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Adomian method for second-order fuzzy differential equation

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Abstract—In this paper, we study the numerical method for solving second-order fuzzy differential equations using Adomian method under strongly generalized differentiability. And, we present an example with initial condition having four different solutions to illustrate the efficiency of the proposed method under strongly generalized differentiability.

Keywords—Fuzzy-valued function, Fuzzy initial value problem, Strongly generalized differentiability, Adomian decomposition method

I. Introduction

The study of fuzzy differential equation (FDE) forms a suitable setting for mathematical modeling of real world problems in which uncertainties or vagueness pervade. The concept of a fuzzy derivative was defined by Chang and Zadeh in [13]. It was followed up by Dubois and Prade in [14], who used the extension principle. The term "fuzzy differential equation" was introduced in 1987 by Kandel and Byatt [21,22]. There have been many suggestions for the definition of fuzzy derivative to study FDE. The first and the most popular approach is using the Hukuhara differentiability for fuzzy-value functions. Under this setting, mainly the existence and uniqueness of the solution of a FDE are studied [13,20,25,28]. This approach has a drawback: the solution becomes fuzzier as time goes by. Hence, the fuzzy solution behaves quite differently from the crisp solution. Seikkala [29] introduced the notion of fuzzy derivative as an extension of the Hukuhara derivative and fuzzy integral, which was the same as what Dubois and Prade [14] proposed. Buckley and Feuring [11]gave a very general formulation of fuzzy first-order initial value problem.They firstly find the crisp solution, fuzzify it and then check to see if it satisfies the FDE. To alleviate the situation, Hllermeier [18] interpreted FDE as a family of differential inclusions. The main shortcoming of using differential inclusions is that we do not have a derivative of a fuzzy-valued function.The strongly generalized differentiability was introduced in [8] and studied in [9,10,12]. In [24] a generalized concept of higher-order differentiability for fuzzy-valued functions is presented to solve nth-order fuzzy differential equations.This concept allows us to resolve the above mentioned shortcoming. Indeed, the strongly generalized derivative is defined for a larger class of fuzzy-valued functions than the Hukuhara derivative. Hence, we use this differentiability concept in the present paper. Under appropriate conditions, the fuzzy initial value problem (FIVP) considered under this interpretation has locally two solutions [9]. Numerical solution of an FDE is obtained now in a natural way, by extending the existing classical methods to the fuzzy case [19]. Some numerical methods for FDE under the Hukuhara differentiability concept such as the fuzzy Euler method, predictor-corrector method, Taylor method and Nystr?m method are presented in [1,2,6,16,24]. In 1990, G. Adomian [4] introduced the Adomian decomposition method and applied it to solve nonlinear equations.Babolian [7] studied the first-order fuzzy initial value problem using this method. In this paper, we develop numerical methods for addressing second-order fuzzy differential equation by Adomian decomposition method using the strongly generalized derivative. The rest of the paper is organized as follows: Section 2 contains the basic material to be used in the rest of paper; In section 3 we define a second-order fuzzy differential equation under strongly generalized differentiability and in section 4 we discuss Adomian decomposition method. In section 5 the proposed method is illustrated by a numerical example and the conclusion and future research is drawn in the last section 6.

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II. Preliminaries

Let us denote by R_f the class of fuzzy subsets of the real axis $u : X \to [0,1]$ satisfying the following properties:

1) u is normal, i.e. $\exists x_0 \in X$ with $u(x_0) = 1$,

2) u is convex fuzzy set(i.e.$u(tx + (1-t)y) \geq \min\{u(x), u(y)\}, \forall t \in [0,1], x, y \in R$),

3) u is upper semicontinuous on R,

4) $\overline{\{x | x \in R, u(x) > 0\}}$ is compact, where $\bar{A}$ denotes the closure of A.

Then R_f is called the space of fuzzy numbers [15]. Obviously $R \subset R_f$. Here $R \subset R_f$ is understood as $R = \chi_{\{x\}} : x$ is usual real number. For $0 < \lambda \leq 1$ denote $[u]^\lambda = \{x \mid u(x) \geq \lambda, x \in R\}$ and $[u]^0 = \{x \mid u(x) > 0, x \in R\}$. Then it is well-known that for any $\lambda \in [0,1]$, $[u]^\lambda$ is a bounded closed interval. The notation $[u]^\lambda = [u_-^\lambda, u_+^\lambda]$ denotes explicitly the $\lambda-$level set of u. One refers to u_-^λ and u_+^λ as the lower and upper branches of u, respectively. The following defition shows when $[u_-^\lambda, u_+^\lambda]$ is a valid $\lambda-$level set.

Definition 2.1 ([29]) The sufficient conditions for $[u_-^\lambda, u_+^\lambda]$ to define the parametric form of a fuzzy number are as follows:

(a) u_-^λ is a bounded monotonic increasing (nondecreasing)left-continuous function on $[0,1]$ and right-continuous for $\lambda = 0$,

(b) u_+^λ is a bounded monotonic decreasing (nonincreasing)left-continuous function on $[0,1]$ and right-continuous for $\lambda = 0$,

(c) $u_-^\lambda \leq u_+^\lambda, \lambda \in [0,1]$.

For $u, v \in R_f$ and $k \in R$, the sum $u+v$ and the product are defined by ku are defined by $[u+v]^\lambda = [u]^\lambda + [v]^\lambda, [kv] = k[v]^\lambda$ for any $\lambda \in [0,1]$, where $[u]^\lambda + [v]^\lambda = \{x+y | x \in [u]^\lambda, y \in [v]^\lambda\}$ means the usual addition of two intervals (subsets) of R and $k[u]^\lambda = \{kx | x \in [u]^\lambda\}$ means the usual product between a scalar and a subset of R, let $D: R_f \times R_f \to R_+ \cup \{0\}$,$D(u,v) = \sup\limits_{\lambda \in [0,1]} max\{|u_-^\lambda - v_-^\lambda|, |u_+^\lambda - v_+^\lambda|\}$ be the Hausdorff distance between fuzzy numbers, where $[u]^\lambda = [u_-^\lambda, u_+^\lambda], [v]^\lambda = [v_-^\lambda, v_+^\lambda]$. The following properties are well-known [17].

(a) $D(u+\omega, v+\omega) = D(u,v), \quad \forall u, v, \omega \in R_f$,

(b) $D(ku, kv) = |k| D(u,v), \quad \forall k \in R, u, v \in R_f$,

(c) $D(u+v, \omega+e) D(u,\omega) + D(v,e), \forall u, v, \omega, e \in R_f$.

and (R_f, D) is a complete metric space.

Definition 2.2 ([26]). Let $x, y \in R_f$. If there exists $z \in R_f$ such that $x = y + z$, then z is called the H-difference of x and y,it is denoted by $z = x -_H y$.

In this paper the " $-_H$" sign stands always for H-difference and let us remark that $x -_H y \neq x + (-y)$. Let us recall the definition of strongly generalized differentiability introduced in [9,10].

Definition 2.3 Let $f : [a,b] \to R_f$ and $t \in [a,b]$. We say that f is strongly generalized differentiable at t, if there exists an element $f'(t) \in R_f$, such that

(1) for all $h > 0$ sufficiently small, $\exists f(t+h) -_H f(t), \exists f(t) -_H f(t-h)$ and the limits (in the metric D) $\lim\limits_{h \to 0} \frac{f(t+h) -_H f(t)}{h} = \lim\limits_{h \to 0} \frac{f(t) -_H f(t-h)}{h} = f'(t)$.

(2) for all $h > 0$ sufficiently small, $\exists f(t-h) -_H f(t), \exists f(t) -_H f(t+h)$ and the limits $\lim\limits_{h \to 0} \frac{f(t-h) -_H f(t)}{-h} = \lim\limits_{h \to 0} \frac{f(t) -_H f(t+h)}{-h} = f'(t)$.

For the sake of simplicity, we say that the fuzzy-valued function f is (1)-differentiable if it satisfies in the Definition 2.3 case (1) and we denote its first derivatives by $D_1^{(1)} f(t)$, we say fuzzy-valued function f is (2)-differentiable if it satisfies in the Definition 2.3 case (2) and we denote its first derivatives by $D_2^{(1)} f(t)$.

Remark 2.1 In the Definition 2.3, (1)-differentiability corresponds to the H-derivative introduced in [29], so this differentiability concept is a generalization of the H-derivative and obviously more general.

In the special case when f is a fuzzy-valued function, we have the following result.

Theorem 2.1([12]) Let $f : [a,b] \to R_f$ be a fuzzy-valued function and denote $[f(t)]^\lambda = [f_-^\lambda(t), f_+^\lambda(t)]$, for $\lambda \in [0,1]$.Then

(1) If f is (1)-differentiable, then $f_-^\lambda(t)$ and $f_+^\lambda(t)$ are differentiable functions and $[D_1^{(1)} f(t)]^\lambda = [(f_-^\lambda(t))', (f_+^\lambda(t))']$.

(2) If f is (2)-differentiable, then $f_-^\lambda(t)$ and $f_+^\lambda(t)$ are differentiable functions and $[D_2^{(1)} f(t)]^\lambda = [(f_+^\lambda(t))', (f_-^\lambda(t))']$.

Now we introduce definitions for second-order derivatives based on the selection of derivative type in each step of differentiation.

For a given fuzzy-valued function f, we have two possibilities (Definition 2.3) to obtain the derivative of f at $t : D_1^{(1)} f(t)$ and $D_2^{(1)} f(t)$. Then for each of these two derivatives, we have again two possibilities $D_1^{(1)}(D_1^{(1)} f(t)), D_2^{(1)}(D_1^{(1)} f(t))$, $D_1^{(1)}(D_2^{(1)} f(t)), D_2^{(1)}(D_2^{(1)} f(t))$ respectively, so the second-order derivative of f has four cases.

Definition 2.4 Let $f : [a,b] \to R_f$ and $t \in [a,b]$. We say that f is second-order strongly generalized differentiable at t, if there exists an element $f''(t) \in R_f$, such that

(i)for all $h > 0$ sufficiently small, $\exists D_1^{(1)} f(t+h) -_H D_1^{(1)} f(t), \exists D_1^{(1)} f(t) -_H D_1^{(1)} f(t-h)$, and the limits (in the metric D)$\lim\limits_{h \to 0} \frac{D_1^{(1)} f(t+h) -_H D_1^{(1)} f(t)}{h} = \lim\limits_{h \to 0} \frac{D_1^{(1)} f(t) -_H D_1^{(1)} f(t-h)}{h} = f''(t)$,

(ii) for all $h > 0$ sufficiently small, $\exists D_1^{(1)} f(t-h) -_H D_1^{(1)} f(t), \exists D_1^{(1)} f(t) -_H D_1^{(1)} f(t+h)$, and the limits $\lim\limits_{h \to 0} \frac{D_1^{(1)} f(t) -_H D_1^{(1)} f(t+h)}{-h} = \lim\limits_{h \to 0} \frac{D_1^{(1)} f(t-h) -_H D_1^{(1)} f(t)}{-h} = f''(t)$,

(iii) for all $h > 0$ sufficiently small, $\exists D_2^{(1)} f(t+h) -_H D_2^{(1)} f(t), \exists D_2^{(1)} f(t) -_H D_2^{(1)} f(t-h)$, and the limits $\lim\limits_{h \to 0} \frac{D_2^{(1)} f(t+h) -_H D_2^{(1)} f(t)}{h} = \lim\limits_{h \to 0} \frac{D_2^{(1)} f(t) -_H D_2^{(1)} f(t-h)}{h} = f''(t)$,

(iv) for all $h > 0$ sufficiently small, $\exists D_2^{(1)} f(t-h) -_H D_2^{(1)} f(t), \exists D_2^{(1)} f(t) -_H D_2^{(1)} f(t+h)$, and the limits $\lim\limits_{h \to 0} \frac{D_2^{(1)} f(t) -_H D_2^{(1)} f(t+h)}{-h} = \lim\limits_{h \to 0} \frac{D_2^{(1)} f(t-h) -_H D_2^{(1)} f(t)}{-h} = f''(t)$.

Remark 2.2 Let $f : [a,b] \to R_f$ and $n, m = 1,2$, if $D_n^{(1)}$ exists on a neighborhood of t as a fuzzy-valued function and it is (m)-differentiable at t, the second derivatives of f are denoted by $D_{n,m}^{(2)}, n, m = 1,2$. One says f is (n,m)-differentiable at t.

Theorem 2.2 Let $D_i^{(1)} f(t) : [a,b] \to R_f, i = 1,2.$ be fuzzy-valued function where $[f(t)]^\lambda = [f_-^\lambda(t), f_+^\lambda(t)]$, then

(I) If $D_1^{(1)} f(t)$ is (1)-differentiable, then $(f_-^\lambda(t))'$ and $(f_+^\lambda(t))'$ are differentiable functions and $[D_{1,1}^{(2)} f(t)]^\lambda = [(f_-^\lambda(t))'', (f_+^\lambda(t))'']$,

(II) If $D_1^{(1)} f(t)$ is (2)-differentiable, then $(f_-^\lambda(t))'$ and $(f_+^\lambda(t))'$ are differentiable functions and $[D_{2,1}^{(2)} f(t)]^\lambda = [(f_+^\lambda(t))'', (f_-^\lambda(t))'']$,

(III) If $D_2^{(1)} f(t)$ is (1)-differentiable, then $(f_-^\lambda(t))'$ and $(f_+^\lambda(t))'$ are differentiable functions and $[D_{1,2}^{(2)} f(t)]^\lambda = [(f_+^\lambda(t))'', (f_-^\lambda(t))'']$,

(IV) If $D_2^{(1)} f(t)$ is (2)-differentiable, then $(f_-^\lambda(t))'$ and $(f_+^\lambda(t))'$ are differentiable functions and $[D_{2,2}^{(2)} f(t)]^\lambda = [(f_-^\lambda(t))'', (f_+^\lambda(t))'']$.

Proof. We present the details only for the case (I), since the other cases are analogous. Since $D_1^{(1)} f(t)$ is (1)-differentiable, for $\lambda \in [0,1]$, from the case (1) of Theorem 2.1, we have $[D_1^{(1)} f(t)]^\lambda = [(f_-^\lambda(t))', (f_+^\lambda(t))']$, Similarly, from the case (2) of Theorem 2.1, we obtain $[D_{1,1}^{(2)} f(t)]^\lambda = [(f_-^\lambda(t))'', (f_+^\lambda(t))'']$, this completes the proof of the theorem.

III. SECOND-ORDER FUZZY DIFFERENTIAL EQUATION

In this section, we study the fuzzy initial value problem for a second-order linear fuzzy differential equation:

$$\begin{cases} x''(t)+a(t)x'(t)+b(t)x(t)=\omega(t), \\ x(0)=c_1, \\ x'(0)=c_2. \end{cases} \tag{1}$$

where,$c_1,c_2\in R_f, a(t),b(t),\omega(t)\in R$. In this paper, we suppose $a(t),b(t)>0$. Our strategy of solving (1) is based on the selection of derivative type in the fuzzy differential equation. We first give the following definition for the solutions of (1).

Definition 3.1 let $x:[a,b]\to R_f$ be a fuzzy-valued function and $n,m=1,2$. One says x is an (n,m)-solution for problem (1). if $D_n^{(1)}x(t), D_{n,m}^{(2)}x(t)$ exist and $D_{n,m}^{(2)}x(t)+a(t)D_n^{(1)}x(t)+b(t)x(t)=\omega(t), x(0)=c_1, D_n^{(1)}x(0)=c_2$.

Let x be an (n,m)-solution for (1). To find it, utilizing Theorems 2.1 and 2.2 and considering the initial values, we can translate (1) to a system of second-order linear ordinary differential equations hereafter, called corresponding (n,m)-system for (1).

Therefore, four ODEs systems are possible for (1), as follows:

(1,1)-system

$$\begin{cases} x_-''(t;\lambda)+a(t)x_-'(t;\lambda)+b(t)x_-(t;\lambda)=\omega(t), \\ x_+''(t;\lambda)+a(t)x_+'(t;\lambda)+b(t)x_+(t;\lambda)=\omega(t), \\ x_-(0;\lambda)=c_{1-}^{\lambda}, x_+(0;\lambda)=c_{1+}^{\lambda}, \\ x_-'(0;\lambda)=c_{2-}^{\lambda}, x_+'(0;\lambda)=c_{2+}^{\lambda}. \end{cases} \tag{2}$$

(1,2)-system

$$\begin{cases} x_+''(t;\lambda)+a(t)x_-'(t;\lambda)+b(t)x_-(t;\lambda)=\omega(t), \\ x_-''(t;\lambda)+a(t)x_+'(t;\lambda)+b(t)x_+(t;\lambda)=\omega(t), \\ x_-(0;\lambda)=c_{1-}^{\lambda}, x_+(0;\lambda)=c_{1+}^{\lambda}, \\ x_-'(0;\lambda)=c_{2-}^{\lambda}, x_+'(0;\lambda)=c_{2+}^{\lambda}. \end{cases} \tag{3}$$

(2,1)-system

$$\begin{cases} x_+''(t;\lambda)+a(t)x_+'(t;\lambda)+b(t)x_-(t;\lambda)=\omega(t). \\ x_-''(t;\lambda)+a(t)x_-'(t;\lambda)+b(t)x_+(t;\lambda)=\omega(t). \\ x_-(0;\lambda)=c_{1-}^{\lambda}, x_+(0;\lambda)=c_{1+}^{\lambda}, \\ x_-'(0;\lambda)=c_{2-}^{\lambda}, x_+'(0;\lambda)=c_{2+}^{\lambda}. \end{cases} \tag{4}$$

(2,2)-system

$$\begin{cases} x_-''(t;\lambda)+a(t)x_+'(t;\lambda)+b(t)x_-(t;\lambda)=\omega(t), \\ x_+''(t;\lambda)+a(t)x_-'(t;\lambda)+b(t)x_+(t;\lambda)=\omega(t), \\ x_-(0;\lambda)=c_{1-}^{\lambda}, x_+(0;\lambda)=c_{1+}^{\lambda}, \\ x_-'(0;\lambda)=c_{2-}^{\lambda}, x_+'(0;\lambda)=c_{2+}^{\lambda}. \end{cases} \tag{5}$$

The previous discussion illustrates the method to solve (1).We first choose the type of solution and translate (1) to a system of ordinary differential equations. Then, we solve the obtained ordinary differential equations system.

Remark 3.1 We see that the solution of fuzzy differential (1) depends upon the selection of derivatives. It is clear that in this new procedure, the unicity of the solution is lost, an expected situation in the fuzzy context.

IV. ADOMIAN METHOD FOR FUZZY DIFFERENTIAL EQUATION

The decomposition method was introduced by Adomian [3,4,5] in the 1980s in order to solve linear and nonlinear functional equations. Adomian has developed a decomposition technique for solving nonlinear functional equations, in this section we solve the fuzzy differential equation under strongly generalized differentiability by Adomain decomposition method and restrictions of the method will be discussed.

We only consider the (2) in section 3 and solve by Adomain decomposition method, since the other cases are analogous. For the sake of simplicity, we denote $x_-(t;\lambda)$ by ^-x, denote $x_+(t;\lambda)$ by $\bar{x}$, denote $a(t),b(t),\omega(t)$ by a,b,ω, respectively.

Let$^-x_1={}^-x, {}^-x_2={}^-x', {}^-x_3={}^-x'', \bar{x}_1=\bar{x}, \bar{x}_2=\bar{x}', \bar{x}_3=\bar{x}''$, $P^-x_1={}^-x_2, P^-x_2={}^-x_3=\omega-a^-x_2-b^-x_1$, $P\bar{x}_1=\bar{x}_2, P\bar{x}_2=\bar{x}_3=\omega-a\bar{x}_2-b\bar{x}_1$, where $P=\frac{\partial(\cdot)}{\partial t}$, then

$$^-x_1={}^-x_1(0)+\int_0^t {}^-x_2dt \tag{6}$$

$$^-x_2={}^-x_2(0)+\int_0^t(\omega-a^-x_2-b^-x_1)dt \tag{7}$$

$$\bar{x}_1=\bar{x}_1(0)+\int_0^t\bar{x}_2dt \tag{8}$$

$$\bar{x}_2=\bar{x}_2(0)+\int_0^t(\omega-a\bar{x}_2-b\bar{x}_1)dt \tag{9}$$

.

where $^-x_1(0)=c_{1-}^{\lambda}, \bar{x}_1(0)=c_{1+}^{\lambda}, {}^-x_2(0)=c_{2-}^{\lambda}, \bar{x}_2(0)=c_{2+}^{\lambda}$. Adomian decomposition method considers the solutions ^-x and $\bar{x}$ as the sum of a series as: $^-x_i=\sum_{j=0}^{\infty}{}^-x_{i,j},\ \bar{x}_i=\sum_{j=0}^{\infty}\bar{x}_{i,j}, i=1,2$. So, we can calculate the terms of $^-x_i=\sum_{j=0}^{\infty}{}^-x_{i,j},\ \bar{x}_i=\sum_{j=0}^{\infty}\bar{x}_{i,j}, i=1,2$..term by term as long as we derive desired accuracy, the more terms the more accuracy. Therefore, we have

$$^-x_{1,0}+{}^-x_{1,1}+\cdots={}^-x_1(0)+\int_0^t({}^-x_{2,0}+{}^-x_{2,1}+\cdots)dt \tag{10}$$

$$^-x_{2,0}+{}^-x_{2,1}+\cdots=\bar{x}_1(0)+\int_0^t[\omega(t)-$$
$$a({}^-x_{2,0}+{}^-x_{2,1}+\cdots)-b({}^-x_{1,0}+{}^-x_{1,1}+\cdots)]dt \tag{11}$$

$$\bar{x}_{1,0}+\bar{x}_{1,1}+\cdots={}^-x_2(0)+\int_0^t(\bar{x}_{2,0}+\bar{x}_{2,1}+\cdots)dt \tag{12}$$

$$\bar{x}_{2,0}+\bar{x}_{2,1}+\cdots=\bar{x}_2(0)+\int_0^t[\omega(t)-$$
$$a(\bar{x}_{2,0}+\bar{x}_{2,1}+\cdots)-b(\bar{x}_{1,0}+\bar{x}_{1,1}+\cdots)]dt \tag{13}$$

By solving (10)-(13), we can calculate the terms of above series. Three other cases are the same.

V. NUMERICAL EXAMPLE

Let us consider the following second-order fuzzy initial value problem

$$\begin{cases} x''(t) + x'(t) + x(t) = t, \\ x(0) = 0, \\ x'(0) = 1. \end{cases} \tag{14}$$

where 0 and 1 are the triangular fuzzy number having λ-level sets $[0]^\lambda = [\lambda - 1, 1 - \lambda]$ and $[1]^\lambda = [\lambda, 2 - \lambda], \lambda \in [0, 1]$, respectively.

By Adomian decomposition method, we obtain the (1,1)-solution of (14) is

$$\begin{cases} x_-(t;\lambda) = (\lambda - 1) + \lambda t + ((1 - 2\lambda)/2)t^2 + (1/6)t^3 \\ x_+(t;\lambda) = (1 - \lambda) + (2 - \lambda)t + ((2\lambda - 3)/2)t^2 + (1/6)t^3 \end{cases} \tag{15}$$

for $t \in [0, (1 + \sqrt{5})/2]$.

The (1,2)-solution of (14) is

$$\begin{cases} x_-(t;\lambda) = (\lambda - 1) + \lambda t + ((2\lambda - 3)/2)t^2 + (1/6)t^3 \\ x_+(t;\lambda) = (1 - \lambda) + (2 - \lambda)t + ((1 - 2\lambda)/2)t^2 + (1/6)t^3 \end{cases} \tag{16}$$

for $t \in [0, +\infty)$.

The (2,1)-solution of (14) is

$$\begin{cases} x_-(t;\lambda) = (\lambda - 1) + \lambda t + (-1/2)t^2 + (1/6)t^3, \\ x_+(t;\lambda) = (1 - \lambda) + (2 - \lambda)t + ((1 - 2\lambda)/2)t^2 + (1/6)t^3 \end{cases} \tag{17}$$

for $t \in [0, +\infty)$.

The (2,2)-solution of (14) is

$$\begin{cases} x_-(t;\lambda) = (\lambda - 1) + \lambda t + (-1/2)t^2 + (1/6)t^3, \\ x_+(t;\lambda) = (1 - \lambda) + (2 - \lambda)t + (-1/2)t^2 + (1/6)t^3. \end{cases} \tag{18}$$

for $t \in [0, +\infty)$.

We only count to three times Adomian polynomials, we obtain the (1,1)-solution of (14) is local existence and others are global existence.

VI. CONCLUSION

In this paper we presented the strongly generalized differentiability for the second order linear differential equation having fuzzy initial conditions. We apply Adomian decomposition method. Note that Adomian method gives explicit formula of solution even for some nonlinear problems that cannot be solved by classical methods and this is an advantage whereas more numerical methods lack this ability. Future research will be concerned with improved Adomian decomposition method and other methods..

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