

New stabilization for switched neutral systems with perturbations

Lianglin Xiong, Shouming Zhong, and Mao Ye

Abstract—This paper addresses the stabilization issues for a class of uncertain switched neutral systems with nonlinear perturbations. Based on new classes of piecewise Lyapunov functionals, the stability assumption on all the main operators or the convex combination of coefficient matrices is avoid, and a new switching rule is introduced to stabilize the neutral systems. The switching rule is designed from the solution of the so-called Lyapunov-Metzler linear matrix inequalities. Finally, three simulation examples are given to demonstrate the significant improvements over the existing results.

Keywords—Switched neutral system, piecewise Lyapunov functional, nonlinear perturbation, Lyapunov-Metzler linear matrix inequality.

I. INTRODUCTION

A Switched system is a dynamical system that consists of a finite number of subsystems and a logical rule which orchestrates switching between these subsystems. Such system has gained a great deal of attention mainly because various real-world systems, such as chemical processing [1], communication networks, traffic control [2]-[4], control of manufacturing systems [5]-[6], automotive engine control and aircraft control [7] can be modeled as switched systems. Large number of excellent papers and monographs on the stability of switched systems have been published [8]-[13] in the past.

Recently, considerable attention has been given to the stability problems arising from neutral systems. And various analysis techniques have been utilized to derive asymptotical stability criteria for the systems by many researchers [14]-[19], while the stabilization problem of neutral systems has also been explored by some researchers [20]-[22]. To the best of our knowledge, it seems that few people have studied the stabilization for switched neutral control systems except for [23]-[25]. Unfortunately, all of these articles investigate the stability for neutral systems provided that all the neutral difference operators be stable, or there exist Hurwitz linear convex combinations of state matrices, which may be conservative to some extent on the stability condition. In addition, the novel switching rule was firstly designed from the solution of so-called Lyapunov-Metzler inequalities for switched linear systems with no delay in [26], and it is verified that is less conservative to stabilize the switched linear systems than some existing ones. However, this method has not been introduced

in the switched delay systems, especially in switched neutral systems. All of those have motivated our research.

In this paper, we are interested in the stabilization synthesis for switched neutral systems with nonlinear perturbations. Firstly, this paper addresses a strategy for the stabilization of the nominal switched neutral systems. Combined with a new class of piecewise Lyapunov functionals and the introduced free weighting matrices, a switching rule ensuring asymptotical stability of the switched neutral systems with nonlinear perturbations is designed from the solution of the so-called Lyapunov-Metzler linear matrix inequalities. Then, by extending the approach to the unforced switched neutral system with time-varying uncertainties, the robust stabilization condition of the unforced system with uncertainties is derived. The proposed results in this paper have two advantages: one is the reduction of computation complexity by introducing free-weighting matrices based on the novel piecewise Lyapunov functionals, and the other is that the proposed state switching rule has no stable restriction on all neutral difference operators of the switched neutral systems or stable convex combination of the state matrices. Finally, three simulation examples are given to demonstrate the main results which are less conservative than some existing ones.

A. Problem statement and preliminaries

Nomenclature

R^n n-dimensional real space

$R^{n \times n}$ set of all real n by n matrices

x^T or A^T transpose of vector x (or matrix A)

$P > 0$ (respectively, $P < 0$) matrix P is symmetric positive (respectively, negative) definite

$P \geq 0$ (respectively, $P \leq 0$) matrix P is symmetric positive (respectively, negative) semi-definite

* the elements below the main diagonal of a symmetric block matrix

Consider the following switched uncertain neutral systems:

$$\Xi_0 : \begin{cases} \dot{x}(t) - C_{\sigma(t)}(t)\dot{x}(t-\tau) = A_{\sigma(t)}(t)x(t) \\ \quad + B_{\sigma(t)}(t)x(t-r) + f_{\sigma(t)}(t, x(t)) \\ \quad + g_{\sigma(t)}(t, \dot{x}(t-\tau)) \\ x(t_0 + \theta) = \varphi(\theta), \forall \theta \in [-\rho, 0] \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the state vector, $r, \tau > 0$ are constant time delays, $H = \max\{\tau, r\}$ and $\varphi(\theta)$ is the initial condition function. $\sigma(t) \in M = \{1, 2, \dots, m\}$ is piecewise constant switching signal. This means that the matrices $(A_{\sigma(t)}(t), B_{\sigma(t)}(t), C_{\sigma(t)}(t), f_{\sigma(t)}(t, x(t)),$

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$g_{\sigma(t)}(t, \dot{x}(t - \tau))$ are allowed to take values, at an arbitrary time, in the finite set

$$\begin{aligned} & (A_{\sigma(t)}(t), B_{\sigma(t)}(t), C_{\sigma(t)}(t), f_{\sigma(t)}(t), g_{\sigma(t)}(t)) \\ & \in \{(A_1(t), B_1(t), C_1(t), f_1(t, x(t)), g_1(t, \dot{x}(t - \tau))), \\ & \dots, (A_m(t), B_m(t), C_m(t), f_m(t, x(t)), g_m(t, \dot{x}(t - \tau)))\}. \end{aligned} \quad (2)$$

The system matrices are assumed to be uncertain and satisfy

$$[A_i(t), B_i(t), C_i(t)] = [A_i, B_i, C_i] + DF(t)[E_{ai}, E_{bi}, E_{ci}], \quad (3)$$

where $A_i, B_i, C_i, D, E_{ai}, E_{bi}, E_{ci}$ are constant matrices with appropriate dimensions for $i \in M$, and $F(t)$ is an unknown, real, and possibly time-varying matrix with Lebesgue measurable elements, satisfying

$$F^T(t)F(t) \leq I. \quad (4)$$

The function $f_i(t, x(t))$, and $g_i(t, \dot{x}(t - \tau))$ represent the nonlinear time-varying perturbations. It is assumed that $f_i(t, 0) = 0$ and $g_i(t, 0) = 0$ ($i = 1, 2, \dots, m$), and

$$\|f_i(t, x(t))\| \leq \beta_{i1} \|x(t)\|, \quad (5a)$$

$$\|g_i(t, \dot{x}(t - \tau))\| \leq \beta_{i2} \|\dot{x}(t - \tau)\|, \quad (5b)$$

where $\beta_{i1} \geq 0$, and $\beta_{i2} \geq 0$ are given constants.

Constraint (5) can be rewritten as following:

$$f_i^T(t, x(t)) f_i(t, x(t)) \leq \beta_{i1}^2 x^T(t) x(t), \quad (6a)$$

$$g_i^T(t, \dot{x}(t - \tau)) g_i(t, \dot{x}(t - \tau)) \leq \beta_{i2}^2 \dot{x}^T(t - \tau) \dot{x}(t - \tau). \quad (6b)$$

Where, for simplicity, $f_i := f_i(t, x(t))$, $g_i := g_i(t, \dot{x}(t - \tau))$.

Before presenting the main result, we first state the following lemmas which will be used in the proof of our main result.

Lemma1([27]) Given matrices $Q = Q^T$, H and E of appropriate dimensions, then

$$Q + HFE + E^T F^T H^T < 0$$

for all F satisfying $F^T(t)F(t) \leq I$, if and only if exists an $\varepsilon > 0$ such that

$$Q + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0$$

Lemma2([28]) For given matrices A_{11}, A_{12}, A_{22} with appropriate dimensions,

$$\begin{bmatrix} A_{11} & A_{12} \\ * & A_{22} \end{bmatrix} < 0$$

holds if and only if $A_{22} < 0, A_{11} - A_{12}A_{22}^{-1}A_{12}^T < 0$.

The aim of this paper is to find a new strategy for the stabilization of the switched uncertain neutral systems.

B. Main results

In this section, we consider the system (1) where the switching rule satisfies (2). A new approach for switching rule which is dependent on the state for the switched neutral systems with nonlinear perturbations will be stated as following.

It is assumed that the state vector $x(t)$ is available for feedback for all $t \geq 0$. That is, our goal is to determine the function $\nu(\bullet) : R^n \rightarrow 1, \dots, m$ such that

$$\sigma(t) = \nu(x(t)) \quad (7)$$

which makes the equilibrium point of (1) asymptotically stable. In this case, it is no necessary to assume each matrix of the set A_1, \dots, A_m is asymptotically stable and require the stable convex combination of A_i, B_i , or $A_i + B_i$.

Let us recall the class of Metzler matrices denoted by \mathcal{M} which consists of all matrices $\Pi \in R^{N \times N}$ with elements π_{ij} , such that

$$\pi_{ij} \geq 0, \forall i \neq j, \sum_{i=1}^N \pi_{ij} = 0, \forall j. \quad (8)$$

Firstly, we study the asymptotical stabilization for the nominal switched neutral system as following

$$\Xi_1 : \begin{cases} \dot{x}(t) - C_i \dot{x}(t - \tau) = A_i x(t) + B_i x(t - \tau) \\ \quad + f_i(t, x(t)) + g_i(t, \dot{x}(t - \tau)) \\ x(t_0 + \theta) = \varphi(\theta), \forall \theta \in [-\rho, 0], \end{cases} \quad (9)$$

$f_i(t, x(t))$ and $g_i(t, \dot{x}(t - \tau))$ satisfy condition (5), the following theorem governs the nominal system Ξ_1 .

Theorem 1. Assume that there symmetric positive definite matrices P_1, \dots, P_m, Q, R , any matrices S_{ij} ($i = 1, \dots, m, j = 1, 2, 3, \dots, 6$) with appropriate dimensions and $\Pi \in R^{N \times N}$, scalars $\varepsilon_1 > 0, \varepsilon_2 > 0, \beta_{i1} > 0$ and $\beta_{i2} > 0$ satisfying the Lyapunov-Metzler Linear matrix inequalities

$$\varphi_i = \begin{pmatrix} \varphi_{i11} & \varphi_{i12} & \varphi_{i13} & \varphi_{i14} & \varphi_{i15} & \varphi_{i16} \\ * & \varphi_{i22} & \varphi_{i23} & \varphi_{i24} & \varphi_{i25} & \varphi_{i26} \\ * & * & \varphi_{i33} & \varphi_{i34} & \varphi_{i35} & \varphi_{i36} \\ * & * & * & \varphi_{i44} & \varphi_{i45} & \varphi_{i46} \\ * & * & * & * & \varphi_{i55} & \varphi_{i56} \\ * & * & * & * & * & \varphi_{i66} \end{pmatrix} < 0 \quad (10)$$

where

$$\varphi_{i11} = P_i A_i + A_i^T P_i + Q + S_{i1}^T A_i + A_i^T S_{i1}$$

$$+ \sum_{j=1}^m \pi_{ji} P_j + \varepsilon_1 \beta_{i1}^2 I,$$

$$\varphi_{i12} = -S_{i1}^T + A_i^T S_{i2},$$

$$\varphi_{i13} = P_i B_i + S_{i1}^T B_i + A_i^T S_{i3},$$

$$\varphi_{i14} = P_i C_i + S_{i1}^T C_i + A_i^T S_{i4},$$

$$\varphi_{i15} = P_i + A_i^T S_{i5} + S_{i1}^T,$$

$$\varphi_{i16} = P_i + A_i^T S_{i6} + S_{i1}^T,$$

$$\varphi_{i22} = -S_{i2}^T - S_{i2} + R,$$

$$\varphi_{i23} = S_{i2}^T B_i - S_{i3},$$

$$\varphi_{i24} = S_{i2}^T C_i - S_{i4},$$

$$\begin{aligned}
\varphi_{i25} &= -S_{i5} + S_{i2}^T, \\
\varphi_{i26} &= -S_{i6} + S_{i2}^T, \\
\varphi_{i33} &= S_{i3}^T B_i + B_i^T S_{i3} - Q, \\
\varphi_{i34} &= S_{i3}^T C_i + B_i^T S_{i4}, \\
\varphi_{i35} &= B_i^T S_{i5} + S_{i3}^T, \\
\varphi_{i36} &= B_i^T S_{i6} + S_{i3}^T, \\
\varphi_{i44} &= S_{i4}^T C_i + C_i^T S_{i4} - R + \varepsilon_2 \beta_{i2}^2 I, \\
\varphi_{i45} &= C_i^T S_{i5} + S_{i4}^T, \\
\varphi_{i46} &= C_i^T S_{i6} + S_{i4}^T, \\
\varphi_{i55} &= S_{i5} + S_{i5}^T - \varepsilon_1 I, \\
\varphi_{i56} &= S_{i6} + S_{i5}^T, \\
\varphi_{i55} &= S_{i6} + S_{i6}^T - \varepsilon_2 I.
\end{aligned}$$

The state-switching control (7) with

$$\nu(x(t)) = \arg \min_{i=1, \dots, m} x^T(t) P_i x(t), \quad (11)$$

makes the equilibrium solution of the systems (9) globally asymptotically stable.

Proof. Firstly, from (6), we obtain for any scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$.

$$\varepsilon_1 [\beta_{i1}^2 x^T(t) x(t) - f_i^T f_i] \geq 0, \quad (12a)$$

$$\varepsilon_2 [\beta_{i2}^2 \dot{x}^T(t-\tau) \dot{x}(t-\tau) - g_i^T g_i] \geq 0. \quad (12b)$$

Choose a new class of piecewise Lyapunov-Krasovskii functional candidate for systems (9) as following:

$$\begin{aligned}
V(t) &= \min_{i=1, \dots, m} \{x^T(t) P_i x(t) + \int_{t-r}^t x^T(s) Q x(s) ds \\
&\quad + \int_{t-\tau}^t \dot{x}^T(s) R \dot{x}(s) ds\}, \quad (13)
\end{aligned}$$

P_i , Q and R are defined in theorem1. In fact, $V(t)$ is not differentiable for all $t \geq 0$. To analyze this aspect, the set $I(x) = \{i : \nu(x) = x^T P_i x\}$ plays a central role since $V(t)$ fails to be differentiable on $x \in R^n$ such that $I(x)$ is composed of more than one element or, in other words, when the result of the minimization indicated in (13) is not unique [29]. For this reason we need to deal with the Dini derivative (see[30]):

$$D^+V(x_t) = \lim_{h \rightarrow 0^+} \sup \frac{V(x_{t+h}) - V(x_t)}{h} \quad (14)$$

Assume, in accordance to (11), that at an arbitrary $t \geq 0$, the state-switching control is given by $\sigma(t) = \nu(x(t)) = i$ for some $i \in I(x(t))$. Hence, from (14) and the system (9), applying the result of Theorem1 of [?], we have

$$\begin{aligned}
D^+V(x_t) &= \lim_{h \rightarrow 0^+} \sup \frac{V(x_t + h\dot{x}_t) - V(x_t)}{h} \\
&= \min_{i \in I(x(t))} \{2x^T(t) P_i [A_i x(t) + B_i x(t-r) \\
&\quad + C_i \dot{x}(t-\tau) + f_i + g_i] + x^T(t) Q x(t) \\
&\quad - x^T(t-r) Q x(t-r) + \dot{x}^T(t) R \dot{x}(t) \\
&\quad - \dot{x}^T(t-\tau) R \dot{x}(t-\tau)\} \quad (15)
\end{aligned}$$

On the other hand, from the systems (9), the following equations are true for any appropriate dimensional matrices, $S_{ij} (i = 1, \dots, m, j = 1, \dots, 6)$

$$\begin{aligned}
&2\{x^T(t) S_{i1}^T + \dot{x}^T(t) S_{i2}^T + x^T(t-r) S_{i3}^T + \dot{x}^T(t-\tau) S_{i4}^T \\
&\quad + f_i^T S_{i5}^T + g_i^T S_{i6}^T\} \{A_i x(t) - \dot{x}(t) + B_i x(t-r) \\
&\quad + C_i \dot{x}(t-\tau) + f_i + g_i\} = 0 \quad (16)
\end{aligned}$$

Then, adding the terms on the left sides of (16) to $D^+V(t)$ with (12), the Dini derivative of Lyapunov functional $V(t)$ along the trajectories of systems (9) is obtained as

$$\begin{aligned}
D^+V(x_t) &\leq 2x^T(t) P_i [A_i x(t) + B_i x(t-r) + C_i \dot{x}(t-\tau)] \\
&\quad + x^T(t) Q x(t) - x^T(t-r) Q x(t-r) \\
&\quad + \dot{x}^T(t) R \dot{x}(t) - \dot{x}^T(t-\tau) R \dot{x}(t-\tau) \\
&\quad + 2\{x^T(t) S_{i1}^T + \dot{x}^T(t) S_{i2}^T + x^T(t-r) S_{i3}^T \\
&\quad + \dot{x}^T(t-\tau) S_{i4}^T\} \{A_i x(t) - \dot{x}(t) \\
&\quad + B_i x(t-r) + C_i \dot{x}(t-\tau)\} \\
&= X^T(t) \varphi_i X(t) - \varepsilon_1 [\beta_{i1}^2 x^T(t) x(t) - f_i^T f_i] \\
&\quad - \varepsilon_2 [\beta_{i2}^2 \dot{x}^T(t-\tau) \dot{x}(t-\tau) - g_i^T g_i] \\
&\quad - \left(\sum_{i=1}^m \pi_{ji} \right) x^T(t) P_j x(t)
\end{aligned}$$

where

$$X^T(t) = [x^T(t) \quad \dot{x}^T(t) \quad x^T(t-r) \quad \dot{x}^T(t-\tau) \quad f_i^T \quad g_i^T]$$

and the inequality holds from the fact that $i \in I(x(t))$. Finally, remembering that (8) is valid for $\Pi \in R^{N \times N}$ and that $x^T(t) P_j x(t) \geq x^T(t) P_i x(t)$ for all $j \neq i = 1, \dots, m$. Therefore, due to the fact that $i \in I(x(t))$, using the Lyapunov-Metzler linear matrix inequalities (10), one can further obtain that the the Dini derivative of Lyapunov functional $V(t)$ along systems (9) satisfies

$$\begin{aligned}
D^+V(x_t) &< -x^T(t) \left(\sum_{i=1}^m \pi_{ji} P_j \right) x(t) \\
&< -\left(\sum_{i=1}^m \pi_{ji} \right) x^T(t) P_i x(t) = 0.
\end{aligned}$$

Then it follows from the Lyapunov-Krasovskii stability theorem that if the conditions given in theorem 1 are met, system (9) is guaranteed to be asymptotically stable.

Remark 1. As a matter of fact, the spectrum radius of the matrix C_i are not necessarily less than 1, and the stability hurwitz combination of each A_i and B_i are also unnecessary. It could be stable using the state switching law for the neutral switched systems which can be illustrated in section4. Furthermore, this class of switched neutral systems is a general case of those studied in [23], [24], [25].

Remark 2. To reduce the conservative of the stability condition brought by the second terms in the Lyapunov functional $V(x_t)$ in (13), this theorem introduce some free matrices which can be selected by solving the LMIs in Theorem1. In addition, as related in [26], Theorem1 does not require the set A_1, \dots, A_m be exclusively composed of

asymptotical stable matrices. The Lyapunov-Metzler linear matrix inequalities introduced in Theorem1 suffer the same difficulty in [26], but fortunately a simple numerical procedure based on line search for π_{ij} can be settled to determine its solution.

Let $f_i(t, x(t)) = g_i(t, \dot{x}(t - \tau)) = 0$, similar to the proof of Theorem 1, we have the following corollary for the switched neutral systems as

$$\Xi_2 : \begin{cases} \dot{x}(t) - C_i \dot{x}(t - \tau) = A_i x(t) + B_i x(t - r), \\ x(t_0 + \theta) = \varphi(\theta), \forall \theta \in [-\rho, 0]. \end{cases} \quad (17)$$

Corollary 1. Assume that there exist symmetric positive definite matrices P_1, \dots, P_m, Q, R , any matrices $S_{ij} (i = 1, \dots, m, j = 1, 2, 3, 4)$ with appropriate dimensions and $\Pi \in R^{N \times N}$ satisfying the Lyapunov-Metzler Linear matrix inequalities

$$\phi_i = \begin{pmatrix} \phi_{i11} & \phi_{i12} & \phi_{i13} & \phi_{i14} \\ * & \phi_{i22} & \phi_{i23} & \phi_{i24} \\ * & * & \phi_{i33} & \phi_{i34} \\ * & * & * & \phi_{i44} \end{pmatrix} < 0 \quad (18)$$

where

$$\phi_{i11} = P_i A_i + A_i^T P_i + Q + S_{i1}^T A_i + A_i^T S_{i1} + \sum_{j=1}^M \pi_{ji} P_j,$$

$$\phi_{i12} = -S_{i1}^T + A_i^T S_{i2},$$

$$\phi_{i13} = P_i B_i + S_{i1}^T B_i + A_i^T S_{i3},$$

$$\phi_{i14} = P_i C_i + S_{i1}^T C_i + A_i^T S_{i4},$$

$$\phi_{i22} = -S_{i2}^T - S_{i2} + R,$$

$$\phi_{i23} = S_{i2}^T B_i - S_{i3},$$

$$\phi_{i24} = S_{i2}^T C_i - S_{i4},$$

$$\phi_{i34} = S_{i3}^T C_i + B_i^T S_{i4},$$

$$\phi_{i33} = S_{i3}^T B_i + B_i^T S_{i3} - Q,$$

$$\phi_{i44} = S_{i4}^T C_i + C_i^T S_{i4} - R.$$

The state-switching control (3) with

$$\nu(x(t)) = \arg \min_{i=1, \dots, m} x^T(t) P_i x(t) \quad (19)$$

makes the equilibrium solution of the systems (17) globally asymptotically stable.

Remark 3. Unlike those switching rules designed from some class of LMIs by means of single Lyapunov approach in [23], [25], the switching rules in this paper are just depended on the state of the switched neutral systems. To some extent, it is not difficult to find that the stabilization condition for the switched neutral systems is superior to those in [23], [25].

From Theorem 1, we can obtain a criterion for the switched neutral system Ξ_0 with time-varying structured uncertainties described in (3) and (4).

Theorem 2. Assume that there symmetric positive definite matrices P_1, \dots, P_m, Q, R , any matrices $S_{ij} (i = 1, \dots, m, j = 1, 2, 3, \dots, 6)$ with appropriate dimensions and

$\Pi \in R^{N \times N}$, scalars $\varepsilon_1 > 0, \varepsilon_2 > 0, \lambda_i > 0, \beta_{i1} > 0$ and $\beta_{i2} > 0$ satisfying the Lyapunov-Metzler Linear matrix inequalities

$$\psi_i = \begin{pmatrix} \psi_{i11} & \psi_{i12} & \psi_{i13} & \psi_{i14} & \psi_{i15} & \psi_{i16} & \psi_{i17} \\ * & \psi_{i22} & \psi_{i23} & \psi_{i24} & \psi_{i25} & \psi_{i26} & S_{i2}^T D \\ * & * & \psi_{i33} & \psi_{i34} & \psi_{i35} & \psi_{i36} & S_{i3}^T D \\ * & * & * & \psi_{i44} & \psi_{i45} & \psi_{i46} & S_{i4}^T D \\ * & * & * & * & \psi_{i55} & \psi_{i56} & S_{i5}^T D \\ * & * & * & * & * & \psi_{i66} & S_{i6}^T D \\ * & * & * & * & * & * & -\lambda_i I \end{pmatrix} < 0 \quad (20)$$

where

$$\psi_{i11} = P_i A_i + A_i^T P_i + Q + S_{i1}^T A_i + A_i^T S_{i1}$$

$$+ \sum_{j=1}^m \pi_{ji} P_j + \varepsilon_1 \beta_{i1}^2 I + \lambda_i E_{ai}^T E_{ai},$$

$$\psi_{i12} = -S_{i1}^T + A_i^T S_{i2},$$

$$\psi_{i13} = P_i B_i + S_{i1}^T B_i + A_i^T S_{i3} + \lambda_i E_{ai}^T E_{bi},$$

$$\psi_{i14} = P_i C_i + S_{i1}^T C_i + A_i^T S_{i4} + \lambda_i E_{ai}^T E_{ci},$$

$$\psi_{i15} = P_i + A_i^T S_{i5} + S_{i1}^T,$$

$$\psi_{i16} = P_i + A_i^T S_{i6} + S_{i1}^T,$$

$$\psi_{i17} = P_i D + S_{i1}^T D$$

$$\psi_{i22} = -S_{i2}^T - S_{i2} + R,$$

$$\psi_{i23} = S_{i2}^T B_i - S_{i3},$$

$$\psi_{i24} = S_{i2}^T C_i - S_{i4},$$

$$\psi_{i25} = -S_{i5} + S_{i2}^T,$$

$$\psi_{i26} = -S_{i6} + S_{i2}^T,$$

$$\psi_{i33} = S_{i3}^T B_i + B_i^T S_{i3} - Q + \lambda_i E_{bi}^T E_{bi},$$

$$\psi_{i34} = S_{i3}^T C_i + B_i^T S_{i4} + \lambda_i E_{bi}^T E_{ci},$$

$$\psi_{i35} = B_i^T S_{i5} + S_{i3}^T,$$

$$\psi_{i36} = B_i^T S_{i6} + S_{i3}^T,$$

$$\psi_{i44} = S_{i4}^T C_i + C_i^T S_{i4} - R + \varepsilon_2 \beta_{i2}^2 I + \lambda_i E_{ci}^T E_{ci}.$$

$$\psi_{i45} = C_i^T S_{i5} + S_{i4}^T,$$

$$\psi_{i46} = C_i^T S_{i6} + S_{i4}^T,$$

$$\psi_{i55} = S_{i5} + S_{i5}^T - \varepsilon_1 I,$$

$$\psi_{i56} = S_{i6} + S_{i5}^T,$$

$$\psi_{i66} = S_{i6} + S_{i6}^T - \varepsilon_2 I.$$

The state-switching control (7) with

$$\nu(x(t)) = \arg \min_{i=1, \dots, m} x^T(t) P_i x(t), \quad (21)$$

makes the equilibrium solution of the systems (1) with (3) and (4) robust globally asymptotically stable.

Proof. Replaced A_i, B_i and C_i with the time-varying structured uncertainty form $A_i + DF(t)E_{ai}$, $B_i + DF(t)E_{bi}$, $C_i + DF(t)E_{ci}$ in Theorem1 respectively, (10) are transformed as following

$$\psi_i = \varphi_i + \Gamma_k^T F^T(t) \Gamma_l + \Gamma_l^T F(t) \Gamma_k \quad (22)$$

where

$$\Gamma^T(k) = [E_{ai} \quad 0 \quad E_{bi} \quad E_{ci} \quad 0 \quad 0]^T,$$

$$\Gamma^T(l) = [P_i D + S_{i1}^T D \quad S_{i2}^T D \quad S_{i3}^T D \quad S_{i4}^T D \quad S_{i5}^T D \quad S_{i6}^T D].$$

By lemma1, necessary and sufficient conditions for (22) for Ξ_0 is that there exist $\lambda_i > 0$ such that

$$\psi_i = \varphi_i + \lambda_i \Gamma_k^T F(t) \Gamma_k + \lambda_i^{-1} \Gamma_l^T \Gamma_l \quad (23)$$

Applying lemma2, we find that (23) is equivalent to (20). That is to say, the state-switching control in (21) makes the equilibrium solution of the uncertain switched neutral systems with nonlinear perturbations (1) robust globally asymptotically stable.

C. Simulation examples

In order to show the effectiveness of the conditions presented in Section 3, in this section, three examples are provided.

Example 1. Consider the switched neutral systems (1) with no perturbation which is equivalent to (17), the parameters of the system are specified as follows,

$$A_1 = \begin{pmatrix} -5 & 1 \\ -0.5 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 1 \\ 1.5 & -2 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} -1.6 & 1.4 \\ 0.8 & -1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -1 & 0.6 \\ -1.5 & -1.2 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} -1.2 & 0.4 \\ 0.69 & -1.01 \end{pmatrix}, \quad C_2 = \begin{pmatrix} -0.6 & 0.1 \\ 0.4 & -0.3 \end{pmatrix},$$

$r = 0.81, \tau = 0.47$, with simple computation, we know that $\rho\{C_1\} > 1$, and each subsystem is unstable. However, choosing $\pi_{11} = -29.43, \pi_{12} = 479.60, \pi_{21} = 29.43, \pi_{22} = -479.60$ and by solving LMIs in Corollary 1, we get

$$P_1 = \begin{pmatrix} 4.7804 & -1.3636 \\ -1.3636 & 0.7614 \end{pmatrix} \times 10^3,$$

$$P_2 = \begin{pmatrix} 5.1579 & -1.6114 \\ -1.6114 & 0.6880 \end{pmatrix} \times 10^3,$$

which means the switched neutral systems (17) is asymptotically stable by Corollary 1. Let $(2, -3)^T$ be the initial state of the equation (17). Figs. 1-2 show the state trajectories of the two subsystem with no switching respectively. Figs. 1-2 also show that each of the subsystems is unstable. Fig. 3 shows that the trajectory of the switched systems with the switched law obtained in this paper, while fig. 4 shows that the phase map of the switched neutral systems. Figs. 3-4 also show that the switched neutral systems with unstable subsystems can reach to stability rapidly using the switching rule in Corollary1. Moreover, the criterion obtained in [23], [24], [25] can not be applied since $C_1 \neq C_2$ and $\rho\{C_1\} > 1$. This shows that our criterion is more effective than that obtained in [23], [24], [25].

Example 2. Consider the another switched neutral systems (17) under the same switching law as in Example 1, which was considered in with

$$A_1 = \begin{pmatrix} -5 & 1 \\ -0.5 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 1 \\ 1.5 & -2 \end{pmatrix},$$

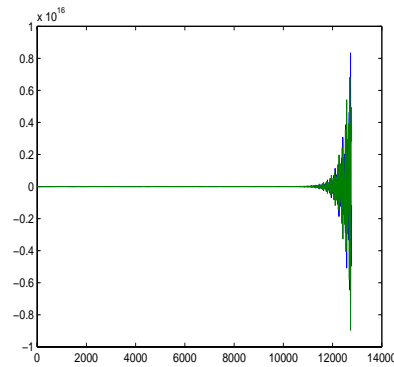


Fig. 1. Behavior of the state component of the switched neutral subsystem1.

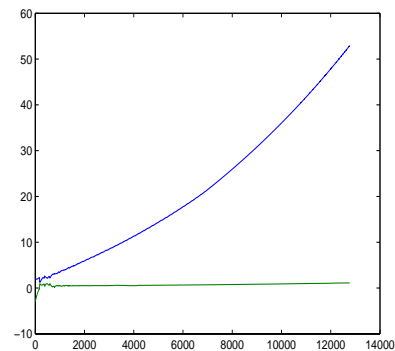


Fig. 2. Behavior of the state component of the switched neutral subsystem2.

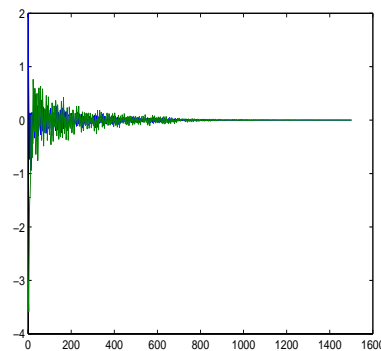


Fig. 3. Behavior of the solution $x(t)$ of the switched neutral systems.

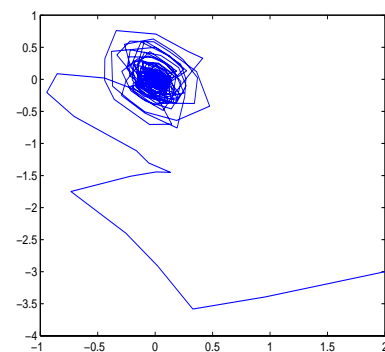


Fig. 4. The phase map of the switched neutral systems.

$$B_1 = \begin{pmatrix} 1.6 & 0.4 \\ 0 & 1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 0.6 \\ 0 & -1.2 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} -0.5 & 0.3 \\ 0.2 & -0.7 \end{pmatrix}, \quad C_2 = \begin{pmatrix} -0.5 & 0.3 \\ 0.2 & -0.7 \end{pmatrix},$$

$r = 0.81, \tau = 0.47$. It is easy to see that, neither $B(i)$ nor $A(i) + B(i)$ have no stable convex combination. However, choosing $\pi_{11} = -12.43, \pi_{12} = 145.52, \pi_{21} = 12.43, \pi_{22} = -145.52$ and by solving LMIs in Corollary 1, we obtain

$$P_1 = \begin{pmatrix} 3.7745 & -0.6240 \\ -0.6240 & 0.2717 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} 4.1190 & -0.7526 \\ -0.7526 & 0.2043 \end{pmatrix},$$

which imply the asymptotical stability of the switched neutral systems (17). However, used the single Lyapunov approach, the criterion obtained in [23], [24], [25] failed because of its no stable convex combination of both $B(i)$ and $A(i) + B(i)$. It also show that our criterion is less conservative than that obtained in [23], [24], [25]. Similar to example 1, its initial state is set as $(2, -3)^T$, and these results are showed in Figs. 5-8. As can be seen from these figures, the switched neutral systems with unstable subsystems converges to the origin very quickly after several switchings.

Example 3. Consider the switched neutral systems with nonlinear perturbations (1), with

$$A_1 = \begin{pmatrix} -5 & 1 \\ -0.5 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 1 \\ 1.5 & -2 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} -1.6 & 1.4 \\ 0.8 & -1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -1 & 0.6 \\ -1.5 & -1.2 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} -0.5 & 0 \\ 0 & -0.1 \end{pmatrix}, \quad C_2 = \begin{pmatrix} -0.5 & 0 \\ 0 & -0.1 \end{pmatrix},$$

$$f_1(x(t)) = (0.07 \sin(2x_1(t)), 0.07 \cos(-3x_2(t)))$$

$$f_2(x(t)) = (-0.06 \cos(5x_1(t)), 0.07 \sin(-6x_2(t)))$$

$$g_1(\dot{x}(t-\tau)) = (-0.03 \cos(5\dot{x}_1(t-\tau)), 0.04 \sin(-6\dot{x}_2(t-\tau)))$$

$$g_2(\dot{x}(t-\tau)) = (-0.05 \cos(7\dot{x}_1(t-\tau)), 0.03 \sin(-2\dot{x}_2(t-\tau)))$$

$r = 0.79, \tau = 0.46$. Obviously, the perturbations f_i and g_i satisfy the condition (5). Choosing $\pi_{11} = -804.43, \pi_{12} = 2950.57, \pi_{21} = 804.43, \pi_{22} = -2950.57$ and by solving LMIs in Theorem 1,

$$P_1 = \begin{pmatrix} 0.3217 & -0.0944 \\ -0.0944 & 0.0438 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} 0.3232 & -0.0947 \\ -0.0947 & 0.0436 \end{pmatrix},$$

are presented, which means that the asymptotical stability of the switched neutral systems (1) is obtained using the switching law in theorem 1. Similar to example 1 and 2, set the initial state as $(2, -3)^T$, Figs. 9-12 illustrate these results respectively. From these figures, one can also see that the switching rule is effective to stabilize the neutral systems with unstable subsystems. Actually, it extend the criterion to some extent in [23], [24], [25].

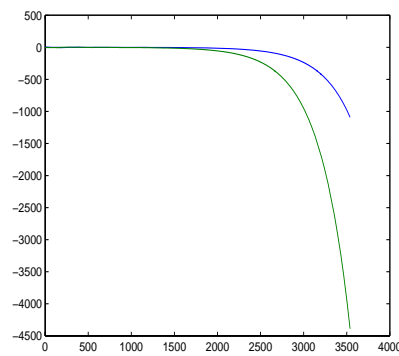


Fig. 5. Behavior of the state component of the switched neutral subsystem1.

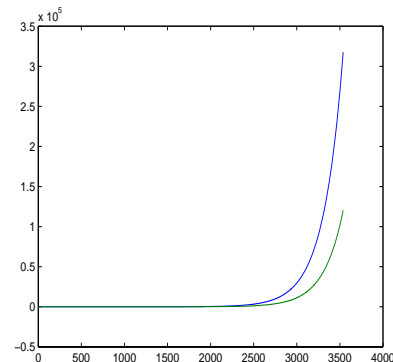


Fig. 6. Behavior of the state component of the switched neutral subsystem2.

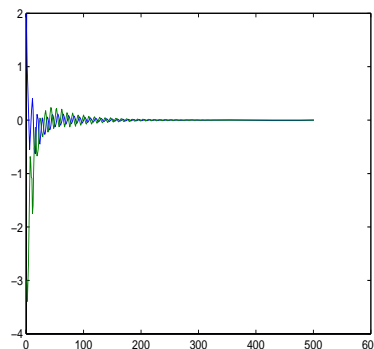


Fig. 7. Behavior of the solution $x(t)$ of the switched neutral systems.

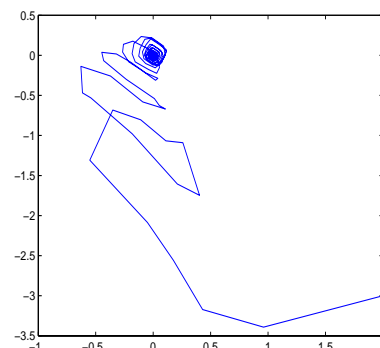


Fig. 8. The phase map of the switched neutral systems.

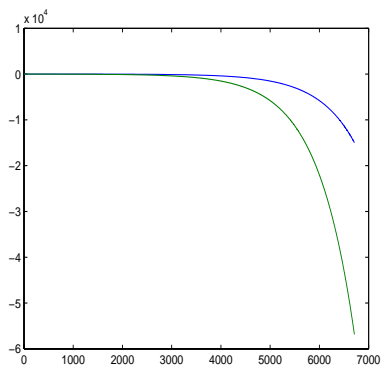


Fig. 9. Behavior of the state component of the switched neutral subsystem1.

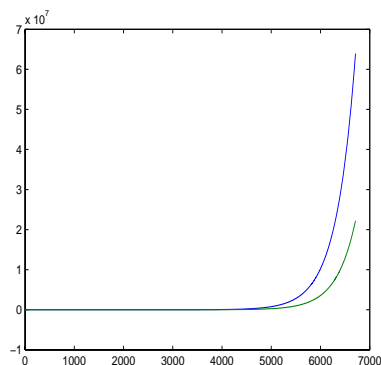


Fig. 10. Behavior of the state component of the switched neutral subsystem2.

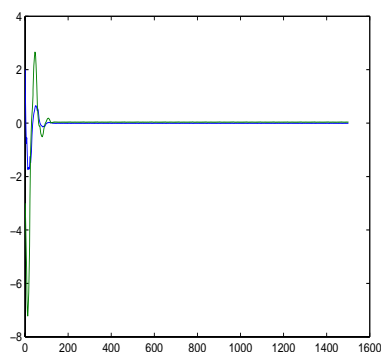
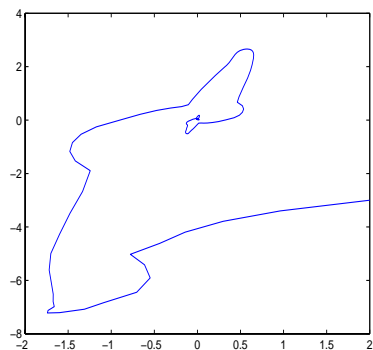
Fig. 11. Behavior of the solution $x(t)$ of the switched neutral systems.

Fig. 12. The phase map of the switched neutral systems.

II. CONCLUSION

Some new switching rules for stabilization of a class of uncertain switched neutral systems with nonlinear perturbation are achieved in this paper. By employing piecewise Lyapunov functionals which are more appropriate for neutral switched systems, more flexible state dependent switching rules for stabilization are given in terms of the so-called Lyapunov-Metzler LMIs. Simulation examples are given to demonstrate the main results.

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REFERENCES

- [1] S. Engell, S. Kowalewski, C. Schulz, O. Strusberg, Continuous-discrete interactions in chemical processing plants, *Proceedings of IEEE*, Vol. 88, No. 7, pp. 1050-1068, 2000.
- [2] R. Horowitz, P. Varaiya, Control design of an automated highway system, *Proceedings of IEEE*, Vol. 88, No. 7, pp.913-925, 2000.
- [3] C. Livadas, J. Lygeros, N.-A. Lynch, High-level modeling and analysis of the traffic alert and collision avoidance system, *Proceedings of IEEE*, Vol. 88, No. 7, pp.926-948, 2000.
- [4] P. Varaiya, Smart cars on smart roads: Problems of control, *IEEE Transactions on Automatic Control*, Vol. 38, No. 2, pp.195-207, 1993.
- [5] D. Pepyne, C. Cassandaras, Optimal control of hybrid systems in manufacturing, *Proceedings of IEEE*, Vol. 88, No. 7, pp.1008-1122, 2000.
- [6] M. Song, T. Tran, N. Xi, Integration of task scheduling, action planning, and control in robotic manufacturing systems, *Proceedings of IEEE*, Vol. 88, No. 7, pp.1097-1107, 2000.
- [7] P. Antsaklis, Special issue on hybrid systems: Theory and applications-A brief introduction to the theory and applications of hybrid systems, *Proceedings of IEEE*, Vol. 88, No. 7, pp.887-897, 2000.
- [8] D. Liberzon, A.-S. Morse, Basic problems in stability and design of switched systems, *IEEE Control Systems Magazine*, Vol. 19, No. 5, pp.59-70, 1999.
- [9] R. DeCarlo, M.-S. Branicky, S. Pettersson, B. Lennartson, Perspectives and results on the stability and stabilizability of hybrid systems, *Proceedings of IEEE*, Vol. 88, No. 7, pp.1069-1082, 2000.
- [10] Z. Sun, S.-S. Ge, Analysis and synthesis of switched linear control systems, *Automatica*, Vol. 41, No. 2, pp.181-195, 2005.
- [11] X.-M. Sun, J. Zhao, D.-J. Hill, Stability and L_2 -gain analysis for switched delay systems: A delay-dependent method, *Automatica*, Vol. 42, No. 10, pp.1769-1774, 2006.
- [12] D. Liberzon, *Switching in systems and control*, Boston: Birkhauser, 2003.
- [13] G.-S. Zhai, X.-P. Xu, H. Lin, A.-N. Michel, Analysis and design of switched normal systems, *Nonlinear Analysis: Theory, Methods & Applications*, Vol. 65, No. 12, pp.2248-2259, 2006.
- [14] W.-H. Chen, W.-X. Zheng, Delay-dependent robust stabilization for uncertain neutral systems with distributed delays, *Automatica*, Vol. 43, No. 1, pp.95-104, 2007.
- [15] Q.-L. Han, A descriptor system approach to robust stability of uncertain neutral systems with discrete and distributed delays, *Automatica*, Vol. 40, No. 10, pp.1791-1796, 2004.
- [16] J.-H. Park, Stability criterion for neutral differential systems with mixed multiple time-varying delay arguments, *Mathematics and Computers in Simulation*, Vol. 59, No. 5, pp.401-412, 2002.
- [17] H. Li, H.-B. Li, S.-M. Zhong, Some new simple stability criteria of linear neutral systems with a single delay, *Journal of Computational and Applied Mathematics*, Vol. 200, No.1, pp.441-447, 2007.

- [18] L.-L. Xiong, S.-M. Zhong, J.-K. Tian, Novel robust stability criteria of uncertain neutral systems with discrete and distributed delays, *Chaos, Solitons & Fractals*, Vol. 40, No.2, pp.771-777, 2009.
- [19] J.-W. Cao, S.-M. Zhong, Y.-Y. Hu, Global stability analysis for a class of neural networks with varying delays and control input, *Applied Mathematics and Computation*, Vol. 189, No. 2, pp.1480-1490, 2007.
- [20] S.-Y. Xu, P. Shi, Y.-M. Chu, Y. Zou, Robust stochastic stabilization and H_∞ control of uncertain neutral stochastic time-delay systems, *Journal of Mathematical Analysis and Applications*, Vol. 314, No. 1, pp.1-16, 2006.
- [21] C. Bonnet, J.-R. Partington, Stabilization of some fractional delay systems of neutral type. *Automatica*, Vol. 43, No. 12, pp. 2047 -2053, 2007.
- [22] J.-H. Park, O. Kwon, On robust stabilization for neutral delay-differential systems with parametric uncertainties and its application, *Applied Mathematics and Computation*, Vol. 162, No. 3, pp.1167-1182, 2005.
- [23] X.-M. Sun, J. Fu, H.-F. Sun, J. Zhao, Stability of linear switched neutral delay systems, *Proceedings of The Chinese Society for Electrical Engineering*, Vol. 25, No. 23, pp.42-46, 2005.
- [24] Y.-P. Zhang, X.-Z. Liu, H. Zhu, Stability analysis and control synthesis for a class of switched neutral systems, *Applied Mathematics and Computation*, Vol. 190, No. 2, pp.1258-1266, 2007.
- [25] D.-Y. Liu, X.-Z. Liu, S.-M. Zhong, Delay-dependent robust stability and control synthesis for uncertain switched neutral systems with mixed delays, *Applied Mathematics and Computation*, Vol. 202, No. 2, pp.828-839, 2008.
- [26] J.-C. Geromel, P. Colaneri, Stability and stabilization of continuous-time switched linear systems, *SIAM Journal on Control and Optimization* Vol. 45, No. 5, pp. 1915-1930, 2006.
- [27] L. Xie, Output feedback control of systems with parameter uncertainty, *International Journal of Control*, Vol. 63, No. 4, pp.741-750, 1996.
- [28] S. Boyd, L.-E. Ghaoui, E. Feron, V. Balakrishnan, *Linear matrix inequalities in systems and control theory*. Philadelphia: SIAM, 1994.
- [29] R. Rockafellar, *Convex Analysis*, Princeton: Princeton Press, 1970.
- [30] K.-M. Garg, *Theory of Differentiation: A Unified Theory of differentiation via new derivate theorems and new derivatives*, Wiley-Interscience, New York, 1998.

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