the organization ease and the traditional request for reducing the number of calculations (of calculation process operations). That is why further we use "conjugation index" term without specifying the type of the index, when the realization peculiarities matter.

III. DECISION MAKING PRINCIPLES

The $N \times 1$-vector $\mathbf{x}_k$ or the $N \times M$-matrix $\mathbf{X}_k$ are formed for each object class (correspondingly for one or more images of the face in this case). Each $\mathbf{x}_k$ vector or column vector belonging to $\mathbf{X}_k$ goes with one preprocessed face image.

Image preprocessing consists of face localization (selection), brightness histogram normalization and adduction to the given scale (to dimension of a vector). It is not necessary to use all the pixels of the image as vector components; we can use the most informative ones, selected by any method.

Each class can be divided into subclasses as it is described in the paper [4]. Each class can be represented by several vectors $\mathbf{x}_{k,j}$ and/or by matrixes $\mathbf{X}_{k,j}$, where j is subclass number. In this case we carry out the merger of the results of testing of the belonging to each subclass (as it is described in the paper [2]) to take the decision about belonging to k-th class. For the simplicity, further we consider the test of belonging to a class, because in this case the test of belonging to class and the test of belonging to subclass are made according to the same principles.

Let us suppose that one of the following $\mathbf{Q}_{k,(*)}$ $N \times N$-matrixes is formed for each class according to relations (1)-(3) :

$$\mathbf{Q}_{k,R} = \mathbf{X}_k \left[\mathbf{X}_k^T \mathbf{X}_k \right]^{-1} \mathbf{X}_k^T , \qquad (6)$$

$$\mathbf{Q}_{k,S} = \mathbf{T}_{0,k} \mathbf{T}_{0,k}^T . \qquad (7)$$

Then the principle of making the decision on rating of $\mathbf{x}_i$ vector to m-th class can be formulated as:

$$R_m = \max_k R_k , \text{ where } R_k = \frac{\mathbf{x}_i \mathbf{Q}_{k,R} \mathbf{x}_i}{\left(\mathbf{x}_i^T \mathbf{x}_i \right)} \qquad (8)$$

or

$$S_m = \min_m S_k , \text{ where } S_k = \frac{\mathbf{x}_i \mathbf{Q}_{k,S} \mathbf{x}_i}{\left(\mathbf{x}_i^T \mathbf{x}_i \right)} . \qquad (9)$$

If the $r_{i,k}^2$ bipartite conjugation index is used we also can calculate the matrix

$$\mathbf{Q}_{k,R} = \frac{\left(\mathbf{x}_k \mathbf{x}_k^T \right)}{\left(\mathbf{x}_k^T \mathbf{x}_k \right)} ,$$

and use (8) principle. But in this case it is better to use relation (1) to save the memory and to reduce calculation expenses.

One of the $\mathbf{Q}_{k,*}$ $N \times N$-matrixes is to be calculated and memorized in advance for realization of described principles of taking the decision for each class using learning sampling of faces. As we can see from the relations (6) and (7), when the number of learning vectors M is comparatively small, it is better to use $\mathbf{Q}_{k,R}$ matrix. If the number M is so large that the null-space dimension $N - M$ is comparatively small (to N dimension), it is better to use $\mathbf{Q}_{k,S}$ matrix.

In the case of using a bipartite conjugation index (1) recognition probability is usually higher, if the index is calculated for each class or subclass. The decision about belonging vector $\mathbf{x}_i$ to k-th class is taken if "the closets" for the index (1) vector $\mathbf{x}_i$ also belongs to this class.

IV. EXPERIMENTS

The experimental research of the qualifiers constructed on the basis of the described above indices of conjugation was made with the use of standard database ORL. The given base contains images of 40 persons. There are 10 various photos with different mime (for each person). Thus, the database contains 400 images. Fig. 1 shows samples of these images (one image per person).

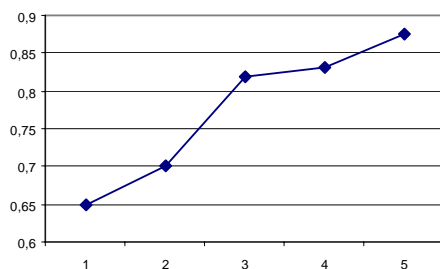Fig. 1 Several face images from ORL Database

The size of original images in ORL database is 112x96 pixels. However, to decrease the computing expenses images of smaller size (80x70 pixels) were used in experiments. Thus, each image was represented by a 5600x1-vector $\mathbf{x}$.

Matrixes $\mathbf{X}_k$ were made of various sets of these vectors for each class. Number of vectors in matrixes was varied from 1 to 5 to reveal the dependence of recognition probability on the number of the elements in a class. Fig. 2 shows an example of five images of one person (class) on which one of 5600x5-matrixes $\mathbf{X}_k$ was generated.
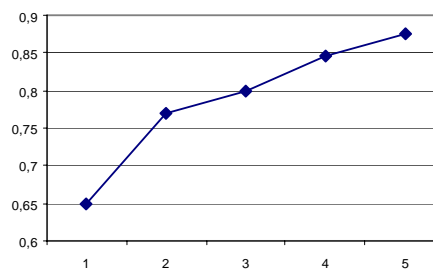

Fig. 2 Example of the images concerning one class

Matrixes $\mathbf{Q}_{k,*}$, which were going to be used in recognition, were formed for each class with use of matrixes $\mathbf{X}_k$ (Eq. (6), (7)). The decision on rating of some vector from control

Experiments have confirmed the almost identical efficiency of all criteria. Therefore the expediency of application of exact conjugation criterion in each case is defined by proportion between dimensions N and M. In particular, if the dimension N of attribute vectors is rather insignificant in comparison with the number of training examples M, i.e. the dimension of null-space is less than the dimension of column-space of matrix $\mathbf{X}_k$ it is better to use a matrix $\mathbf{Q}_{k,S}$ (7).

The described recognition method can be successfully applied to the distributed systems of video control and video observation. In this case the distributed database of persons which can replenish without conversion training system can be used. Service-oriented ideology, discussed in work [4], can be used in realization of such system.

a)

b)

Fig. 3 Dependence of recognition probability on the size of training set by using the following parameters:
a) bipartite conjugation; b) conjugation with a space stretched on the vectors from a class

sample to m-th class was made by rules (8), (9). 200 persons (5 images per person) were included to control set. It should be emphasized that these persons were not used in formation of matrixes $\mathbf{Q}_{k,*}$.

Fig. 3a and 3b show the dependence of recognition probability on a number of images in a class (vectors in a matrix $\mathbf{X}_k$), received in experiment for the qualifier constructed using criteria (1), (2). As one would expect, by increasing the number of the examples representing each class the recognition probability increases. Besides, the comparison of curves shows that recognition quality of the qualifiers is almost identical when the numbers of the learning samples representing different classes are about the same.

V. CONCLUSION

In this work the opportunity of construction of the qualifiers for face-recognition systems based on conjugation criteria is investigated. The linkage between the bipartite conjugation, the conjugation with a subspace and the conjugation with the null-space is shown. The unified solving rule is investigated. It makes the decision on the rating of face to a class considering the linkage between conjugation values.

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