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A study on exclusive breastfeeding using over-dispersed statistical models

Naushad Mamode Khan, Cheika Jahangeer, and Maleika Heenaye-Mamode Khan

Abstract—Breastfeeding is an important concept in the maternal life of a woman. In this paper, we focus on exclusive breastfeeding. Exclusive breastfeeding is the feeding of a baby on no other milk apart from breast milk. This type of breastfeeding is very important during the first six months because it supports optimal growth and development during infancy and reduces the risk of obliterating diseases and problems. Moreover, in Mauritius, exclusive breastfeeding has decreased the incidence and/or severity of diarrhea, lower respiratory infection and urinary tract infection. In this paper, we give an overview of exclusive breastfeeding in Mauritius and the factors influencing it. We further analyze the local practices of exclusive breastfeeding using the Generalized Poisson regression model and the negative-binomial model since the data are over-dispersed.

Keywords—Exclusive breastfeeding, Regression model, Generalized Poisson, Negative binomial.

I. Introduction

Breast milk is the most ideal food for an infant and if adequately supplied, it should meet most of the nutritional requirements. In fact, the recommended practice by WHO [4] and AAP [7] is exclusive breastfeeding for the first six months of life followed by nutritionally adequate and safe complementary foods with continued breastfeeding up to two years of age or beyond. Breastfeeding of infants provides advantages with regards to general health, growth, and development, while significantly decreasing the risk for a large number of acute and chronic diseases such as respiratory infection, bacterial meningitis and botulism. Other studies have also shown possible protective effect of human milk feeding against sudden infant death syndrome, insulin dependant diabetes mellitus, Crohn's disease, ulcerative colitis, lymphoma, allergic diseases and chronic digestive diseases [1]. Moreover, exclusive breastfeeding also improves the motor and language skills as compared to infants who have not been breastfed [8]. Modernization and the fast changing evolution have led to a decrease in both the incidence and duration of exclusive breastfeeding [14].

Mauritius being a small island is also affected by this issue where factors such as maternal age, employment, length of maternity leave, place of antenatal treatment, information obtained on breastfeeding, type of delivery and place of delivery have accounted for a decrease in the incidence of exclusive breastfeeding. Moreover, a decline in exclusive breastfeeding pattern has been noticed since 2002 amongst mauritian mothers,i.e, the prevalence of exclusive breastfeeding at 4 months was 34.2 percent in 2002 [6]. In this paper, we use the Generalized Poisson regression model(GPR) [2], [3] and the negative-binomial regression models (NB) to analyze the practices of exclusive breastfeeding based on a random subset of data collected from a survey on breastfeeding in Mauritius over the period 2007-2009. The organization of the paper is as follows: In section 2, we describe the factors influencing exclusive breastfeeding in Mauritius since 2006. We develop the GPR and NB models in section 3 and 4 respectively. Estimation of parameters via the quasi-likelihood estimation technique is presented in section 5. In section 6, we analyze the exclusive breastfeeding data and present the results and conclusion.

Naushad Mamode Khan is a PhD student in the Department of Mathematics, Faculty of Science, University of Mauritius(e-mail: alimamode@yahoo.co.uk)
Maleika Heenaye-Mamode Khan is a lecturer in the Department of Computer Science and Engineering, University of Mauritius (e-mail: m.mamodekhan@uom.ac.mu)
Cheika Jahangeer holds an MSc with Distinction in Health Sciences (dietetics)(e-mail: cheika1108@hotmail.com)

II. Factors influencing exclusive breastfeeding

Maternal age has been considered as a factor that can adversely affect breastfeeding rates among mothers. Lower maternal age has been considered as factors that can adversely affect breastfeeding rates [14]. Employment, maternity leave and the length of maternity leave are very influential on the incidence of exclusive breastfeeding and thus affect mother's choice of feeding practice. Despite the fact that the working mothers may be aware of the benefits of breastfeeding, many of them are rather reluctant to practice exclusive breastfeeding as compared to unemployed mothers. In Mauritius, according to the 2003 report from the Pay Research Bureau, only 12 weeks of maternity leaves are granted to public officers for 3 confinements only [10]. However, working outside the home and being a full-time worker is related to shorter duration of breastfeeding. Other studies have also reported that one of the most important reasons for mothers to stop breastfeeding at 6 months or earlier was ”returning to work” [5]. Several studies have shown that information on breastfeeding can influence a mother's choice of feeding practice. Other authors have stated that health education could improve the present status on infant feeding practices [12]. The lack of proper information on breastfeeding sometimes acts as a barrier to its practice though women are strongly determined to breastfeed. It was reiterated that continual support using a nutrition education 'communication mix' is prone to be more effective to result in positive behavior change towards infant feeding practices [16]. Support needs to be given to breastfeeding mothers to encourage breastfeeding beyond the first month and also the education of mothers and grandmothers is very important to establish good infant feeding practices [13]. The type of delivery may have a negative effect on breastfeeding initiation. Moreover, caesarian section seems to be a big barrier for the rightly timed initiation of breastfeeding. Caesarian section is becoming an increasingly common practice in the private hospitals among the upper and middle income groups, and this seems to be an obstacle to successful breastfeeding [15]. The place of antenatal treatment and the place of delivery can also have an impact on the feeding practices of mothers. There are two types of hospital set-up in Mauritius namely the public hospitals and the private hospitals. Both differ in the ways in which antenatal care, perinatal and postnatal care are being provided. Enthusiasm, support and pediatricians involvement are also very essential in the promotion and practice of breastfeeding towards the achievement of optimal infant and child health, growth and development [1].

A survey was carried out since 2006 with 25,000 mothers having an infant between the age of 5 and 24 months old. We choose a random subset of this data consisting of 6500 mothers. We noted that the average practices of exclusive breastfeeding during the first six months of the baby's life is approximately 250 while the variance is 356. This indicates that the data is over-dispersed. To model such data under a regression set-up, we use the GPR model following Famoye and K.P Singh [2] and the NB model. In the next sections, we provide an overview of the two models and develop two sets of quasi-likelihood estimating equations to estimate the regression and over-dispersion parameters.

III. Generalized Poisson Regression model

(GPR) let y_i be a count response and X_i be a p-dimensional vector of covariates for subject $i(i = 1, \ldots, I)$. Let β be the vector of regression parameters such that β_j $(j = 1, 2, \ldots, p)$ is the regression effect of the j^{th} covariate on the incidence of exclusive breastfeeding among mauritian mothers. The density function of y_i is given by

$$f(y_i, \theta_i, \alpha) = (\frac{\theta_i}{1+\alpha\theta_i})^{y_i} \frac{(1+\alpha y_i)^{y_i-1}}{y_i!} \exp[\frac{-\theta_i(1+\alpha y_i)}{1+\alpha_{\theta_i}}] \quad (1)$$

, $y_i = 0, 1, 2, \ldots$; where $\theta_i = \exp(X_i^T\beta)$. The mean of y_i is given by θ_i and the variance of y_i is given by $\theta_i(1 + \alpha\theta_i)^2$, $\alpha < 0$ represents count data with under-dispersion. Since GPR does not belong to the family of exponential dispersion, it becomes difficult to apply the popular maximum likelihood estimation technique (MLE) to estimate the parameters. Moreover, the partial derivatives of β and α are quite complicated [3]. Thus, we propose to use the quasi-likelihood estimation (QLE) technique [11] to estimate the regression and over-dispersion parameters.

IV. Negative binomial regression model (NB)

We assume y_{it} follows negative binomial distribution with probability mass function

$$f(y_i) = \frac{\Gamma(c^{-1}+y_i)}{\Gamma(c^{-1})y_i!}(\frac{1}{1+c\theta_i})^{c^{-1}}(\frac{c\theta_i}{1+c\theta_i})^{y_i} \quad (2)$$

i.e

$$Y_i \sim \text{NeBin}(1/c, c\theta_i) \quad (3)$$

where c is the over-dispersion parameter and the expectation and variance are given by

$$E(Y_i) = \theta_i = \exp(x_i^T\beta), Var(Y_i) = \theta_i + c\theta_i^2 \quad (4)$$

where $c > 0$.

V. Quasi-likelihood estimation technique

Wedderburn [11] developed a quasi-likelihood estimation technique (QLE) to estimate parameters under generalized linear model. In this section, we extend his approach and develop two marginal QLEs under GPR. The first QLE is to estimate the vector of regression parameters β based on observations y_i while the second QLE is to estimate the dispersion index α.

A. GPR equations

The QLE to estimate β is given by

$$\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}^{-1}(y_i - \theta_i) = 0, \quad (5)$$

where $V_{i,\beta} = \theta_i(1 + \alpha\theta_i)^2$. $D_{i,\beta} = \frac{\partial\theta_i}{\partial\beta^T} = \theta_i X_i^T$ is a $p \times 1$ matrix. The QLE to estimate α is given by

$$\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}^{-1}(y_i^2 - \eta_i) = 0, \quad (6)$$

where $\eta_i = \theta_i(1 + \alpha\theta_i)^2 + \theta_i^2$ and $D_{i,\alpha} = 2\theta_i^2(1 + \alpha\theta_i)$. $V_{i,\alpha}$ is the variance of Y_i^2 and is calculated using

$$V_{i,\alpha} = E(Y_i^4) - E(Y_i^2)^2 \quad (7)$$

where

$$E(Y_i^4) = \frac{3\theta_i^2}{(1-\alpha\theta_i)^6} + \theta_i(\frac{15}{(1-\alpha\theta_i)^2} - \frac{20}{(1-\alpha\theta_i)} + 6)\frac{1}{(1-\alpha\theta_i)^5} \quad (8)$$

following [3] and Johnson and Kotz [9]. The Newton-Raphson technique is then applied to the two estimating equations. The iterative equations are given as follows: At the rth iteration,

$$\left(\hat{\beta}_{r+1}\right) = \left(\hat{\beta}_r\right) + [\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}{}^{-1} D_{i,\beta}]_r^{-1} [\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}{}^{-1}(y_i - \theta_i)]_r \quad (9)$$

$$\left(\hat{\alpha}_{r+1}\right) = \left(\hat{\alpha}_r\right) + [\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}{}^{-1} D_{i,\alpha}]_r^{-1} [\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}{}^{-1}(y_i^2 - \eta_i)]_r \quad (10)$$

where $\hat{\beta}_r$ and $\hat{\alpha}_r$ are the values of $\hat{\beta}$ and $\hat{\alpha}_r$ at the r^{th} iteration. $[.]_r$ is the value of the expression at the r^{th} iteration. The estimators are consistent and under mild regularity conditions, for $I \to \infty$, it may be shown that $I^{\frac{1}{2}}((\hat{\beta}) - (\beta))^T$ has an asymptotic normal distribution with mean 0 and covariance matrix $I[\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}{}^{-1} D_{i,\beta}]^{-1} [\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}{}^{-1}(y_i - \theta_i)(y_i - \theta_i)^T V_{i,\beta}{}^{-1} D_{i,\beta}][\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}{}^{-1} D_{i,\beta}]^{-1}$ and $I^{\frac{1}{2}}((\hat{\alpha}) - (\alpha))^T$ has an asymptotic normal distribution with mean 0 and covariance matrix

$I[\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}{}^{-1} D_{i,\alpha}]^{-1}[\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}{}^{-1}(y_i^2 - \eta_i)(y_i^2 - \eta_i)^T V_{i,\alpha}{}^{-1} D_{i,\alpha}][\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}{}^{-1} D_{i,\alpha}]^{-1}$ The algorithm to estimate the parameters works as follows: For an initial estimate of β and α, we iterate equation (2) until convergence, then use the updated β to update α in equation (3). We then replace the updated β and α in equation (2) and iterate until convergence. Having obtained the new β, we replace in equation (3) to obtain a new α and the cycle continues until both values converge.

B. NB equations

The QLE to estimate β is given by

$$\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}^{-1}(y_i - \theta_i) = 0, \quad (11)$$

where $V_{i,\beta} = \theta_i(1 + c\theta_i)^2$. $D_{i,\beta} = \frac{\partial\theta_i}{\partial\beta^T} = \theta_i X_i^T$ is a $p \times 1$ matrix. The QLE to estimate c is given by

$$\sum_{i=1}^{I} D_{i,c}^T V_{i,c}^{-1}(y_i^2 - \eta_i) = 0, \quad (12)$$

where $\eta_i = \theta_i(1+c\theta_i)+\theta_i^2$ and $D_{i,c} = \theta_{it}^2$. $V_{i,\alpha}$ is the variance of Y_i^2 and is calculated using

$$V_{i,\alpha} = E(Y_i^4) - E(Y_i^2)^2 \quad (13)$$

where

$$V_{i,\alpha} = \theta_{it}+(6+7c)\theta_{it}^2+(4+16c+12c^2)\theta_{it}^3+(4c+10c^2+6c^3)\theta_{i1}^4 \quad (14)$$

The iterative equations are given as follows: At the rth iteration,

$$\left(\hat{\beta}_{r+1}\right) = \left(\hat{\beta}_r\right)+[\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}{}^{-1} D_{i,\beta}]_r^{-1}[\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}{}^{-1}(y_i-\theta_i)]_r \quad (15)$$

$$\left(\hat{c}_{r+1}\right) = \left(\hat{c}_r\right)+[\sum_{i=1}^{I} D_{i,c}^T V_{i,c}{}^{-1} D_{i,c}]_r^{-1}[\sum_{i=1}^{I} D_{i,c}^T V_{i,c}{}^{-1}(y_i^2-\eta_i)]_r \quad (16)$$

where $\hat{\beta}_r$ and $\hat{c}_r$ are the values of $\hat{\beta}$ and $\hat{c}_r$ at the r^{th} iteration. $[.]_r$ is the value of the expression at the r^{th} iteration. The estimators are consistent and under mild regularity conditions, for $I \to \infty$, it may be shown that $I^{\frac{1}{2}}((\hat{\beta})-(\beta))^T$ and $I^{\frac{1}{2}}((\hat{c})-(c))^T$ have asymptotic normal distributions.

VI. Results and Conclusions

In this section, we use the GPR and NB regression models described in section II and III and estimate the regression parameters via the quasi-likelihood techniques in section IV. The covariates are the intercept term, age of the mothers, length of maternity leave, place of antenatal treatment, information on infant feeding practices, type of delivery and place of delivery.

These results are obtained by taking small initial values of the regression parameters. The entry in brackets represent the standard errors of each estimate. The negative value of the age factor indicates that age has an adverse effect on the practice of exclusive breastfeeding. This has been particularly observed among young mothers of less than 18 years old. The positive estimate of the length of maternity leave shows that as the number of days of maternity leave increases, it is more likely that the mothers will adopt a better infant feeding practice and the incidence of exclusive breastfeeding will increase. The estimated value of the place of antenatal treatment reflects the current situation of the private and public health institutions in Mauritius. In fact, in the public health sector, more information on proper infant feeding practices is being dispersed as compared to the private health sector, thus justifying the negative sign. In the same way, the estimate of the place of delivery is negative because there is a disparity at the level of the private and public health institutions where only the latter have adopted the Baby Friendly Hospital Initiative (BFHI), thereby encouraging proper breastfeeding initiation and successful exclusive breastfeeding for the six months. The regression estimate corresponding to the type of delivery indicates that mothers undergoing caesarian section are less likely to practise exclusive breastfeeding. The information parameter estimate justifies that mothers who have been well informed about proper feeding practices are more likely to practise exclusive breastfeeding for the recommended time. The estimates of the standard error of the NB model are quite lower than under the GPR model.

TABLE I
Estimates of the regression parameters based on QLE approach for the breastfeeding data: GPR model

Intercept	1.1732	(0.3428)
Age	-2.2501	(0.0662)
Length of maternity leave	1.3411	(0.0531)
Place of antenatal treatment	-3.1206	(0.1975)
Information	10.1012	(0.0989)
Type of delivery	-1.2516	(0.1321)
Place of delivery	-2.4544	(0.1028)
$\hat{\alpha}$	2 .312	(0.1898)

TABLE II
Estimates of the regression parameters based on QLE approach for the breastfeeding data: NB model

Intercept	1.1785	(0.3541)
Age	-2.3211	(0.0588)
Length of maternity leave	1.2351	(0.0525)
Place of antenatal treatment	-3.2210	(0.1871)
Information	12.2212	(0.0910)
Type of delivery	-1.2226	(0.1111
Place of delivery	-2.3044	(0.0999)
$\hat{c}$	2.5611	(0.1771)

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