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The Link between Unemployment and Inflation Using Johansen's Co-Integration Approach and Vector Error Correction Modelling

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***Abstract*—**In this paper bi-annual time series data on unemployment rates (from the Labour Force Survey) are expanded to quarterly rates and linked to quarterly unemployment rates (from the Quarterly Labour Force Survey). The resultant linked series and the consumer price index (CPI) series are examined using Johansen's cointegration approach and vector error correction modeling. The study finds that both the series are integrated of order one and are cointegrated. A statistically significant co-integrating relationship is found to exist between the time series of unemployment rates and the CPI. Given this significant relationship, the study models this relationship using Vector Error Correction Models (VECM), one with a restriction on the deterministic term and the other with no restriction.

A formal statistical confirmation of the existence of a unique linear and lagged relationship between inflation and unemployment for the period between September 2000 and June 2011 is presented. For the given period, the CPI was found to be an unbiased predictor of the unemployment rate. This relationship can be explored further for the development of appropriate forecasting models incorporating other study variables.

***Keywords*—**Forecasting, lagged, linear, relationship.

I. Introduction

REFERENCE [8], carried out several formal cointegration tests for the relationships between inflation, unemployment and labour force change rate and obtained an overall confidence in the existence of a true linear and lagged link between the variables. According to [7], the links between inflation and unemployment demonstrate various and even opposite dependencies. In the USA, this dependence is characterized by a positive influence of inflation on unemployment. Effectively, low inflation in the USA leads low unemployment by three years. Studies have also shown that there is very little evidence to support a significant relationship between real wage growth and industry- level employment growth [4].

In explaining the relationship between inflation and unemployment, [9] found that although there are periods where there is a clear trade-off between inflation and unemployment there are periods where both inflation and unemployment change in the same direction. Further, using a panel from ten different OECD countries, from 1950 to 2005, [1] researchers applied panel cointegration methodologies to find statistical evidence for a relationship between these real wages and employment variables. A study has also shown that real wages has a short run negative impact on employment but in the long run this relationship was shown to be positive [12].

In this study, the two variables (unemployment rate and CPI), being non-stationary I (1), were found to be cointegrated in a statistical sense. This means that their residual time series in the vector error correction model (VECM) representation proves to be stationary. The models for both a VECM model with a restriction on the deterministic term and one with no restriction is presented.

In interpreting the two models one must consider that, modeling cointegrated series is difficult because of the need to model systems of equations in which one has to simultaneously specify the deterministic terms and how they enter, determine the lag length, and ensure a congruent representation [6].

II. Data

The Labour Force Survey (LFS) was introduced in September 2000 and was published on a biannual basis until March 2008 when the Quarterly Labour Force Survey (QLFS) was introduced. This resulted in a discontinuous series that made analysis of unemployment estimates very difficult.

In this study, the LFS biannual unemployment rate time series from September 2000 to September 2007 and the QLFS quarterly unemployment rate time series for the period March 2008 to June 2011 were combined and adjusted for the purpose of analysis. The biannual unemployment rates were converted to quarterly rates using the SAS procedure PROC EXPAND. The procedure uses the SPLINE method by fitting a cubic spline curve to the input values. A cubic spline is a segmented function consisting of third-degree (cubic) polynomial functions joined together so that the whole curve and its first and second derivatives are continuous.

The CPI rates (an economic indicator of inflation) were used for matching quarters with the data for the corresponding unemployment rates.

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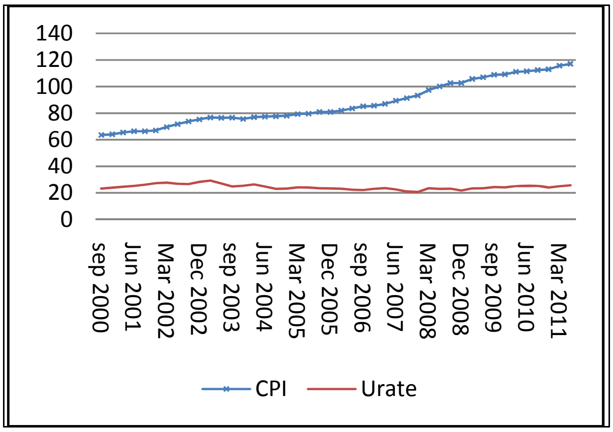

Fig. 1 Graphical representation of unemployment rates and CPI

The graphical representation (Fig. 1) shows the plot of the expanded LFS series combined with the QLFS series and the plot of the CPI for the corresponding months at the end of each quarter.

From the graphs we observe that there are no large changes in the data or outliers that will influence the estimates with a large weight and, hence, potentially bias the estimates, so there is no need for deterministic components, such as intervention dummies in the model specification.

III. Methodology and Results

A. Unit Root Test

The first step in the time series analysis was to determine whether the two series are stationary or non-stationary in nature. If the time series are I (1), they have to be characterized by the presence of a unit root and their first difference by the absence of unit roots [6].

The Augmented Dickey Fuller (ADF) unit root test was used to determine whether the series was stationary or non-stationary. The Dickey- Fuller tests for non stationarity of each of the series is shown below (Table I). The null hypothesis is to test a unit root. The ADF test constructs a model with higher order lag terms and tests the significance of the parameter estimates using a non-standard *t-test*. The model used for this routine is $\Delta y_t = \alpha_1 y_{t-1} + \beta_1 \Delta y_{t-1} + \ldots + \beta p_{-1} \Delta y_{t-p+1} + \varepsilon_t$, where the *t-test* checks significance of the α_1a term. This procedure is available in most time series analysis software packages.

Consequently, both series have a unit root and their first differences do not have any. Thus, the variables URate and CPI are first order difference stationary and are integrated of order I (1).

TABLE I
DICKEY-FULLER UNIT ROOT TEST

Variable	Type	Rho	Pr<Rho	Tau	Pr<Tau
Urate	Zero Mean	0.02	0.6828	0.07	0.7002
	Single Mean	-9.54	0.1271	-2.08	0.2553
	Trend	-12.61	0.2292	-2.29	0.4286
CPI	Zero Mean	0.61	0.826	3.99	0.9999
	Single Mean	0.52	0.9747	0.92	0.9948
	Trend	-2.88	0.9361	-1.12	0.9131

IV. Cointegration Test

The Johansen and Juliusλ_{trace} cointegration statistic test for testing the null hypothesis that there are at most r cointegrated vectors is used versus the alternative Hypothesisof more than r cointegratedvectors. Where:

$$\lambda_{trace} = -T \sum_{\lambda=r+1}^{k} log(1-\lambda_i) \quad (1)$$

Theory holds that two time series variables will be cointegrated if they have a long term or equilibrium relationship between them [5]. Given that both series are I (1) imply that their linear combination is I (0). Johansen's test has a number of desirable properties, including the fact that all test variables are treated as endogenous variables [11]. The maximum lag length was set to 7 quarters and an autoregressive order of p=6 were selected based on the partial correlation matrices and partial canonical matrices [10]. The SAS procedure PROCVARMAX was used to test for cointegration and model fitting. The results of the cointegration tests are shown below (Table II).

Tables II A & B show the output from the VARMAX procedure based on the model specified (an intercept term is assumed). In the cointegration rank test using trace (Table II A), we observe that there is no separate drift in the ECM and the process has a constant drift before differencing. These trace statistics are based on the alternate hypothesis (H1) that there is a separate drift and no separate linear trend in the VECM.

The cointegration rank test using trace under restriction (Table II B) shows the trace statistics based on the null hypothesis (H0) that there is no separate drift in the VECM but a constant enters only via the error correction term.

In both cases the series are cointegrated with rank=1 because the trace statistics are smaller than the critical values. In the unrestricted case, Johansen's trace statistic has a value of 16.076 which is greater than the 5% critical value of 15.34, therefore we reject r=0. Further, the test for r=1 versus r>1 does not reject r=1. Thus, Johansen's test indicates a single (r=1) cointegrating vector.

The study proceeds to determine which result, either the model with restriction or the model with no restriction, is appropriate depending on the significance level. Since the cointegration rank is chosen to be 1and the p-value is 0.0549, the hypothesis H0 cannot be rejected at 5% significance level but can be rejected at the significance level of 10% (Table III).

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Since U Rate and CPI are cointegrated, according to the Granger representation theorem a cointegrated series can be represented by a vector error correction model (VECM) [2].

For H0, a VECM (6) model with a restriction on the deterministic term will be used and a similar model with no restriction will be used for H1.

A Vector Error Correction Model (VECM) of order p can be written as:

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta_1 y_{t-1} + \ldots + \Delta_{p-1} y_{t-p+1} + \varepsilon_t t \in Z \qquad (2)$$

where

yt is a k x 1 random vector

the sequence yt is a Var(p) process

yt ~ CI(1)

$\Pi = \alpha\beta^t$ where α is the adjustment coefficient and β the cointegrating vector

$\Gamma_{1\ldots}, \Gamma_k$ are fixed coefficient matrices

ε_t is a k x 1 white noise process

The results of the fitted VECM (6) model with a restriction on the deterministic term are shown below.

TABLE IIA
COINTEGRATION RANK TEST USING TRACE

H0: Rank=r	H1: Rank>r	Eigenvalue	Trace	5% Critical Value	Drift in ECM	Drift in Process
0	0	0.2939	16.076	15.34	Constant	Linear
1	1	0.0723	2.8521	3.84		

TABLE IIB
COINTEGRATION RANK TEST USING TRACE UNDER RESTRICTION

H0: Rank=r	H1: Rank>r	Eigenvalue	Trace	5% Critical Value	Drift in ECM	Drift in Process
0	0	0.4215	27.335	19.99	Constant	Constant
1	1	0.158	6.5366	9.13		

TABLEIII
HYPOTHESIS TEST OF THE RESTRICTION

Rank	Eigenvalue	RestrictedEigenvalue	DF	Chi-Square	Pr > ChiSq
0	0.2939	0.4215	2	11.26	0.0036
1	0.0723	0.158	1	3.68	0.0549

TABLE IV
ALPHA AND BETA ESTIMATES

Long-Run Parameter Beta Estimates When RANK=1		Adjustment Coefficient Alpha Estimates When RANK=1
Variable	1	1
URate (Y_1)	1.000	0.00768
CPI (Y_2)	0.02331	-0.26592
1	-31.09951	---

The estimates of the long run parameter β, and the adjustment coefficient, α, are given in the table above. Since the cointegration rank is 1 in the bivariate system, α and β^t are (2x1) and (1x3) vectors respectively. The estimated cointegrating vector is $\beta^t = [1, 0.02331, -31.09951]$.The first element of β is 1 since $y_1$is specified as the normalised variable. The impact matrix is: $\pi = \alpha\beta^t$ becomes

$$\begin{bmatrix} 0.00768 \\ -0.26592 \end{bmatrix} [1.000 \quad 0.023 \quad -31.09 = \begin{bmatrix} 0,07680 & 0,00018 & -0,23885 \\ -0,26592 & -0,00620 & 8,27006 \end{bmatrix}$$

The long run relationship of the series is

$$\beta^t Y_t = [1 \quad 0.023 \; -31.099] \begin{bmatrix} y_{1,} \\ y_2 \\ 1 \end{bmatrix} = Y_{1t} + 0.023 Y_{2t} - 31.099$$

$$Y_{1t} = 31.099 - 0.023\, Y_{2t}$$

Based on the output of the Varmax procedure the model can be written using (2):

$$\Delta y_t = \begin{bmatrix} 0,0768 & 0,00018 & -0,23885 \\ -0,26592 & -0,00620 & 8,27006 \end{bmatrix} \begin{bmatrix} y_1, t_{-1} \\ y_2, t_{-1} \\ 1 \end{bmatrix} + \begin{bmatrix} 0,12398 & -0,02172 \\ -0,04139 & 0,28424 \end{bmatrix} \Delta y_{t-1} + \begin{bmatrix} -0,20272 & 0,27722 \\ 0,52267 & 0,12452 \end{bmatrix} \Delta y_{t-2} + \begin{bmatrix} -0,41346 & 0,16301 \\ -0,05825 & -0,01603 \end{bmatrix} \Delta y_{t-3} + \begin{bmatrix} 0,58462 & -0,21349 \\ 0,33003 & 0,13659 \end{bmatrix} \Delta y_{t-4} + \begin{bmatrix} 0,01262 & -0,17129 \\ 0,54388 & -0,47806 \end{bmatrix} \Delta y_{t-5} + \epsilon_t$$

V. MODEL DIAGNOSTICS

Checking the assumptions of the model, (i.e., checking the white-noise requirement of the residuals, and so on), is not only crucial for correct statistical inference, but also for the economic interpretation of the model as a description of the behavior of rational agents [6]. Various tests such as tests for autocorrelation in the squares are able to detect model failures [3].

The univariate equations are found to be a good fit for the data based on the model F statistics and R-square statistics. The regression of Δ U Rate resulted in a model F test 3.7 and R-square of 0.64. Similarly the regression of Δ CPI resulted in a model F test of 4.3 and R-square of 0.674 (Table V).

The residuals are checked for normality and autoregressive conditional heteroskedasticity or ARCH effects. The model also tests whether the residuals are correlated. The Durbin-Watson test statistics are both near 2 for both residual series and the series does not deviate from normal and are homoscedastic. The results also show that there are no ARCH effects on the residuals since the "no ARCH" hypothesis cannot be rejected given the F values (Table VI).

There are no AR effects on the residuals - for both residual series the autoregressive model fit to the residuals show no significance indicating that the residuals are uncorrelated (Table VII).

TABLE V
UNIVARIATE MODEL ANOVA DIAGNOSTICS

Variable	R-Square	Standard Deviation	F Value	Pr > F
URate	0.6400	0.6644	3.7	0.0028
CPI	0.6735	0.5951	4.3	0.0010

TABLE VI
UNIVARIATE MODEL WHITE NOISE DIAGNOSTICS

Variable	Durbin Watson	Normality		ARCH	
		Chi-Square	Pr > ChiSq	F Value	Pr > F
URate	1.9628	1.71	0.4256	1.12	0.2981
CPI	1.7549	1.27	0.5287	0.56	0.4585

TABLE VII
UNIVARIATE MODEL AR DIAGNOSTICS

Variable	AR1		AR2		AR3		AR4	
	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F
URate	0	0.9733	0.01	0.9866	0.01	0.998	0.26	0.900
CPI	0.19	0.6673	0.07	0.9361	0.25	0.8621	0.41	0.803

TABLE VIII
FITTED MODEL FOR H1: VECM (6) MODEL WITH NO RESTRICTION

Long-Run Parameter Beta Estimates When RANK=1		Adjustment Coefficient Alpha Estimates When RANK=1
Variable	1	1
URate (Y_1)	1.000	-0.11375
CPI (Y_2)	-0.00191	-0.26517

The estimates of the long –run parameter β, and the adjustment coefficient, α, are given in the table above. Since the cointegration rank is 1 in the bivariate system, α and β are two dimensional vectors. The estimated cointegrating vector is β^t = [1 -0.00191].The first element of β is 1 since y_1 is specified as the normalised variable. The impact matrix is: $\pi = \alpha\beta^t$, becomes

$$\begin{bmatrix} -0.11375 \\ -0.26517 \end{bmatrix} [\,1.000 \;\; -0.00191\,] = \begin{bmatrix} -0.11375 & 0.00022 \\ -0.26517 & 0.00051 \end{bmatrix}$$

The long run relationship of the series is

$$\beta^t Y_t = [\,1 \;\; -0.00191\,] \begin{bmatrix} y1 \\ y2 \end{bmatrix} = Y_{1t} - 0.00191\, Y_{2t}$$

$$Y_{1t} = 0.00191\, Y_{2t}$$

Based on the output of the Varmax procedure the model can be written using (2):

$$\Delta y_t = \begin{bmatrix} -0{,}11375 & 0{,}00022 \\ -0{,}26517 & 0{,}00051 \end{bmatrix} y_{t-1} + \begin{bmatrix} 0{,}14007 & -0{,}03246 \\ -0{,}07601 & 0{,}29286 \end{bmatrix} \Delta y_{t-1} + \begin{bmatrix} -0{,}12656 & 0{,}27726 \\ 0{,}52433 & 0{,}11732 \end{bmatrix} \Delta y_{t-2} + \begin{bmatrix} -0{,}34873 & 0{,}16542 \\ -0{,}07007 & -0{,}00901 \end{bmatrix} \Delta y_{t-3} + \begin{bmatrix} 0{,}57513 & -0{,}17569 \\ 0{,}31159 & 0{,}13770 \end{bmatrix} \Delta y_{t-4} + \begin{bmatrix} 0{,}10943 & -0{,}16003 \\ 0{,}56371 & -0{,}51045 \end{bmatrix} \Delta y_{t-5} + \begin{bmatrix} 2.61048 \\ 7.66795 \end{bmatrix} + \epsilon_t$$

The univariate equations are found to be a good fit for the data based on the model F statistics and R-square statistics. The regression of Δ URate resulted in a model F test 4.33 and R-square of 0.675. Similarly the regression of Δ CPI resulted in a model F test of 4.5 and R-square of 0.676 (Table IX).

The residuals are checked for normality and ARCH effects. The model also tests whether the residuals are correlated. The Durbin-Watson test statistics are both near 2 for both residual series and the series does not deviate from normal and are homoscedastic. The results also show that there are no ARCH effects on the residuals (Table X).

There are no AR effects on the residuals for both residual series the autoregressive model fit to the residuals show no significance indicating that the residuals are uncorrelated (Table XI).

TABLE IX
UNIVARIATE MODEL ANOVA DIAGNOSTICS

Variable	R-Square	Standard Deviation	F Value	Pr > F
URate	0.6751	0.6292	4.33	0.001
CPI	0.676	0.5927	4.35	0.0009

TABLE X
UNIVARIATE MODEL WHITE NOISE DIAGNOSTICS

Variable	Durbin Watson	Normality		ARCH	
		Chi-Square	Pr > ChiSq	F Value	Pr > F
URate	1.9978	2.55	0.2791	3.35	0.0757
CPI	1.7753	1.28	0.5262	0.53	0.4713

TABLE XI
UNIVARIATE MODEL AR DIAGNOSTICS

Variable	AR1		AR2		AR3		AR4	
	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F
URate	0.06	0.8149	0.45	0.6418	0.26	0.8565	0.35	0.8426
CPI	0.11	0.7392	0.04	0.9634	0.25	0.8575	0.42	0.7894

A. Testing Weak Exogeneity

Results from the weak exogeneity test indicate that the unemployment rate (URate) is a weak exogeneity of the consumer price index (CPI), whereas the CPI is not a weak exogeneity of URate (Table XII).

TABLE XII
TESTING WEAK EXOGENEITY OF EACH VARIABLE

Variable	DF	Chi-Square	Pr > ChiSq
URate	1	1.94	0.164
CPI	1	10.02	0.0016

VI. CONCLUSION

The expected result of the above analysis consists in a formal statistical confirmation of the existence of a unique linear and lagged relationship between inflation and unemployment for the period between September 2000 and June 2011. Hence, the two variables, being non-stationary I (1), are cointegrated; i.e. their residual time series in the VECM representation has been proved to be stationary. Johansen's cointegration techniques were applied to investigate the long run relationship between inflation and unemployment. The results indicate the existence of one cointegrating vector amongst the variables. Further, the weak exogeneity results indicate that the unemployment rate is a weak exogeneity of the CPI, whereas the CPI is not a weak exogeneity of the unemployment rate. This relationship found in this study can be explored further for the development of appropriate forecasting models incorporating other study variables.

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