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Septic B-spline collocation method for solving one-dimensional hyperbolic telegraph equation

Marzieh Dosti and Alireza Nazemi

Abstract—Recently, it is found that telegraph equation is more suitable than ordinary diffusion equation in modelling reaction diffusion for such branches of sciences. In this paper, a numerical solution for the one-dimensional hyperbolic telegraph equation by using the collocation method using the septic splines is proposed. The scheme works in a similar fashion as finite difference methods. Test problems are used to validate our scheme by calculate L_2-norm and L_∞-norm. The accuracy of the presented method is demonstrated by two test problems. The numerical results are found to be in good agreement with the exact solutions.

Keywords—B-spline; collocation method; second-order hyperbolic telegraph equation; difference schemes.

I. INTRODUCTION

WE Let the following be the second-order linear hyperbolic telegraph equation in one-space dimension:

$$\frac{\partial^2 u}{\partial t^2} + 2\alpha\frac{\partial u}{\partial t} + \beta^2 u = \frac{\partial^2 u}{\partial x^2} + f(x,t),\ a \le x \le b,\ t \ge 0, \quad (1)$$

subject to initial conditions

$$u(x,0) = f_0(x), \quad a < x < b, \quad (2)$$
$$\frac{\partial u(x,0)}{\partial t} = f_1(x), \quad a < x < b, \quad (3)$$

and Dirichlet boundary conditions

$$u(a,t) = g_0(t), \quad u(b,t) = g_1(t), \quad (4)$$
$$u_x(a,t) = g_2(t), \quad u_x(b,t) = g_3(t), \quad (5)$$
$$u_{xx}(a,t) = g_4(t), \quad u_{xx}(b,t) = g_5(t), \quad (6)$$

where α and β are known constant coefficients. We assume that $f_0(x)$, $f_1(x)$ and their derivatives are continuous functions of x, and $g_p(t)$, $p = 0,...,5$ and their derivatives are continuous functions of t. Both the electric voltage and the current in a double conductor, satisfy the telegraph equation, where x is distance and t is time. For $\alpha > 0$, $\beta = 0$ Eq. (1) represents a damped wave equation and for $\alpha > \beta > 0$, it is called telegraph equation.

The hyperbolic partial differential equations model the vibrations of structures (e.g. buildings, beams and machines) and are the basis for fundamental equations of atomic physics. Equations of the form Eq. (1) arise in the study of propagation of electrical signals in a cable of transmission line and wave phenomena. Interaction between convection and diffusion or reciprocal action of reaction and diffusion describes a number of nonlinear phenomena in physical, chemical and biological process [1]-[4]. In fact the telegraph equation is more suitable than ordinary diffusion equation in modeling reaction diffusion for such branches of sciences. For example biologists encounter these equations in the study of pulsate blood flow in arteries and in one- dimensional random motion of bugs along a hedge [5]. Also the propagation of acoustic waves in Darcy-type porous media [6], and parallel flows of viscous Maxwell fluids [7] are just some of the phenomena governed [8]-[9] by Eq. (1).

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The theory of spline functions is very active field of approximation theory, boundary value problems and partial differential equations, when numerical aspects are considered. Among the various classes of splines, the polynomial spline has been received the greatest attention primarily because it admits a basis of B-splines [10]-[14] which can be accurately and efficiently computed. As the piecewise polynomial, B-spline have also become a fundamental tool for numerical methods to get the solution of the differential equations. In this paper, numerical solution of the hyperbolic telegraph equation by using the septic B-spline collocation scheme is proposed. The collocation method together with B-spline approximations represents an economical alternative since it only requires the evaluation of the unknown parameters at the grid points. As is known, the success of the B-spline collocation method is dependent on the choice of B-spline basis. The septic B-spline basis has been used to build up the approximation solutions for some differential equations. For instance see [15]-[17].

In what follows, it is shown that how we use the B-spline collocation method to approximate the solution of the the hyperbolic telegraph equation in section 2. To demonstrate the efficiency of the proposed method, numerical experiments are carried out for several test problems and results are given in section 3. In section 4 the conclusion is given in the last Section. Finally some references are introduced at the end. Note that we have computed the numerical results by Matlab programming.

II. UNIVARIATE B-SPLINE QUASI-INTERPOLANTS

We consider a mesh $a = x_0 < x_1 < ... < x_N = b$ as a uniform partition of the solution domain $a \le x \le b$ by the knots x_j, and $h = x_{j+1} - x_j$, $j = -3, -2, -1, 0, ..., N, N+1, N+2, N+3$. Let the septic B-spline function $\phi_m(x)$ at these knots are given by:

$$\phi_m(x) =$$

TABLE I
THE VALUES OF $\phi_m, \phi_m^{'}, \phi_m^{''}, \phi_m^{'''}$.

x	x_{m-4}	x_{m-3}	x_{m-2}	x_{m-1}	x_m	x_{m+1}	x_{m+2}	x_{m+3}	x_{m+4}
ϕ_i	0	1	120	1191	2416	1191	120	1	0
$\phi_i^{'}$	0	$\frac{7}{h}$	$\frac{392}{h}$	$\frac{1715}{h}$	0	$\frac{-1715}{h}$	$\frac{-392}{h}$	$\frac{-7}{h}$	0
$\phi_i^{''}$	0	$\frac{42}{h^2}$	$\frac{1008}{h^2}$	$\frac{630}{h^2}$	$\frac{-3360}{h^2}$	$\frac{630}{h^2}$	$\frac{1008}{h^2}$	$\frac{42}{h^2}$	0
$\phi_i^{'''}$	0	$\frac{210}{h^3}$	$\frac{1680}{h^3}$	$\frac{-3990}{h^3}$	0	$\frac{3990}{h^3}$	$\frac{-16800}{h^3}$	$\frac{-210}{h^3}$	0

$$\frac{1}{h^7}\begin{cases} (x-x_{m-4})^7, & x\in[x_{m-4},x_{m-3}],\\ (x-x_{m-4})^7-8(x-x_{m-3})^7, & x\in[x_{m-3},x_{m-2}],\\ (x-x_{m-4})^7-8(x-x_{m-3})^7+ & \\ 28(x-x_{m-2})^7, & x\in[x_{m-2},x_{m-1}],\\ (x-x_{m-4})^7-8(x-x_{m-3})^7+ & \\ 28(x-x_{m-2})^7-56(x-x_{m-1})^7, & x\in[x_{m-1},x_m],\\ (x_{m+4}-x)^7-8(x_{m+3}-x)^7+ & \\ 28(x_{m+2}-x)^7-56(x_{m+1}-x)^7, & x\in[x_m,x_{m+1}],\\ (x_{m+4}-x)^7-8(x_{m+3}-x)^7+ & \\ 28(x_{m+2}-x)^7, & x\in[x_{m+1},x_{m+2}],\\ (x_{m+4}-x)^7-8(x_{m+3}-x)^7, & x\in[x_{m+2},x_{m+3}],\\ (x_{m+4}-x)^7, & x\in[x_{m+3},x_{m+4}],\\ 0, & \text{otherwise,} \end{cases}$$

where the set of splines, $\{\phi_{-3},\phi_{-2},\phi_{-1},\phi_0,...,\phi_N,\phi_{N+1},\phi_{N+2},\phi_{N+3}\}$ forms a basis over the region of solution $a\le x\le b$. This means that the values of the septic B-spline function $\phi_m(x)$, and all its first, second, and third derivatives vanish outside the interval $[x_{m-4},x_{m+4}]$. One can easily verify that the values of $\phi_m(x)$ and its derivatives are as shown in Table 1. Our numerical treatment for solving Eq. (1) using the collocation method with septic splines is to find an approximate solution $U_N(x,t)$ to the exact solution $u(x,t)$ in the form:

$$U_N(x,t)=\sum_{m=-3}^{N+3}\delta_m(t)\phi_m(x), \tag{7}$$

where $\delta_m(t)$ are time dependent quantities to be determined using the boundary conditions:

$$U_N(a,t)=g_0(t),\qquad U_N(b,t)=g_1(t), \tag{8}$$
$$(U_{x)_N}(a,t)=g_2(t),\quad (U_{x)_N}(b,t)=g_3(t), \tag{9}$$
$$(U_{xx})_N(a,t)=g_4(t),\quad (U_{xx)_N}(b,t)=g_5(t). \tag{10}$$

For every x by using the Taylor expansion in the time direction, using the notation $u_i=u(x,t_i)$ where $t_i=t_{i-1}+\Delta t$, we have the following difference schemes

$$\frac{\partial^2 u(x,t_i)}{\partial^2 t}=\frac{u_{i+1}-2u_i+u_{i-1}}{(\Delta t)^2}+O((\Delta t)^2), \tag{11}$$

$$\frac{\partial u(x,t_i)}{\partial t}=\frac{u_{i+1}-u_{i-1}}{2\Delta t}+O((\Delta t)^2), \tag{12}$$

$$u(x,t_i)=\frac{u_{i+1}+2u_i+u_{i-1}}{4}+O((\Delta t)^2), \tag{13}$$

$$\frac{\partial^2 u_{xx}(x,t_i)}{\partial^2 t}=\frac{u_{i+1}^{''}+u_{i-1}^{''}}{2}+O((\Delta t)^2). \tag{14}$$

Now, let us discretize Eq. (1) according to schemes (11)-(14) in the following form

$$\frac{u_{i+1}-2u_i+u_{i-1}}{(\Delta t)^2}+2\alpha\frac{u_{i+1}-u_{i-1}}{2\Delta t}+\beta^2\frac{u_{i+1}+2u_i+u_{i-1}}{4}$$
$$=\frac{u_{i+1}^{''}+u_{i-1}^{''}}{2}+f(x,t_i). \tag{15}$$

Rearranging Eq. (15) we have

$$(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})u_{i+1}-\frac{(\Delta t)^2}{2}u_{i+1}^{''}=(2-\frac{\beta^2(\Delta t)^2}{2})u_i+$$
$$(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_{i-1}+\frac{(\Delta t)^2}{2}u_i^{''}+(\Delta t)^2 f(x,t_i), \tag{16}$$

and the initial conditions are given in Eqs. (2) and (3) as follows

$$u(x,0)=f_0(t)=u_0, \tag{17}$$
$$u_t(x,0)=\frac{u_1-u_0}{\Delta t}=f_1(x), \tag{18}$$
$$u_1=u_0+\Delta t f_1(x). \tag{19}$$

Substituting Eq. (19) into Eq. (16) then is obtained as follows

$$i=1,$$
$$(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})u_2-\frac{(\Delta t)^2}{2}u_2^{''}=(2-\frac{\beta^2(\Delta t)^2}{2})u_1-$$
$$(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_0+\frac{(\Delta t)^2}{2}u_1^{''}+(\Delta t)^2 f(x,t_1), \tag{20}$$

$$i=2,$$
$$(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})u_3-\frac{(\Delta t)^2}{2}u_3^{''}=(2-\frac{\beta^2(\Delta t)^2}{2})u_2-$$
$$(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_1+\frac{(\Delta t)^2}{2}u_2^{''}+(\Delta t)^2 f(x,t_2), \tag{21}$$

...
...
...

$$i=n-1,$$
$$(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})u_n-\frac{(\Delta t)^2}{2}u_n^{''}=(2-\frac{\beta^2(\Delta t)^2}{2})u_{n-1}-$$
$$(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_{n-2}+\frac{(\Delta t)^2}{2}u_{n-1}^{''}+(\Delta t)^2 f(x,t_{n-1}). \tag{22}$$

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TABLE II
RESULTS AT $\Delta t = 0.001$ AND $\Delta x = 0.005$ IN EXAMPLE 4.1.

t	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 1$
L_∞	2.774×10^{-4}	7.0782×10^{-4}	1.3848×10^{-3}	3.0930×10^{-3}	1.3424×10^{-2}
L_2	3.3189×10^{-8}	2.3067×10^{-7}	8.208×10^{-7}	3.237×10^{-6}	3.2782×10^{-5}

The approximate solution of Eqs. (20)-(22) are sought in the form of the B-spline functions $U_N(x,t)$, it follows that

$$\begin{aligned}
&i = 1,\\
&(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})(U_N)_2-\frac{(\Delta t)^2}{2}(U_N)_2^{''}=\\
&(2-\frac{\beta^2(\Delta t)^2}{2})u_1-(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_0+\\
&\qquad\frac{(\Delta t)^2}{2}u_1^{''}+(\Delta t)^2 f(x,t_1),
\end{aligned}\tag{23}$$

$$\begin{aligned}
&i = 2,\\
&(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})(U_N)_3-\frac{(\Delta t)^2}{2}(U_N)_3^{''}=\\
&(2-\frac{\beta^2(\Delta t)^2}{2})u_2-(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_1+\\
&\qquad\frac{(\Delta t)^2}{2}u_2^{''}+(\Delta t)^2 f(x,t_2),
\end{aligned}\tag{24}$$

$$\dots$$
$$\dots$$
$$\dots$$

$$\begin{aligned}
&i = n-1,\\
&(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})(U_N)_n-\frac{(\Delta t)^2}{2}(U_N)_n^{''}=\\
&(2-\frac{\beta^2(\Delta t)^2}{2})u_{n-1}+(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_{n-2}+\\
&\qquad\frac{(\Delta t)^2}{2}u_{n-1}^{''}+(\Delta t)^2 f(x,t_{n-1}),
\end{aligned}\tag{25}$$

and boundary conditions (8)-(10) can be written as

$$\sum_{m=-3}^{N+3}\delta_m(t)\phi_m(x_0)=g_0(t),\quad \text{for}\quad x=a,\ t\geq 0,\tag{26}$$

$$\sum_{m=-3}^{N+3}\delta_m(t)\phi_m(x_N)=g_1(t),\quad \text{for}\quad x=b,\ t\geq 0,\tag{27}$$

$$\sum_{m=-3}^{N+3}\delta_m(t)\phi_m^{'}(x_0)=g_2(t),\quad \text{for}\quad x=a,\ t\geq 0,\tag{28}$$

$$\sum_{m=-3}^{N+3}\delta_m(t)\phi_m^{'}(x_N)=g_3(t),\quad \text{for}\quad x=b,\ t\geq 0,\tag{29}$$

$$\sum_{m=-3}^{N+3}\delta_m(t)\phi_m^{''}(x_0)=g_4(t),\quad \text{for}\quad x=a,\ t\geq 0,\tag{30}$$

$$\sum_{m=-3}^{N+3}\delta_m(t)\phi_m^{''}(x_N)=g_5(t),\quad \text{for}\quad x=b,\ t\geq 0.\tag{31}$$

The spline solution of Eq. (23) with the boundary conditions (26)-(31) are obtained by solving to the following matrix equation. The value of spline functions at the knots $\{x_j\}_{j=0}^N$ are determined using Table 1. Then the B-spline method in matrix form can be written as follows:

$$AX = B,\tag{32}$$

where $X = [\delta_{-3}, \delta_{-1}, ..., \delta_N, \delta_{N+3}]$, while $A \in I\!R^{(N+7)\times(N+7)}$ and $B \in I\!R^{(N+7)}$ are obtained from left and right hand sides of Eqs. (23) and (26)-(31), respectively as follows

$$A =$$

$$\begin{bmatrix}
1 & 120 & 1191 & 2416 & 1191 & 120 & 1 & \dots & 0\\
\frac{4}{h} & \frac{392}{h} & \frac{1715}{h} & 0 & \frac{-1715}{h} & \frac{-392}{h} & \frac{-7}{h} & \dots & 0\\
\frac{42}{h^2} & \frac{1008}{h^2} & \frac{630}{h^2} & \frac{-3360}{h^2} & \frac{630}{h^2} & \frac{1008}{h^2} & \frac{42}{h^2} & \dots & 0\\
r_1 & r_2 & r_3 & r_4 & r_3 & r_2 & r_1 & \dots & 0\\
\vdots & & & & & & & & \\
0 & \dots & r_1 & r_2 & r_3 & r_4 & r_3 & r_2 & r_1\\
0 & \dots & 1 & 120 & 1191 & 2416 & 1191 & 120 & 1\\
0 & \dots & \frac{4}{h} & \frac{392}{h} & \frac{1715}{h} & 0 & \frac{-1715}{h} & \frac{-392}{h} & \frac{-7}{h}\\
0 & \dots & \frac{42}{h^2} & \frac{1008}{h^2} & \frac{630}{h^2} & \frac{-3360}{h^2} & \frac{630}{h^2} & \frac{1008}{h^2} & \frac{42}{h^2}
\end{bmatrix}.$$

and

$$B = \begin{bmatrix}
g_0(t_1)\\
g_2(t_1)\\
g_4(t_1)\\
(\Delta t)^2 f(x_0,t_1)+(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_0(x_0)\\
+\frac{(\Delta t)^2}{2}u_1^{''}(x_0)+(2-\frac{\beta^2(\Delta t)^2}{4})u_1(x_0)\\
\vdots\\
(\Delta t)^2 f(x_N,t_1)+(\alpha\Delta t-1-\frac{\beta^2(\Delta t)^2}{4})u_0(x_N)\\
+\frac{(\Delta t)^2}{2}u_1^{''}(x_N)+2-\frac{\beta^2(\Delta t)^2}{4})u_1(x_N)\\
g_1(t_1)\\
g_3(t_1)\\
g_5(t_1)
\end{bmatrix}$$

where

$$\begin{aligned}
r_1 &= (1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})-\frac{21(\Delta t)^2}{h^2},\\
r_2 &= 120(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})-\frac{504(\Delta t)^2}{h^2},\\
r_3 &= 1191(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})-\frac{315(\Delta t)^2}{h^2},\\
r_4 &= 2416(1+\alpha\Delta t+\frac{\beta^2(\Delta t)^2}{4})+\frac{1680(\Delta t)^2}{h^2}.
\end{aligned}$$

It is easy to see that, the same approximation can be applied the other Eqs. (24) and (25) together with the corresponding boundary conditions (26)-(31). We solved $n-1$ times the system (32) by means of a home-made program which is based on singular value decomposition (SVD) method [18] and in each step obtain $u(x_0,t_i), u(x_1,t_i), ..., u(x_N,t_i)$ $(i = 1, ..., n-1)$.

TABLE III
RESULTS AT $\Delta t = 0.001$ AND $\Delta x = 0.01$ IN EXAMPLE 4.2.

t	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 1$
L_∞	3.2×10^{-3}	1.5×10^{-3}	6.5137×10^{-4}	2.9426×10^{-4}	1.4555×10^{-4}
L_2	2.0865×10^{-7}	6.1177×10^{-7}	1.3864×10^{-7}	4.1007×10^{-8}	1.0665×10^{-8}

The condition number of A

$$\kappa_s(A) = \|A\|_s \|A^{-1}\|_s, \qquad s = 1, 2, \infty,$$

depends on α, β, distance of collocation points and Δt Therefore a small perturbation in initial data may produce large amount of perturbation in the solution. Also the conditio number grows with N for fixed values of α and β. Generall for a fixed number of collocation points N, smaller values o α and β produce better approximations, but the matrix A wil be more ill-conditioned.

III. NUMERICAL EXAMPLES

We now obtain the numerical solutions of hyperbolic tele graph equation for two problems. The accuracy of the numer ical method is measured by computing the difference betwee the analytic and numerical solutions at each mesh point, an use these to compute the L_2- and L_∞-error norms. We repor the root mean square error L_2 and maximum error L_∞ errors

$$L_2 = |U - U_N|^2 = h \sum_{j=0}^{N} |U_j - (U_N)_j|^2,$$
$$L_\infty = |U - U_N|_\infty = \max_j |U_j - (U_N)_j|.$$

Example 4.1: We consider the hyperbolic telegraph Eq. (1) with $\alpha = 2$, $\beta = 1$ and $f(x,t) = -2\alpha \sinh(x)\sin(t) + (\beta^2 - 2)\sinh(x)\cos(t)$ and $0 \le x \le 1$. The initial conditions are given by

$$u(x,0) = \sinh(x),$$
$$u_t(x,0) = 0,$$

and the boundary conditions

$$u(0,t) = u_{xx}(0,t) = 0,$$
$$u(2,t) = u_{xx}(1,t) = \cos(t)\sinh(1),$$
$$u_x(0,t) = \cos(t),$$
$$u_x(1,t) = \cos(t)\cosh(1).$$

The analytical solution of this example is $u(x,t) = \cos(t)\sinh(x)$. The space-time graph of the numerical solution up to $t = 1$ is presented in Figure 1. The graph of analytical and estimated solutions for some different times and $x \in [0,1]$ is presented in Figure 2. The accuracy of the B-spline method is measured by using the L_2 and L_∞ errors. The errors are reported in Table 2.

Example 4.2: Consider the hyperbolic telegraph Eq. (1) with $\alpha = 4$, $\beta = 2$, $f(x,t) = (3 - 4\alpha + \beta^2)\exp(-2t)\sinh(x)$ and $0 \le x \le 1$. The initial conditions are given by

$$u(x,0) = \sinh(x),$$
$$u_t(x,0) = -2\sinh(x),$$

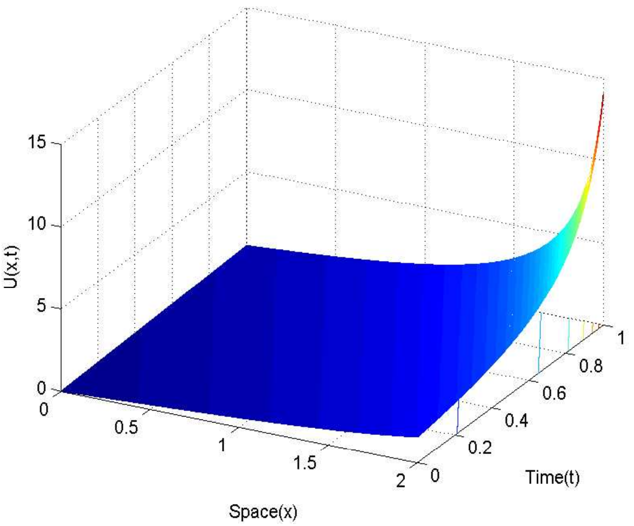

Fig. 1. Three-dimensional plot, with $\Delta t = 0.001$ and $\Delta x = 0.005$ in Example 4.1.

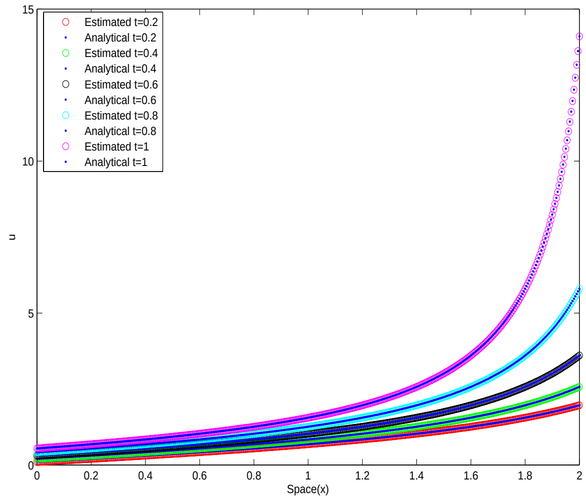

Fig. 2. Comparisons between numerical and analytical solutions of Eq. (1) in $t = 0.2s$, $t = 0.4s$, $t = 0.6s$, $t = 0.8s$, $t = 1s$, with $\Delta t = 0.001$ and $\Delta x = 0.005$ in Example 4.1.

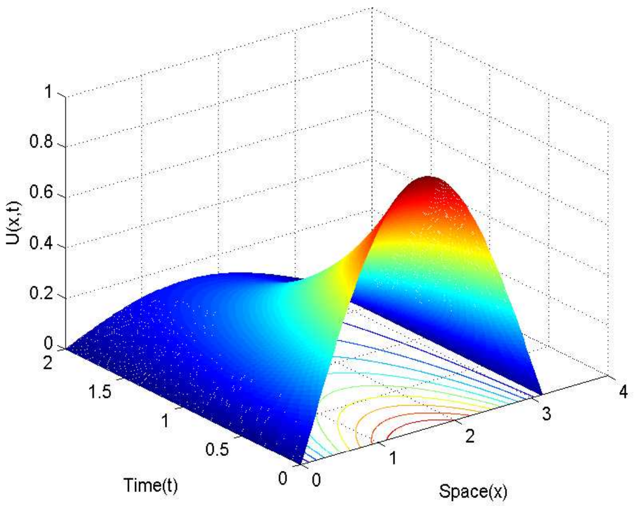

Fig. 3. Three-dimensional plot, with $\Delta t = 0.001$ and $\Delta x = 0.01$ in Example 4.2.

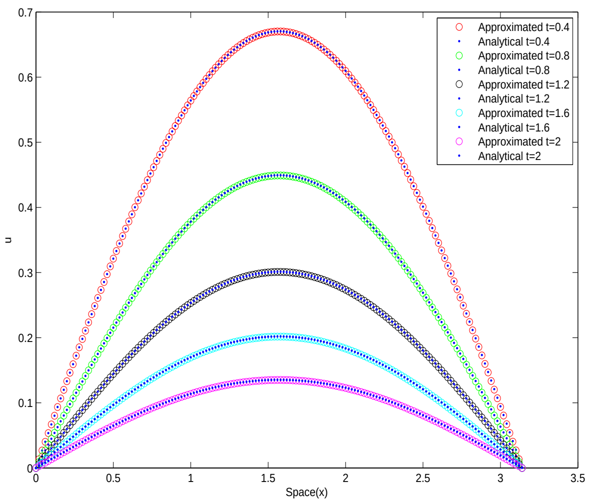

Fig. 4. Comparisons between numerical and analytical solutions of Eq. (1) in $t = 0.2s$, $t = 0.4s$, $t = 0.6s$, $t = 0.8s$, $t = 1s$, with $\Delta t = 0.001$ and $\Delta x = 0.01$ in Example 4.2.

and the boundary conditions

$$\begin{aligned}
&u(0,t) = u_{xx}(0,t) = 0,\\
&u(1,t) = u_{xx}(1,t) = \exp(-2t)\sinh(1),\\
&u_x(0,t) = \exp(-2t),\\
&u_x(1,t) = \exp(-2t)\cosh(1),
\end{aligned}$$

The analytical solution of this example is $u(x,t) = \exp(-2t)\sinh(x)$. The root-mean-square error L_2 and maximum error L_∞ are presented in Table 3. The space-time graph of the estimated solution up to $t = 1$ is shown in Figure 3. The graph of analytical and estimated solutions for some different times and $x \in [0,1]$ is presented in Figure 4.

IV. Conclusion

In this paper a numerical treatment for telegraph equation is proposed using a collection method with the septic B-splines. The numerical solutions are compared with the exact solution by finding the L_2 and L_∞ errors. The obtained approximate numerical solutions maintain good accuracy compared with the exact solutions. Most importantly, septic B-spline methods are especially advisable for obtaining numerical solutions of differential equations when higher continuity of the solutions exist.

References

[1] D. M. Pozar, Microwave Engineering. NewYork: Addison-Wesley, 1990.
[2] A. Mohebbi, M. Dehghan, High order compact solution of the one-space-dimensional linear hyperbolic equation, Numerical Methods for Partial Differential Equations, 24 (2008) 1222–1235.
[3] A. Jeffrey, Advanced Engineering Mathematics. Harcourt Academic Press, 2002.
[4] A. Jeffrey, Applied Partial Differential Equations. NewYork: Academic Press, 2002.
[5] R. K. Mohanty, New unconditionally stable difference schemes for the solution of multi-dimensional telegraphic equations, Computer Mathematics, 86 (2008) 2061 - 2071.
[6] H. Pascal, Pressure wave propagation in a fluid flowing through a porous medium and problems related to interpretation of Stoneley's wave attenuation in acoustical well logging, Engineering Science, 24 (1986) 1553–1570.
[7] G. Bohme, Non-Newtonian Fluid Mechanics, NewYork: North-Holland, 1987.
[8] D. J. Evans, H. Bulut, Thenumerical solution of thetelegraph equation by the alternating group explicit method, Computer Mathematics, 80 (2003) 1289– 1297.
[9] P. M. Jordan, M. R. Meyer, A. Puri, Causal implications of viscous damping in compressible fluid flows. Phys Rev, 62 (2000) 7918–7926.
[10] L. L. Schumaker, Spline Functions: Basic Theory, Krieger Publishing Company, Florida, 1981.
[11] J. M. Ahlberg, E. N. Nilson, J. L. Walsh, The Theory of Splines and Their Applications, Academic Press, NewYork, 1967.
[12] P. M. Prenter, Splines and Variational Methods, New York: John Wiley, 1975.
[13] C. De Boor, A Practical Guide to Splines, Springer-Verlag, New York, 1978.
[14] G. Micula, Handbook of Splines, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1999.
[15] M. A. Ramadan, T. S. El-Danaf, F. E.I. Abd Alaal, A numerical solution of the Burgers' equation using septic B-splines, Chaos, Solitons and Fractals, 26 (2005) 795-804.
[16] T. S. A. El-Danaf, Septic B-spline method of the Korteweg-de Vries-Burger's equation, Communications in Nonlinear Science and Numerical Simulation, 13 (2008) 554-566.
[17] A. A. Soliman, M. H. Hussien, Collocation solution for RLW equation with septic spline, Applied Mathematics and Computation, 161 (2005) 623-636.
[18] A.Quarteroni, R. Sacco, F. Saleri, Numerical mathematics, Springer-Verlag, Berlin, 2007.