On Solution of Interval Valued Intuitionistic Fuzzy Assignment Problem Using Similarity Measure and Score Function

Gaurav Kumar, Rakesh Kumar Bajaj

Abstract—The primary objective of the paper is to propose a new method for solving assignment problem under uncertain situation. In the classical assignment problem (AP), z_{pq} denotes the cost for assigning the qth job to the pth person which is deterministic in nature. Here in some uncertain situation, we have assigned a cost in the form of composite relative degree F_{pq} instead of z_{pq} and this replaced cost is in the maximization form. In this paper, it has been solved and validated by the two proposed algorithms, a new mathematical formulation of *IVIF* assignment problem has been presented where the cost has been considered to be an *IVIFN* and the membership of elements in the set can be explained by positive and negative evidences. To determine the composite relative degree of similarity of *IVIFS* the concept of similarity measure and the score function is used for validating the solution which is obtained by Composite relative similarity degree method. Further, hypothetical numeric illusion is conducted to clarify the method's effectiveness and feasibility developed in the study. Finally, conclusion and suggestion for future work are also proposed.

Keywords—Assignment problem, Interval-valued Intuitionistic Fuzzy Sets, Similarity Measures, score function.

I. Introduction

ASSIGNMENT problems deal with the question how to assign n objects to m other objects in an injective fashion in the best possible way. An assignment problem is completely specified by its two components: the assignments - which represent the underlying combinatorial structure and the objective function to be, optimized which models "the best possible way". To find solutions of assignment problems, various algorithms such as linear Programming [1]-[4], Hungarian algorithm [5], neural network [6] and genetic algorithm [7] have been developed. Over the past 50 years, many variations of the classical assignment problems are proposed e.g. bottleneck assignment problem, generalized assignment problem, quadratic assignment problem etc. But in real life situation, the parameters of AP are imprecise number instead of fixed real numbers. Zadeh [8] introduced the notion of fuzzy sets to deal with vague situation in real life. In recent years, fuzzy transportation and fuzzy assignment problems have received much concentration. Lin and Wen [9] proposed an efficient algorithm based on the labeling method for solving the linear fractional programming case. Atanassov [10]-[12] introduced a generalized concept of fuzzy sets i.e. intuitionistic fuzzy sets. Then Atanassov introduced prominent form by combining the concept of intuitionistic fuzzy sets and interval-valued fuzzy sets i.e., interval–valued intuitionistic fuzzy sets *IVIFS* [12] and this concept is widely used in multi criterion decision making problems, medical diagnosis etc. [13]–[17]. Bustince and Burillo [18] showed that vague sets are intuitionistic fuzzy sets. Further, they studied some distance measures, similarity measures, and correlation measures for *IVIFS* and applied the introduced concepts to many real-life problems in pattern recognition, decision making, etc. In addition to this, there are other applications which involve the information entropy of *IVIFS* [19]–[21]. Xu [22] defined the concept of the degree of similarity between interval-valued intuitionistic fuzzy sets and defines some distances measures between two *IVIFS* and proposed an approach for decision making with interval-valued intuitionistic fuzzy information [23], [24]. Some authors [25], [26] proposed different methods for decision making under intuitionistic or interval-valued intuitionistic fuzzy environment. Jahan [27] used the linear assignment method to rank the materials of a given engineering component in accordance with several criteria. Mukherjee and Basu [28] proposed an algorithm to solve Intuitionistic Fuzzy Assignment Problem by Similarity Measures and Score Functions.

The paper is organized as follows: In Section II, we recall some basic notions related to Interval-valued intuitionistic fuzzy sets and Interval-valued Fuzzy Intuitionistic Fuzzy number. In Section III, we develop a new methodology to solve assignment problem in interval-valued intuitionistic fuzzy environment. In Section IV, by using the concept of positive and negative ideal for interval-valued intuitionistic fuzzy sets, we develop two algorithms to solve by *IVIFAP*. Section V provides an illustrative example and finally, in Section VI, we conclude the paper.

II. Preliminaries

A. Interval-Valued Intuitionistic Fuzzy Sets and Interval-Valued Fuzzy Intuitionistic Fuzzy Number

Let T be a finite non-empty set i.e. $T = \{t_1, t_2, t_3, ..., t_n\}$. Let $R[0,1]$ be all subintervals of $[0,1]$. An *IVIFS* X in T is defined with the form,

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$$X = \{(t, \mu_X(t), \gamma_X(t) / t \in T)\} ;$$

where $\mu_X : T \to R[0,1], \gamma : T \to R[0,1]$ with the condition $0 \leq \sup(\mu_X(t)) + \sup(\gamma_X(t)) \leq 1$ for all $t \in T$. The intervals $\mu_X(t)$ and $\gamma_X(t)$ denotes the membership and non-membership degree of t to X, respectively.

Let $X = \{(t, [\mu_X^L(t), \mu_X^U(t)], [\gamma_X^L(t), \gamma_X^U(t)]) / t \in T\}$; where

$$\mu_X^L(t) = \inf(\mu_X(t)), \quad \mu_X^U(t) = \sup(\mu_X(t)),$$
$$\gamma_X^L(t) = \inf(\gamma_X(t)), \quad \gamma_X^U(t) = \sup(\gamma_X(t)).$$

If $\mu_X^L(t) = \mu_X^U(t)$ and $\gamma_X^L(t) = \gamma_X^U(t)$ then $IVIFS$ X reduces to an IFS. For an $IVIFS$ X, the pair $\{(\mu_X(t), \gamma_X(t))\}$ is called an Interval-valued Intuitionistic fuzzy number as $([\alpha, \beta], [\eta, \delta])$; where $[\alpha, \beta] \subset [0,1], [\eta, \delta] \subset [0,1], \ \beta + \delta \leq 1$ and Ω is denoted as the set of all $IVIFS$. Let $A = ([\alpha_1, \beta_1], [\eta_1, \delta_1])$ and $B = ([\alpha_2, \beta_2], [\eta_2, \delta_2])$ be any two $IVIFS$ in Ω. The following expressions are defined as follow:

(P1) $A.B = B.A$

(P2) $\lambda(A + B) = \lambda A + \lambda B, \lambda \geq 0$

(P3) $\lambda_1 A + \lambda_2 A = (\lambda_1 + \lambda_2)A, \lambda_1, \lambda_2, A \geq 0$

B. Score Function:

In order to make comparisons between two $IVIFS$, Xu [24], [25] introduced a concept of score function in 2007. The score function is applied to compare the grades of $IVIFS$. This function shows that greater is the value, the greater is the interval-valued intuitionistic fuzzy sets $IVIFS$ and by using this concept alternatives can be ranked. Let there is an interval-valued intuitionistic fuzzy number denoted by $X = \{(t, [\mu_X^L(t), \mu_X^U(t)], [\gamma_X^L(t), \gamma_X^U(t)]) / t \in T\}$, then score function is given by

$$\Pi(X) = \frac{(\mu_X^L + \mu_X^U - \gamma_X^L - \gamma_X^U)}{2} \tag{1}$$

where $\Pi(X) \in [-1,1]$.

Let X_1 and X_2 be any two interval-valued intuitionistic fuzzy values and

$$\Pi(X_1) = \frac{(\mu_{X_1}^L + \mu_{X_1}^U - \gamma_{X_1}^L - \gamma_{X_1}^U)}{2}$$

and

$$\Pi(X_2) = \frac{(\mu_{X_2}^L + \mu_{X_2}^U - \gamma_{X_2}^L - \gamma_{X_{21}}^U)}{2}$$

be the score of X_1 and X_2, respectively. If $\Pi(X_1) \angle \Pi(X_2)$, then we can say X_1 is smaller than X_2 and denoted by $X_1 \prec X_2$.

C. Similarity Measures of IVIFS

Based on the Euclidean distance, we use the following distance function which deals with similarity measures of $IVIFS$:

$$\Psi(M,N) = 1 - \left[\frac{1}{4n}\sum_{i=1}^{m}\left(|\mu_N^L - \mu_M^L| + |\mu_N^U - \mu_M^U| + |\gamma_N^L - \gamma_M^L| + |\gamma_N^U - \gamma_M^U| + |\pi_N^L - \pi_M^L| + |\pi_N^U - \pi_M^U|\right)\right] \tag{2}$$

Above function satisfies all properties of similarity function and the similarity between two sets M and N is directly proportional to the value of $\Psi(M, N)$ i.e. if the value of above mentioned function is large then the similarity between two sets is also larger.

III. MATHEMATICAL MODELS

A. Model Based On Crisp Assignment Problem

From the above section we analysed that assignment problem is a special type of transportation problem in which the objective is to assign a number of origin to an equal number of destination at a minimum cost (or maximum profit). Mathematical formulation of a crisp assignment problem is as below:

$$\text{Total Cost:} \ T = \sum_{p=1}^{m}\sum_{q=1}^{m} z_{pq}\tau_{pq} \tag{3}$$

where z_{pq} indicates the cost for assigning the qth destination to the pth origin and τ_{pq} indicates the characteristic function and derived as:

$$\tau_{pq} = \begin{cases} 1, \text{if the origin } p^{th} \text{ is assigned the destination } q^{th} \\ 0, \text{ otherwise} \end{cases} \tag{4}$$

where $p, q = 1,2,3,...,m$.

The objective of assignment problem is to minimize the cost. Therefore, the total cost of assignment problem in minimized form can be written as under:

$$\text{Total Minimum Cost:} \ \text{Min}(T) = \sum_{p=1}^{m}\sum_{q=1}^{m} z_{pq}\tau_{pq} \tag{5}$$

Subjected to: $\quad \sum_{p=1}^{m}\tau_{pq} = 1, \quad \text{where } q = 1,2,3,...,m$

And $\quad \sum_{q=1}^{m}\tau_{pq} = 1, \quad \text{where } p = 1,2,3,...,m$

Also τ_{pq} satisfies condition (4). According to the crisp assignment problem assumptions cost must be deterministic in nature i.e., exact but in the realistic situations the value of cost is not exact. In such conditions, we can replace z_{pq} by F_{pq} in classical assignment problem and this cost should be in the maximization form if we evaluate the preferences for assigning the qth destination to the pth origin in the form of composite relative degree of similarity to the ideal solution.

B. Model Based On Interval-valued Intuitionistic Fuzzy Assignment Problem:

In this case, the preferable model for assignment problem can be written as

$$\text{Total Maximum Cost: Max } T = \sum_{p=1}^{m} \sum_{q=1}^{m} F_{pq} \tau_{pq} \qquad (6)$$

subjected to: $\sum_{p=1}^{m} \tau_{pq} = 1$ and $\sum_{q=1}^{m} \tau_{pq} = 1$, where

$p = 1, 2, 3, ..., m$ and $q = 1, 2, 3, ..., m$

From the above section, the value of z_{pq} is not deterministic and giving an imprecise value of cost. As we know Interval-valued intuitionistic fuzzy sets is a generalization of intuitionistic fuzzy set and is a more realistic description involving more uncertainty compared to the crisp and fuzzy concept. So, we can consider z_{pq} as an Interval-valued intuitionistic fuzzy number.

Interval-valued intuitionistic fuzzy number is given by $X = \left\{ [\mu_{pq}^L, \mu_{pq}^U], [\gamma_{pq}^L, \gamma_{pq}^U] \right\}, p, q = 1, 2, 3, ..., m$. Here μ_{pq}^L, μ_{pq}^U denotes the degree of acceptance and $\gamma_{pq}^L, \gamma_{pq}^U$ denotes the degree of rejection of assigning the qth destination to the pth origin. In this problem we have used the degree of acceptance and degree of rejection instead of cost where the problem seems more realistic then latter.

On substituting the value of cost in terms of interval-valued intuitionistic fuzzy number in the above models (6) we obtain,

$$MaxT = \sum_{p=1}^{m} \sum_{q=1}^{m} \{ [\mu_{pq}^L, \mu_{pq}^U], [\gamma_{pq}^L, \gamma_{pq}^U] \} \tau_{pq}$$

with condition, $\mu_{pq}^U + \gamma_{pq}^U \leq 1$.

Therefore, mathematical formulation of assignment problem can be written as follows:

$$MaxT_1 = \sum_{p=1}^{m} \sum_{q=1}^{m} \{ [\mu_{pq}^L, \mu_{pq}^U] \} \tau_{pq} \qquad (7)$$

with condition $\mu_{pq}^U + \gamma_{pq}^U \leq 1$

$$MinT_2 = \sum_{p=1}^{m} \sum_{q=1}^{m} \{ [\gamma_{pq}^L, \gamma_{pq}^U] \} \tau_{pq} \qquad (8)$$

with condition $\mu_{pq}^U + \gamma_{pq}^U \leq 1$

Subject to $(\mu_{pq}^U + \gamma_{pq}^U - 1) \tau_{pq} \leq 0, (\mu_{pq}^U) \tau_{pq} \geq (\gamma_{pq}^U) \tau_{pq};$

$$\sum_{p=1}^{m} \tau_{pq} = 1 \text{ and } \sum_{q=1}^{m} \tau_{pq} = 1,$$

where $p = 1, 2, 3, ..., m$ and $q = 1, 2, 3, ..., m$

IV. PROPOSED ALGORITHM FOR INTERVAL-VALUED INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM

Under the idea of positive and negative ideal for interval-valued intuitionistic fuzzy sets, we propose following algorithms to solve interval-valued intuitionistic fuzzy assignment problem, which are based on the concept of Similarity measures and Score function of interval-valued intuitionistic fuzzy sets:

Algorithm Based On Score Function:

In this algorithm, we calculate the optimal solution of *IVIFS* by using the concept of Score function.

Step 1. Utilize (1) to compute score function matrix of given cost matrix.

Step 2. Considering the obtained Score function matrix as the profit matrix and solve the profit matrix to get optimal solution by using Hungarian method.

Algorithm based on Similarity Measures:

In this algorithm, we calculate the relative degree of similarity for the jobs with respect to the persons by applying the concept of similarity measures of *IVIFS* .

Step 1. Construct interval valued intuitionistic fuzzy decision matrix $M = (I_{pq})_{mxn}$; where, $\{I_{pq}\} = \{ [\mu_{pq}^L, \mu_{pq}^U], [\gamma_{pq}^L, \gamma_{pq}^U] \}$ these numbers must satisfies the condition, $\mu_{pq}^L, \mu_{pq}^U, \gamma_{pq}^L, \gamma_{pq}^U \in [0,1]$.

Step 2. Compute positive and negative ideal solution for each task based on interval-valued intuitionistic fuzzy number, defined as follows for $q = 1, 2, 3, ..., m$ respectively:

$$I_{IDEAL}^+ = \left([\alpha_q^+, \beta_q^+], [\eta_q^+, \delta_q^+] \right) = \begin{bmatrix} \left([\max\alpha_{pq}, \max\beta_{pq}] [\min\eta_{pq}, \min\delta_{pq}] \right) q \in A \\ \left([\min\alpha_{pq}, \min\beta_{pq}] [\max\eta_{pq}, \max\delta_{pq}] \right) q \in B \end{bmatrix};$$

$$I_{IDEAL}^- = \left([\alpha_q^-, \beta_q^-], [\eta_q^-, \delta_q^-] \right) = \begin{bmatrix} \left([\min\alpha_{pq}, \min\beta_{pq}] [\max\eta_{pq}, \max\delta_{pq}] \right) q \in A \\ \left([\max\alpha_{pq}, \max\beta_{pq}] [\min\eta_{pq}, \min\delta_{pq}] \right) q \in B \end{bmatrix};$$

$$(9)$$

where A be the collection of benefit attribute and B be the collection of cost attributes

Step 3. Compute degree of similarity for both positive and negative ideal with respect to their alternatives by using (1) as follows:

Degree of similarity of positive ideal and negative ideal *IVIFS* and the alternatives I_p is defined as:

$$\Psi(I_p, I_{IDEAL}^+)=1-\left[\frac{1}{4n}\sum_{p=1}^{m}\left(\left|\mu_{I^+}^L-\mu_{I_p}^L\right|+\left|\mu_{I^+}^U-\mu_{I_p}^U\right|+\left|\gamma_{I^+}^L-\gamma_{I_p}^L\right|+\left|\gamma_{I^+}^U-\gamma_{I_p}^U\right|+\left|\pi_{I^+}^L-\pi_{I_p}^L\right|+\left|\pi_{I^+}^U-\pi_{I_p}^U\right|\right)\right]$$

(10)

$$\Psi(I_p, I_{IDEAL}^-)=1-\left[\frac{1}{4n}\sum_{p=1}^{m}\left(\left|\mu_{I^-}^L-\mu_{I_p}^L\right|+\left|\mu_{I^-}^U-\mu_{I_p}^U\right|+\left|\gamma_{I^-}^L-\gamma_{I_p}^L\right|+\left|\gamma_{I^-}^U-\gamma_{I_p}^U\right|+\left|\pi_{I^-}^L-\pi_{I_p}^L\right|+\left|\pi_{I^-}^U-\pi_{I_p}^U\right|\right)\right]$$

(11)

where $p=1,2,3,...,m: q=1,2,3,...,m$.

Repeat Step2 to Step4 for other column of cost matrix and compute relative similarity measure Δ_{pq} corresponding to I_p alternatives. Resultant matrix obtained by column operation is denoted by $\langle A\rangle$. Similarly repeat Step2 to Step4 for rows of interval-valued intuitionistic fuzzy matrix and compute relative similarity measures Δ_{pq} corresponding to I_p alternatives and resultant matrix obtained by column operation is denoted by $\langle B\rangle$.

Step 4. Utilize (10) and (11), to compute relative similarity measure Δ_{pq} corresponding to the alternatives I_p as follows:

$$\Delta_{pq}=\frac{\Psi(I_p, I_{IDEAL}^+)}{\Psi(I_p, I_{IDEAL}^+)+\Psi(I_p, I_{IDEAL}^-)}, \quad p=1,2,3,...,m$$

(12)

Step 5. Compute composite matrix $[H]_{mxm}$ by using $Comp(\langle A\rangle.\langle B\rangle)=H$. This composite matrix will represent the preference that q^{th} destination is chosen by p^{th} origin. Further, obtained assignment problem can be solved by suitable method i.e. Hungarian Method.

V. ILLUSTRATIVE EXAMPLE

In order to test the robustness of proposed algorithm and applicability of Interval-valued Intuitionistic Fuzzy Assignment problem we provide the following example:

Example: Let us consider an Interval-valued Intuitionistic Fuzzy Assignment Problem having three persons and three jobs and the estimated time to perform the job q^{th} by p^{th} person is given below in the form of Interval-valued Intuitionistic Fuzzy Assignment number

$$M=\begin{bmatrix}\{[0.40,0.50],[0.32,0.40]\} & \{[0.67,0.78],[0.14,0.20]\} & \{[0.50,0.65],[0.13,0.22]\} \\ \{[0.52,0.60],[0.10,0.17]\} & \{[0.56,0.23],[0.23,0.28]\} & \{[0.65,0.70],[0.20,0.25]\} \\ \{[0.62,0.72],[0.20,0.25]\} & \{[0.35,0.45],[0.33,0.43]\} & \{[0.55,0.63],[0.28,0.32]\}\end{bmatrix}$$

satisfies condition $0\le\mu_{pq}^U+\gamma_{pq}^U\le 1$ and $\mu_{pq}^U\ge 0, \gamma_{pq}^U\ge 0$.

Enumerate the optimal assignment for the Interval-valued Intuitionistic Fuzzy Assignment Problem. Solution:

Solution Based On Score Function: Apply Algorithm-I on above Interval-valued Intuitionistic Fuzzy Assignment Problem and the score function matrix is as under:

TABLE I
SCORE FUNCTION

jobs → persons ↓	J_1	J_2	J_3
A_1	0.09	0.55	0.40
A_2	0.42	–0.08	–0.07
A_3	0.44	0.04	0.29

From the above table, the optimal assignments are:

Job J_1 is assigned to person A_2; Job J_2 is assigned to person A_1; Job J_3 is assigned to person A_3.

Solution Based On similarity measures: Apply Algorithm-II i.e. based on similarity measures, on given Interval-valued Intuitionistic Fuzzy Assignment Problem (IVIFAP) to find the optimal solution of assignment problem. By using Step2 to Step4, find degree of similarity measures of positive and negative ideal of *IVIFS* w.r.t. the alternatives (row-wise and column-wise) and we tabulate the obtained values in following Tables II to V:

TABLE II
$\Psi(I_p, I_{IDEAL}^+)$ ROW-WISE FOR POSITIVE IDEAL

jobs → persons ↓	J_1	J_2	J_3
A_1	0.86	0.96	0.92
A_2	0.88	0.96	0.99
A_3	1.00	0.86	0.96

TABLE III
$\Psi(I_p, I_{IDEAL}^-)$ ROW-WISE FOR NEGATIVE IDEAL

jobs → persons ↓	J_1	J_2	J_3
A_1	1.00	0.87	0.91
A_2	0.94	0.97	0.95
A_3	0.86	1.00	0.90

TABLE IV
$\Psi(I_p, I_{IDEAL}^+)$ COLUMN WISE FOR POSITIVE IDEAL

jobs → persons ↓	J_1	J_2	J_3
A_1	0.88	0.91	0.92
A_2	1.00	0.00	0.83
A_3	0.95	0.97	0.93

TABLE V

$\Psi \left(I_p, I_{\overline{IDEAL}} \right)$ COLUMN WISE FOR NEGATIVE IDEAL

jobs → persons ↓	J_1	J_2	J_3
A_1	1.00	0.88	0.89
A_2	0.83	0.89	1.00
A_3	0.93	0.90	0.98

TABLE VI

RELATIVE SIMILARITY MATRIX A (ROW-WISE) FOR JOBS W.R.T. PERSONS

jobs → persons ↓	J_1	J_2	J_3
A_1	0.462	0.524	0.502
A_2	0.483	0.497	0.510
A_3	0.537	0.469	0.516

TABLE VII

RELATIVE SIMILARITY MATRIX B (COLUMN-WISE) FOR JOBS W.R.T. PERSONS

Jobs → Persons ↓	J_1	J_2	J_3
A_1	0.468	0.522	0.497
A_2	0.531	0.513	0.479
A_3	0.516	0.492	0.508

Further, obtain composite relative similarity measure matrix and solve obtained matrix by Hungarian method to obtain the optimal assignments of Interval-valued Intuitionistic Fuzzy assignment problem. Hence, the optimal allocation is given by the following Table VIII:

TABLE VIII

COMPOSITE MATRIX $(A.B)$ REPRESENTING THE COMPOSITE RELATIVE DEGREE

score function	J_1	J_2	J_3
A_1	0.734	0.737	0.756
A_2	0.759	0.756	0.774
A_3	0.747	0.752	0.771

VI. CONCLUSIONS

In this paper, we proposed two algorithms-one based on degree of similarity measures and another based on the score function to get the optimal assignment for the Interval-valued Intuitionistic Fuzzy Assignment Problem. We have also introduced more specified and realistic objective function for assignment problem which has vague information. We utilize the interval-valued intuitionistic fuzzy positive ideal solution and negative ideal solution to compute the coefficient of relative closeness of alternatives and a illustrative example has been taken to show the validity and practicability of the proposed approach.

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