Piezoelectric Transducer Modeling: with System Identification (SI) Method

Nora Taghavi, and Ali Sadr

Abstract—System identification is the process of creating models of dynamic process from input- output signals. The aim of system identification can be identified as " to find a model with adjustable parameters and then to adjust them so that the predicted output matches the measured output". This paper presents a method of modeling and simulating with system identification to achieve the maximum fitness for transformation function. First by using optimized KLM equivalent circuit for PVDF piezoelectric transducer and assuming different inputs including: sinuside, step and sum of sinusides, get the outputs, then by using system identification toolbox in MATLAB, we estimate the transformation function from inputs and outputs resulted in last program. Then compare the fitness of transformation function resulted from using ARX,OE(Output-Error) and BJ(Box-Jenkins) models in system identification toolbox and primary transformation function form KLM equivalent circuit.

Keywords—PVDF modeling, ARX, BJ(Box-Jenkins), OE(Output-Error), System Identification.

I. INTRODUCTION

SYSTEM identification is the art and methodology of making matemathical models from dynamic systems base on the Input/Output data.The emphasize is on the discrete-time system models which their data are sampled.Base of the system identification are the standard basic and satatistic technics including well known Least Mean Square method, Recursive Least Square, Batch Least Square, Linear methods such as BJ, ARMAX,ARX,OE and Non-Linear methods such as black-box model, gray-box model, physical model and dynamical methods [1]. The common usage of piezoelectric materials is in microphones, oscillators, ultrasound transducers, transducers.

II. SYSTEM IDENTIFICATION

Now days we use model based controller design. The first step in designing the controller is to model the plant and controller. System identification is the process of creating models of dynamic process from input-output signals. The aim of system identification can be identified as " to find a model with adjustable parameters and then to adjust them so

that the predicted output matches the measured output". Two important questions are:

- Which model parameterization is to be used
- How to know if the fitted model is good

Most of SID techniques have their roots in statistical methods like Least square fitting, maximum likelihood estimation etc.

It should be noted that there are non-parametric techniques for system identification like spectral analysis, correction analysis and transient analysis.

The various steps in system identification are:

1. Experiment setup and data collection
2. Data preprocessing
3. Model structure selection
4. Parametric estimation
5. Validation

There are generally 2 methods:

- First principles modeling
- System identification

In first principles modeling the mathematical model of the system identification is constructed from knowledge of system working. The parameters of the mathematical model are determined by iterative process.

In situations where system is a black box we go for system identification procedures where we will use various model structures before selecting one that gives optimum fit for test data.

Nora Taghavi is with Iran University of Science and Technology (IUST), Narmak, Tehran 16844, Iran (phone: +98(21)22802184; e-mail: nrtaghavi@gmail.com).

Ali Sadr is with Iran University of Science and Technology (IUST), Narmak, Tehran 16844, Iran (phone: +98(21)73912614; e-mail: sadr@iust.ac.ir).
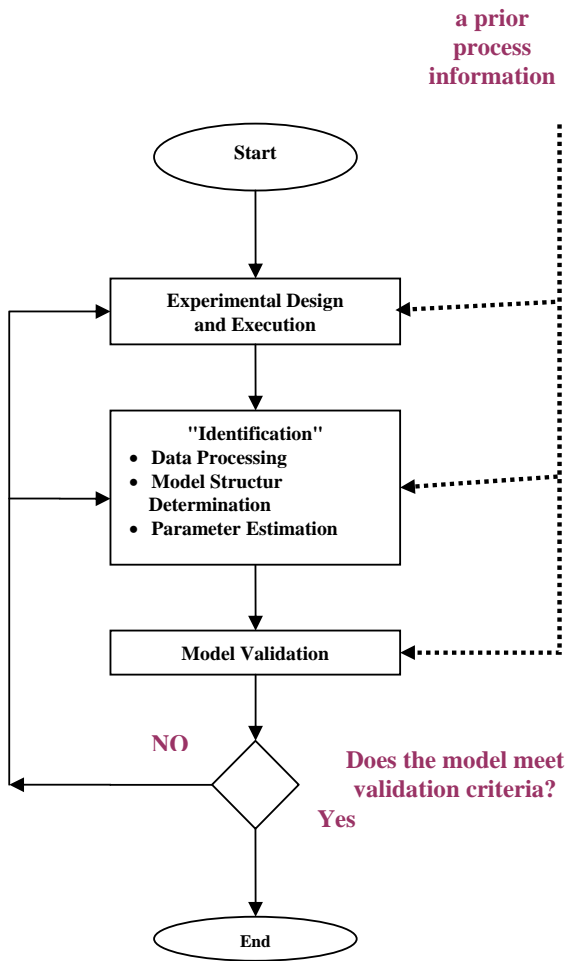
A. Stages of System Identification

Fig. 1 Stages of system identification
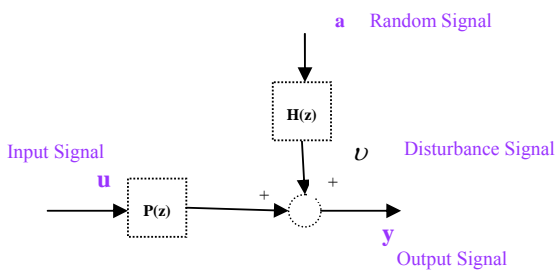
B. System Identification Structure

Fig. 2 System identification structure.P(z) and H(z) are discrete-time (z-domain) transfer functions

B. SID Experiment Set up

System identification is done using the input output test data. The sole information is test data from which all parameters of system transfer function is to be identified. The test data must incorporate all the properties of the system. So the way system identification experiment is performed is very crucial. Following points must be noted:

The input signal must expose all relevant properties of the system. Suppose it is pure sinoside of a particular frequency. It will give information of system frequency response at the frequency. The input signal must have as many input frequencies as the order of the system. Following inputs can be considered:

- Step/Pulse Inputs
- Gaussian White noise
- Random Binary Signal (RBS)
- Pseudo-Random Binary Signal (PRBS)
- Multi-level Pseudo-Random Signals
- Multisine Inputs

A variety of model structures are available to assist in modeling a system. The choice of model structure is based upon an understanding of the system identification method and insight and understanding into the system undergoing identification [5,6]. Even then it is often beneficial to test a number of structures to determine the best one. Model types are as following:

- Parametric Model Structures
- AR Model
- ARX Model
- ARMAX Model
- Box-Jenkins Model
- Output-Error Model
- State-Space Model

Here we assume ARX, BJ, OE models, and describe them as following:

1. Parametric Model Structures

Parametric models describe systems in terms of different equations and transfer functions. A general-linear polynomial model or the general-linear model is as following.

$$y(n) = q^{-k}G(q^{-1},\theta)u(n) + H(q^{-1},\theta)e(n)$$

u(n) and y(n) are the input and output of the system respectively.

e(n) is zero-mean white noise, or the disturbance of the system.

$G(q^{-1},\theta)$ is the transfer function of the deterministic part of the system.

$H(q^{-1},\theta)$ is the transfer function of the stochastic part of the system.

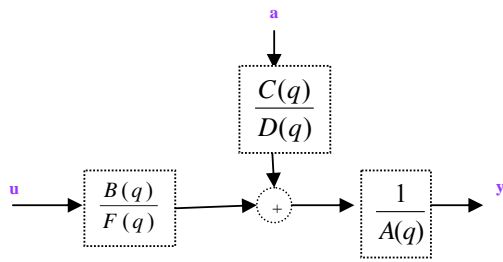The general-linear model structure is shown in Fig. 3.

Fig. 3 General-Linear Model Structure

Simpler models that are a subset of the General Linear model structure are possible. By setting one or more of A(q), B(q), C(q) or D(q) polynomials equal to 1, you can create these simpler models such as ARX, Box-Jenkins, and output-error structures[7].

2. ARX Model

The ARX model, shown in Fig. 4, is the simplest model incorporating the stimulus signal. The estimation of the ARX model is the most efficient of the polynomial estimation methods because it is the result of solving linear regression in analytic form. Moreover, the solution is unique. In other words, the solution always satisfies the global minimum of the loss function. The ARX model therefore is preferable, especially when the model order is high. The disadvantage of the ARX model is that disturbances are part of the system dynamics.
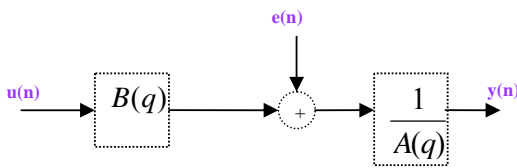

Fig. 4 ARX Model Structure

3. Box-Jenkins Model

The Box-Jenkins (BJ) structure provides a complete model with disturbance properties modeled separately from system dynamics.
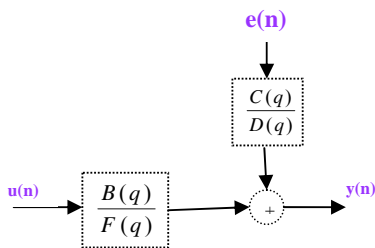

Fig. 5 Box-Jenkins Model Structure

4. Output-Error Model

The Output-Error (OE) model structure describes the system dynamics separately. No parameters are used for modeling the disturbance characteristics [4,8].
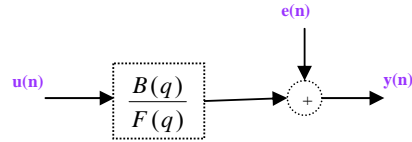

Fig. 6 OE Model Structure

III. PIEZOELECTRIC MATERIAL

There are two classes of piezoelectric materials used in vibration control, ceramics and polymers. The best well-known piezoceramic is Lead Zirconate Titanate, widely used as an actuator and sensor including ultrasound applications that is good for high-accuracy. Generally piezopolymers used as a sensors which the best well-known is Lead Lantanum Zirconate Titanate (PVDF).

There are several ways to show and analysis the piezoelectric transducers equivalent models, we choose KLM optimized equivalent circuit to simulate and modeling the transducer.Thus, here is a brief description of theorical equation of it. Usually it's hard to design transducers with wide-boundary operation and also predict their operation without referring to numerical calculation. But it's possible to some physical aspects of this type of transducers by using another equivalent circuit that is well-known as a KLM equivalent circuit.

Here, we assume the piezoelectric as amaterial that can emit bi-axes wave. KLM equivalent circuit is shown below. These modes are sontinousely simulated by the current ($j\omega DA$) in the transducer[2,3].
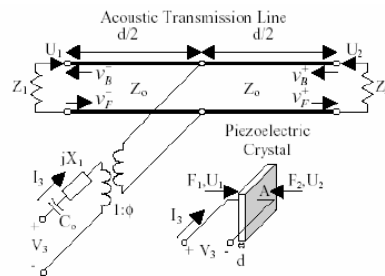

Fig. 7 KLM equivalent circuit

A. Related Equations

In this model, V_3 and I_3 are sequentially voltage and current that are used to piezoelectric to produce Acoustic forces F and Paticle velocities V in two-side piezo. Model parameters are Transducer thickness l, piezo area A, Acoustic Transmission line Impedance Zc. Z_1 and Z_2 Impedances, environment

Radiation Impedances are in bi-axial crystal. Finally value of circuit elements is calculated as following:

$$C_0 = \frac{\varepsilon^s A}{l} \qquad (1)$$

$$X_0 = -\frac{C_0}{k_T^2} \frac{1}{\sin(\omega/\omega_0)} \qquad (2)$$

$$\varphi = k_T (\pi/\omega_0 C_0 Z_c)^{1/2} \sin(\omega/2\omega_0) \qquad (3)$$

$$k_T = \sqrt{\frac{e_{33}^2}{c_D \varepsilon^s}} \qquad (4)$$

In above equation, piezoelectric dielectric constant ε^s, electromechanical coupling constant k_T, half of transducer frequency wave length (equall with $\pi c/T$), stiffened elastic constant C^D, piezoelectric force constant e_{33} is in thickness direction.

By using electrical impedances and an impedance caused by acoustic load that is known as radiation impedance Z_a, total electrical impedance Z_{in} that is seen in electrical input is calculated from below equation:

$$Z_{in} = \frac{1}{j\omega C_0} + \frac{1}{j\omega X_0} + Z_a \qquad (5)$$

where,

$$Z_a = \varphi^2 Z_{sh} \qquad (6)$$

Z_a is radiation impedance that is converted shant impedance and calculated as following:

$$Z_a = \frac{Z_{L1} Z_{L2}}{Z_{L1} + Z_{L2}} \qquad (7)$$

KLM model successfully can be used for transducers that are made from piezo electric elements with small internal losses for example: quartz and piezoceramic material such as PZT.
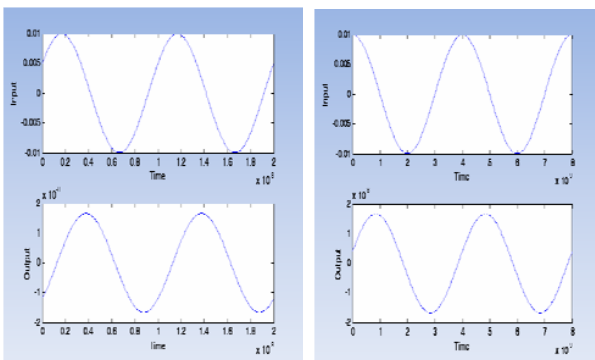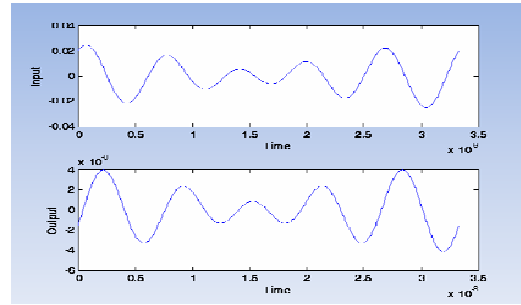

Fig. 8 Outputs for sinuside inputs
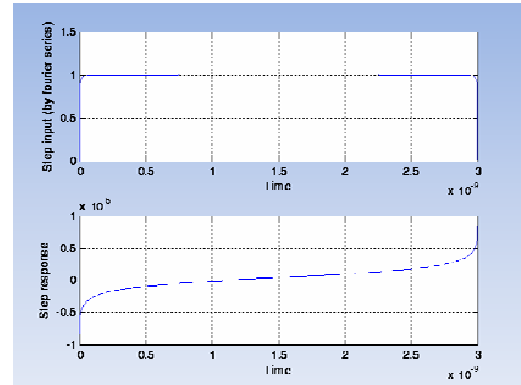

Fig. 9 Output for multi-sinuside input

Fig. 10 Output for step input

IV. FIND OUT TRANSMISSION FUNCTION USING SYSTEM IDENTIFICATION

Here we find out the transmission function with programming in MATLAB and using ident to work with Geraphical User Interface (GUI) and simultantly compare the result with the transmission fuction of the KLM optimized circuit and find out which model can give us the maximum fitness[9].

- For the single sinuside inputs we change the parameters such as frequency, phase and amplitude and found out whit assuming phase and amplitude as constant parameters and changenig the frequency we don't have describable change on the result.

Examples:

1. Assume that Ap=10e-3, Ph0=pi/6 are constant parameters and we are changing the frequency from f=500e6 to f=700e6, thus Fig. 11 illustrates the results.
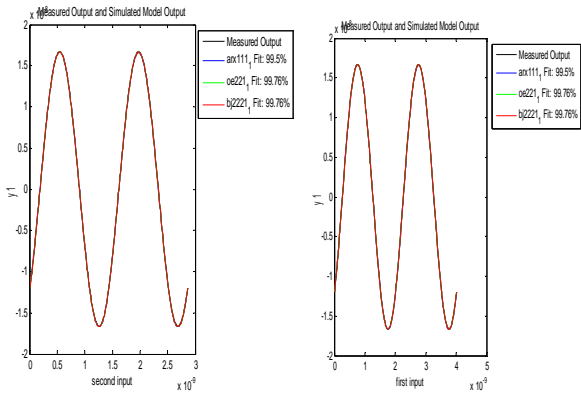
Fig. 11 Changing the frequency from f=500e6 to f=700e6

2. With assuming amplitude (Ap=10e-3) and frequency (f=500e6) as constant parameters and changing phase from Ph0=pi/6 to Ph0=pi, we have:

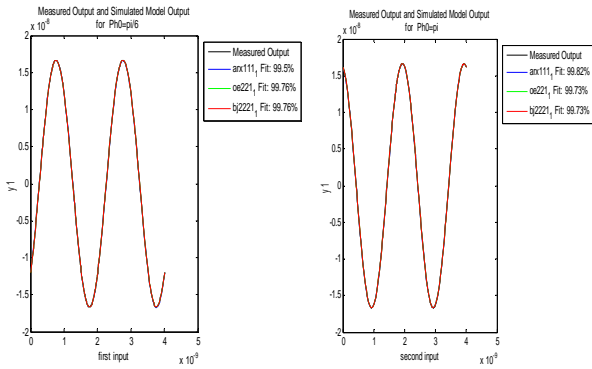Fig. 12 Changing the phase from Ph0=pi/6 to Ph0=pi

3. With assuming frequency (f=500e6) and phase (Ph0=pi/6) as constant parameters and changing amplitude from Ap=10e-3 to Ap=100e-3 and to Ap=20e-3, we don't have any change in results:
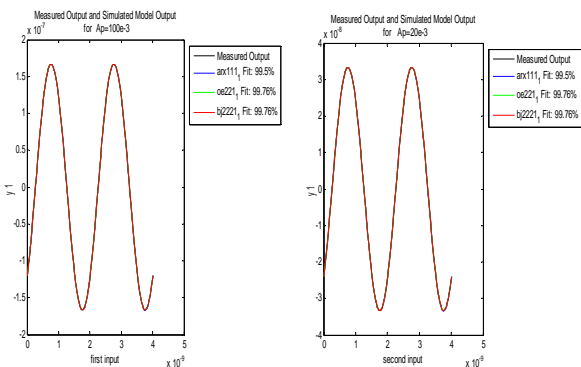

Fig. 13 Changing the amplitude from Ap=10e-3 to Ap=100e-3(left) and to Ap=20e-3(right)

• For the **multi-sinuside** input the results are so different in comparison with the single inputs as with assuming amplitude and as constant parameters and decreasing the phase , the fitness will increase, but increasing the phase for some phases the fitness is decreased   and for some others, the fitness is increased as following:
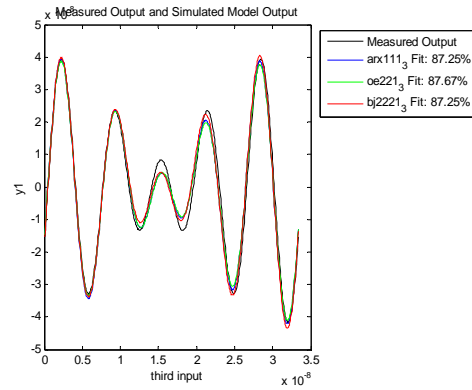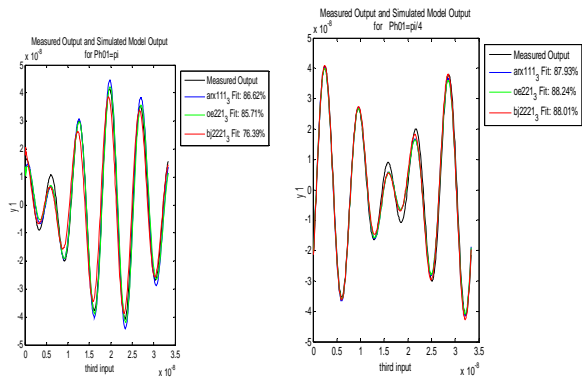

Fig. 14 The original result

Fig. 15 Decreasing the phase from Ph0=pi/3 to Ph0=pi/6(left), to Ph0=pi/4(right)

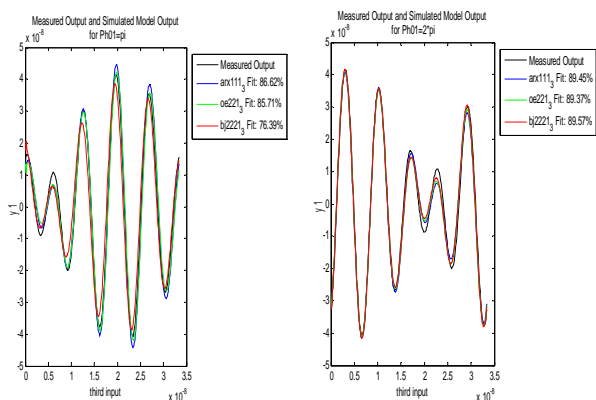
Fig. 16 Increasing the phase from Ph0=pi/3 to Ph0=pi(left) result in fitness decrease, to Ph0=2*pi (right) result in fitness increase

From above result we find the ph01=pi/6 as best choice. Now we do the same work for the ph02:

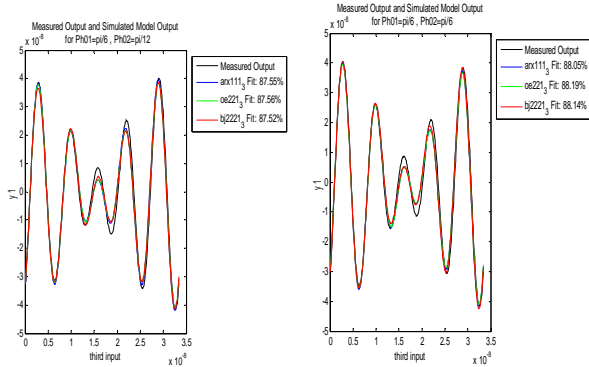Fig. 17 Changing the phase from Ph02=pi/4 to Ph0=pi/12(left), to Ph0=pi/6 (right)

Hence, choosing ph01=pi/6 and ph02=pi/4result in the best fitness for the multi-sinuside input. Coressponding that theKLM tarnsmission function is closed to the OE model, it provides the best fitness compare to the other models.

- For the step input changing the parameters don't have any affect on the result, thus we use the ident in MATLAB and attempp to find the maximum fitness by changing the coefficients orders of different system models(ARX,BJ,OE) polymonials.

TABLE I
COMPARISON OF FITNESS PERCENTAGE WITH THE BEST ORDER

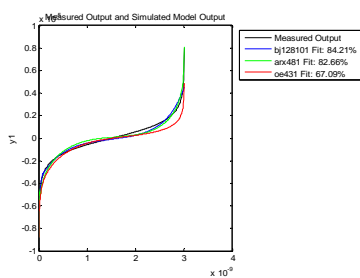| Model Type	ARX	BJ	Units OE
order	4 8 1	1 2 8 10 1	4 3 1
fitness	% 82/66	%84/21	%67/09

Fig. 18 The otput of step input with the best order

V. CONCLUSION

As the result show, it's easy to understand that the OE model is choosed as a best model with achieving %99.7 fitness for the single sinuside input and % 88.71 for the multi-sinuside input, we mentioned earlier that the OE model is supposed to be the best model as its transmission function is closed to the KLM transmission function. Hence we expect it would be best model for every input such as step and it is, but in comparison with the other models here, results shows that the BJ model gives us the higher fitness, so the BJ model is choosed as a best model with %84.21for step input.

REFERENCES

[1] (Flynn and Sanders) Flynn, A.M.; Sanders, S.R. 2002." Fundamental limits on energy transfer and circuit considerations for piezoelectric transformers". IEEE Transactions on Power Electronics, vol.17, (no.1), IEEE, Jan. 2002. p.-14.
[2] (Ikeda1990)Ikeda.T "Fundamentals of piezoelectricity", Oxford University Press.
[3] R.L.Lin, F. C. Lee, and E. Baker, " Characterization of piezoelectric transformers" in Proc, IEEE VPEC, 2000, pp. 219-225.
[4] S. Nitin, "Introduction of System Identification",DOI= http://www.Tataelxsi.co.in
[5] Lennart Ljung , "Nonlinear Black-Box Modeling in System Identification". Dept of Electrical Engineering , Linkoping University, Sweden.2001.
[6] R. Haber and L. Keviczky , " Nonlinear system identification – input-output modeling approach, Vol 1 and 2. Kluwer Academic Publisher (2004) 901-903.
[7] J.S. Lew, J.N.Juang and R.W. Longman. "Comparison of Several System Identification Methods for Flexıble Structures", NASA Technical Memorandum, March 1991.
[8] A. Juditsku, H. Hjalmarsson, A. Benveniste, B. Deylon,L. Ljung, J.Sjoberg, and Q.Zhang." Nonlinear black-box modeling in system identification: Mathematical foundations," Automatics, 31, 1995.
[9] MATLAB Toolbox