

# Different Approaches for the Design of IFIR Compaction Filter

Sheeba V.S and Elizabeth Elias

**Abstract**—Optimization of filter banks based on the knowledge of input statistics has been of interest for a long time. Finite impulse response (FIR) Compaction filters are used in the design of optimal signal adapted orthonormal FIR filter banks. In this paper we discuss three different approaches for the design of interpolated finite impulse response (IFIR) compaction filters. In the first method, the magnitude squared response satisfies Nyquist constraint approximately. In the second and third methods Nyquist constraint is exactly satisfied. These methods yield FIR compaction filters whose response is comparable with that of the existing methods. At the same time, IFIR filters enjoy significant saving in the number of multipliers and can be implemented efficiently. Since eigenfilter approach is used here, the method is less complex. Design of IFIR filters in the least square sense is presented.

**Keywords**—Principal Component Filter Bank, Interpolated Finite Impulse Response filter, Orthonormal Filter Bank, Eigen Filter.

## I. INTRODUCTION

THE optimal energy compaction filters have received considerable attention due to the fact that they are the main building blocks in the design of principal component filter banks (PCFB) [1]. If the analysis/synthesis filters used in a filter bank (Fig.1) are constrained to satisfy an orthonormality condition [2], but are of unconstrained order, then the optimal filter bank for a number of objectives, happens to be a principal component filter bank for the input power spectral density (psd)  $S_{xx}(z)$ . They satisfy the principal component property i.e., they minimize the mean-squared error caused by reconstruction, after dropping the  $P$  lowest variance subbands for any  $P < M$ , where  $M$  is the number of channels in the filter bank. If PCFB exists for a class of filter banks, then, for an input psd  $S_{xx}(e^{j\omega})$ , the subband variance vector majorizes [3] the subband variance vector of any other filter bank from the same class. PCFB has also been shown to be optimal for any concave objective function of subband variances [4]. Such filter banks are simultaneously optimal for a number of objectives such as coding gain or multiresolution. But PCFB exists only for three special classes of filter banks, (i) a class of 2 channel paraunitary (PU) filter banks (ii) a class of all unconstrained PU filter banks and (iii) a class of orthogonal transform coders. The optimal analysis or synthesis filters in the PCFB turn out to be a series of

compaction filters [1] which are ideal band pass filters and hence are unrealizable.

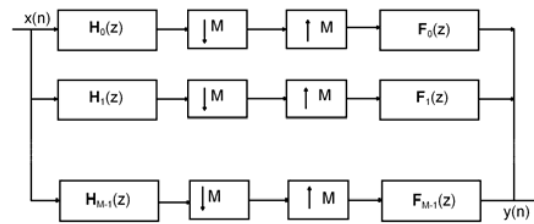


Fig. 1 Uniform maximally decimated M-channel filter bank.

PCFB does not exist for FIR PU filter banks [4], and hence the optimization of such filter banks becomes more difficult. But such filter banks are optimal only for one specific objective unlike unconstrained order filter banks. There are several approaches for the design of FIR compaction filter like Optimizing product filter [5], State space approach [6], Iterative gradient technique [7], etc. But most of these are computationally intensive when the filter order is high.

In this paper, the approach taken to obtain an FIR compaction filter is to find the one which best approximates the unconstrained or infinite order optimum compaction filter in the least mean square sense. Since the compaction filter maximizes the output variance subject to Nyquist ( $M$ ) constraint[1], it has supports on the high energy region of the input power spectrum and consequently the optimum compaction filter can be a multiband filter rather than lowpass. This implies that higher order FIR filters are required to approximate such a multiband filter. Interpolated finite impulse response (IFIR) approach is an efficient way to design and implement narrow band filters [8]. With IFIR approach, higher order FIR compaction filter can be designed with a good reduction in the number of multipliers [8]. The main attraction of this method is that the filter can be implemented efficiently since the compaction filter is an optimum solution for many objectives in communication and signal processing.

We focus on three methods for the design of IFIR compaction filter using eigen filter approach. In the first method, the designed filters satisfy Nyquist constraint approximately. In the second and third methods, the magnitude squared response of the designed filter satisfies the Nyquist ( $M$ ) property exactly. The input signal  $x(n)$  is assumed to be cyclo wide sense stationary of period  $M$  (CWSS ( $M$ )) so that its  $M$ -fold blocked version  $\mathbf{x}(n)$  [9] is wide sense stationary with power spectral density  $S_{xx}(z)$ .

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Simulation results provided show that these methods can yield FIR compaction filters whose response is comparable to that of the filters designed by the existing methods. Furthermore, since the eigenfilter approach is used here, the method is less complex. The filter banks designed with these filters are useful for various applications including signal compression, signal denoising, digital communication etc

The paper is organized as follows. The theory of IFIR compaction filter is given in section 2. Section 3 discusses three different approaches for the design of IFIR compaction filter which includes model filter and image suppressor filter. Section 4 gives simulation results.

II. IFIR COMPACTION FILTER

IFIR approach is an efficient method of design of compaction filters since it can reduce the number of multipliers needed for the realization of the filter. Here, compaction filter is realized as a cascade (Fig. 2) of a model filter P(z) and an image suppressor filter F(z), where L is the interpolation factor

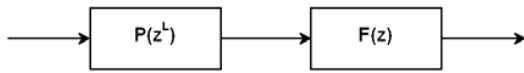


Fig. 2 Cascade implementation of IFIR filters

$$H(z)=P(z^L)F(z) \tag{1}$$

The transition band width of the model filter is L times that of the overall filter. The image filter F(z) is designed to remove the unwanted spectral image of the passband of the model filter. The coefficients h(n) of H(z) are related to the coefficients p(n) and f(n) by

$$h(n) = \sum_m p(m) \cdot f(n - Lm) \tag{2}$$

If N, N<sub>p</sub> and N<sub>f</sub> are the orders of h(n), p(n) and f(n) respectively, N = L N<sub>p</sub> + N<sub>f</sub>. We express all the functions in vector form and relate them through matrix.

$$\mathbf{h} = \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(N) \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} p(0) \\ p(1) \\ \vdots \\ p(N_p) \end{bmatrix} \quad \mathbf{f} = \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N_f) \end{bmatrix} \tag{3}$$

$$\text{Let } p'(n) = \begin{cases} p(n/L) & \text{if } n = Lk \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

Then h(n) = p'(n)\*f(n). This relation can be written in matrix form

$$\mathbf{h}=\mathbf{F}\mathbf{p}' \tag{5}$$

where F is of the dimension (N+1) x (LN<sub>p</sub> + 1) given by

$$\mathbf{F} = \begin{bmatrix} f(0) & 0 & \dots & 0 \\ f(1) & f(0) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f(N_f) & f(N_f-1) & \ddots & f(0) \\ 0 & f(N_f) & \ddots & f(1) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & f(N_f) \end{bmatrix} \tag{6}$$

each column of F consists of the vector f.

p' can be expressed as

$$\mathbf{p}'=\mathbf{S}\mathbf{p} \tag{7}$$

where S is (LN<sub>p</sub> + 1) x (N<sub>p</sub> + 1) matrix. For L = 2 this is defined as

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{8}$$

Substituting (7) in (5),

$$\mathbf{h}=\mathbf{F}\mathbf{p}'=(\mathbf{F}\mathbf{S})\mathbf{p}=\mathbf{A}\mathbf{p} \tag{9}$$

For L=2, matrix A becomes

$$\mathbf{A} = \begin{bmatrix} f(0) & 0 & \dots & 0 \\ f(1) & 0 & \dots & 0 \\ f(2) & f(0) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f(N_f) & f(N_f-2) & \ddots & f(0) \\ 0 & f(N_f-1) & \ddots & f(1) \\ 0 & f(N_f) & \ddots & \vdots \\ \vdots & \vdots & \ddots & f(N_f-1) \\ 0 & 0 & \dots & f(N_f) \end{bmatrix} \tag{10}$$

The optimum FIR compaction filter, which is the filter corresponding to the largest subband variance in the FIR orthonormal filter banks, satisfies

$$|H_k(e^{j\omega})|^2|_{\downarrow M}=1 \text{ (Nyquist constraint)} \tag{11}$$

where M is the decimation factor. This constraint implies [1],[10] **unit energy** property

$$\int_0^{2\pi} |H_k(e^{j\omega})|^2 d\omega / 2\pi = 1 \tag{12}$$

as well as the **boundedness** property

$$|H_k(e^{j\omega})|^2 \leq M \tag{13}$$

If no order-constraint is made on H(z) then optimum compaction filter is in general an ideal bandpass filter[1],[7],

i.e. the magnitude response of the ideal compaction filter must satisfy

$$|D(e^{j\omega})|^2 = \begin{cases} M & \omega \in \omega_x \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

where  $\omega_x$  is the set of frequencies defined as follows

$$\omega_x = \left\{ \omega \in [0, 2\pi) : S_{xx}(e^{j\omega}) \geq S_{xx}(e^{j(\omega + \frac{2\pi l}{M})}) \forall 1 \leq l \leq M-1 \right\} \quad (15)$$

The design of IFIR compaction filter is discussed in the next section.

### III. THE DESIGN OF IFIR COMPACTION FILTER

In this section we consider the design of the compaction filter by three approaches. In the first method eigen filter approach is used subject to the conditions (12) and (13) so that causal FIR filter  $H(z)$  is approximated in the least square sense with optimum brickwall compaction filter. Then the designed filters will be able to satisfy the condition (11) approximately. In the last two methods eigen filter approach is used to obtain the spectral factor of the  $M^{\text{th}}$  band filter. In these methods Nyquist constraint is exactly satisfied.

#### 3.1 Least Square Design of IFIR Compaction Filter

In this approach we discuss the least square design of IFIR compaction filter minimizing both passband and stopband error. The optimum compaction filter  $D(e^{j\omega})$ , which is the filter corresponding to the largest subband variance in the PCFB [1], satisfies Nyquist constraint exactly. Here a causal FIR filter  $H(e^{j\omega})$  is approximated with this optimum brickwall compaction filter in the least mean square sense by weighted eigen filter approach, and at the same time the conditions (12) and (13) are also met. It is shown in [11] that this approach can be used to design FIR filters having arbitrary magnitude and phase responses with error as low as  $10^{-2}$ . If  $H(e^{j\omega})$  can be approximated as  $D(e^{j\omega})$ , with very small value of mean squared error, and at the same time satisfy the two necessary conditions for orthonormality,  $H(e^{j\omega})$  will satisfy Nyquist constraint approximately. By adjusting the weighting function  $W(e^{j\omega})$  used in the design and simulation of these filters, the error in the approximation can be reduced to a very small value. This is verified using an example in section 4. In this approach when the filter order is increased, mean squared error reduces and the response becomes very close to the ideal one. Also, spectral factorization is not required. The disadvantage of this approach is that the Nyquist constraint can only be approximately satisfied. Since  $H(z)$  is realized using IFIR approach as a cascade of  $P(z)$  and  $F(z)$ , the unit energy constraint  $\mathbf{h}^\dagger \mathbf{h} = 1$  is equivalent to

$$\mathbf{p}^\dagger \mathbf{p} = \mathbf{I} \quad (16)$$

$$\mathbf{A}^\dagger \mathbf{A} = \mathbf{I} \quad (17)$$

Hence the model filter  $P(z)$  is designed to satisfy (16) and the image filter is designed to satisfy (17)

#### 3.1.1 Design of $P(z)$ – Model Filter

We consider the design of real coefficient FIR filter that approximates a desired arbitrary function in the least square sense using eigenfilter method [11]. The desired function is selected in such a way that, its interpolated response matches the response of optimum brickwall compaction filter in the absence of images. Let  $D(e^{j\omega})$ ,  $P(e^{j\omega})$ ,  $W(e^{j\omega})$  denote the complex valued desired function, designed function and an arbitrary weighting function respectively. The objective function to be minimized is formulated as

$$E_{LS} = \frac{1}{\pi} \int_R |W(e^{j\omega})| |\gamma D(e^{j\omega}) - P(e^{j\omega})|^2 d\omega \quad (18)$$

where  $R$  is the region  $0 \leq \omega \leq \pi$ , excluding the transition band,  $\gamma = P(e^{j\omega_0}) / D(e^{j\omega_0})$  (19)

where  $\omega_0$  is the reference frequency. Frequency response  $P(z)$  takes the form

$$P(e^{j\omega}) = \sum_{n=0}^{N_p} p(n) e^{-j\omega n} = \mathbf{p}^T \mathbf{e}(e^{j\omega}) \quad (20)$$

where  $\mathbf{e}(e^{j\omega}) = [1 e^{-j\omega} \dots e^{-jN_p\omega}]^T$ . By substituting (19) and (20) in (18) the total error to be minimized can be expressed in the quadratic form,

$$E_{LS} = \mathbf{p}^T \mathbf{E} \mathbf{p} = \mathbf{p}^T (\mathbf{E}_p + \mathbf{E}_s) \mathbf{p} \quad (21)$$

The elements of  $\mathbf{E}_p$  are given by

$$[\mathbf{E}_p]_{m,n} = \frac{1}{\pi} \sum_{k=1}^{M_p} \int_{\omega_p(k,1)}^{\omega_p(k,2)} \left[ \operatorname{Re} \left\{ \left( \frac{D(e^{j\omega})}{d_0} \mathbf{e}_0 - \mathbf{e} \left( \frac{D(e^{j\omega})}{d_0} \mathbf{e}_0 - \mathbf{e} \right)^\dagger \right) \right\} \right] |W(e^{j\omega})| d\omega \quad (22)$$

where  $M_p$  is the number of pass bands,  $\omega_p(k,1)$ ,  $\omega_p(k,2)$  are the band edges of the  $k^{\text{th}}$  passband and  $d_0 = D(e^{j\omega_0})$ . In terms of the parameters  $p(n)$ ,  $D(e^{j\omega_0}) = \mathbf{p}^T \mathbf{e}_0$ ,  $\mathbf{e}_0 = \mathbf{e}(e^{j\omega_0})$ . Elements of  $\mathbf{E}_s$  are given by

$$[\mathbf{E}_s]_{m,n} = \frac{1}{\pi} \sum_{k=1}^{M_s} \int_{\omega_s(k,1)}^{\omega_s(k,2)} \cos(m-n)\omega |W(e^{j\omega})| d\omega \quad (23)$$

where  $M_s$  is the number of stop bands,  $\omega_s(k,1)$ ,  $\omega_s(k,2)$  are the band edges of the  $k^{\text{th}}$  stopband. In this case,  $\mathbf{p}$  is the real vector consisting of the filter coefficients and  $\mathbf{E}$  is real, symmetric and positive definite matrix depending upon the specifications. Then by Rayleigh principle [3], the vector  $\mathbf{p}$  that minimizes the least square error  $E_{LS}$  is the eigen vector corresponding to the minimum eigen value of  $\mathbf{E}$  so that the solution can be readily obtained. We assume  $\mathbf{p}^T \mathbf{p} = 1$  to satisfy unit energy constraint. The weighting function  $W(e^{j\omega})$  is suitably adjusted so that the frequency response approximating optimum infinite order compaction filter and satisfying unit energy and boundedness property is obtained.

It is shown in [11] that by using this design method, the error in the magnitude response as low as  $10^{-2}$  can be achieved by iteratively adjusting the weighting function  $W(e^{j\omega})$ .

3.1.2 Design of  $F(z)$  – Image Suppressor Filter

The image suppressor filter is designed to attenuate the unwanted passband replicas of  $P(z^L)$ . The transition bandwidth of  $F(z)$  can be wider and therefore it requires lower order. The design of this filter is done in the least mean square sense subject to a number of constraints, which are chosen to satisfy the unit energy property of orthonormal filter banks. The optimization problem for a real coefficient image suppressor filter becomes

$$\text{Min } \frac{1}{\pi} \int_0^\pi |\gamma I(e^{j\omega}) - F(e^{j\omega})|^2 |W(e^{j\omega})| d\omega \quad (24)$$

$$\text{subject to } \mathbf{A}^T \mathbf{A} = \mathbf{I}_{N_p+1} \quad (25)$$

where  $I(e^{j\omega})$  is the desired response of the image suppressor filter.  $\gamma$  is as defined in (19). Since each column of  $\mathbf{A}$  consists of the vector  $\mathbf{f}$  and  $LN_p$  number of zero entries, the number of constraints in (25) is less.

The optimization problem can be solved by any constrained nonlinear programming method starting from an initial value. Since the only purpose of the image suppressor filter is to eliminate images, this filter need not be a sharp filter. Therefore the design of this filter is not difficult. We have tried the optimization problem with different initial values. Even with a random initial value, the optimization converges successfully in a short time. The implementation of this filter is easy using appropriate functions available in Matlab.

The coefficients of  $H(e^{j\omega})$  which satisfy **unit energy** and **boundedness** constraints can be obtained from (9). As the filter  $H(e^{j\omega})$  approximates optimum compaction filter in the least square sense, this filter satisfies Nyquist(M) constraint approximately as we have shown in the simulation results.

3.2 IFIR Compaction filter as a spectral factor of the  $M^{\text{th}}$  band filter

There are two ways in which the IFIR compaction filter can be designed as the spectral factor of the  $M^{\text{th}}$  band filter

**Method 1**

Let  $G_p(z)$  and  $G_f(z)$  represent the product filters of  $P(z)$  and  $F(z)$  respectively. If  $G_f(z)$  is designed as Nyquist(L) and  $G_p(z)$  as Nyquist(M/L) it can be verified that  $H(z) = P(z^L)F(z)$  satisfies Nyquist(M) constraint. Hence the model filter  $P(z)$  is designed as the spectral factor of  $(M/L)^{\text{th}}$  band filter and  $F(z)$  as the spectral factor of the  $L^{\text{th}}$  band filter separately. The eigen filter approach[12] can be used to find out the spectral factors of the  $m^{\text{th}}$  band filter ( $m$  can be either  $L$  or  $M/L$ ) as described in [13]. Let  $F(z)$  be any FIR transfer function which can be factorized as

$$F(z) = F_0(z)F_1(z) \quad (26)$$

where  $F_1(z)$  has all zeros on the unit circle and  $F_0(z)$  has none on the unit circle. If  $F_0(z)$  is the spectral factor of  $G_{F_0}(z)$ , then, from (26) we can write

$$G_F(z) = G_{F_0}(z)G_{F_1}(z) = G_{F_0}(z)F_1^2(z) \quad (27)$$

Now the filter  $F(z)$  can be designed using eigen filter approach by minimizing the stopband energy under the condition that  $G_f(z)$  is an  $m^{\text{th}}$  ( $m=L$  in the case of image filter) band filter. If  $F(z)$  is a real coefficient FIR filter, the objective function to be minimized is formulated as

$$\varepsilon = \frac{1}{\pi} \sum_{k=1}^{M_s} \int_{\omega_s(k,1)}^{\omega_s(k,2)} |F_0(e^{j\omega})|^2 |F_1(e^{j\omega})|^2 d\omega \quad (28)$$

This can be written as

$$\varepsilon = \frac{1}{\pi} \sum_{k=1}^{M_s} \int_{\omega_s(k,1)}^{\omega_s(k,2)} |G_{F_0}(e^{j\omega})| |F_1(e^{j\omega})|^2 d\omega \quad (29)$$

The coefficients of  $G_{F_0}(z)$  and  $F_1(z)$  can be computed iteratively.  $F_1(z)$  is calculated by minimizing (29) by weighted eigen filter method[12] with weights given by  $G_{F_0}(z)$ . The coefficients of  $G_{F_0}(z)$  are computed under the constraint that  $G_f(z)$  is Nyquist(L). These steps are repeated starting with  $G_{F_0}(e^{j\omega})$  assumed to be unity for all  $\omega$  till the solution converges and the desired output response is obtained. In this design, both  $G_{F_0}(z)$  and  $F_1(z)$  are treated as linear phase FIR transfer functions. The coefficients of  $F_0(z)$  can be computed as the spectral factor of  $G_{F_0}(z)$  and image filter  $F(z)$  can be obtained using (26). The spectral factorization step involves much lower order polynomial which is quite easier than finding out the spectral factor of higher order polynomial  $G_f(z)$ .

In this method  $P(z)$  and  $F(z)$  are separately designed using the above approach and filter  $H(z)$  is calculated using (9). This method can be applied for the design of compaction filter, provided  $L$  is a factor of  $M$ .

**Method 2**

In this method,  $H(z)$  is designed as the spectral factor of the  $M^{\text{th}}$  band filter without going for the design of individual filters  $P(z)$  and  $F(z)$ . The image filter  $F(z)$  is designed to remove the unwanted spectral image of the passband of the model filter to a level below the desired stopband level. The specifications for its design are obtained directly from the desired specifications. The stopband regions of this filter should entirely encompass the undesired spectral images as well as parts of the transition bands which could contribute to decreased attenuation in the stopbands of the overall design. The passbands of this filter does not contribute much to the overall design. It can be selected to be of very nominal width. Equation (1) can be written as

$$H(z) = P(z^L)F(z) = P_0(z^L)P_1(z^L)F(z) = P_0(z^L)Q(z) \quad (30)$$

$$\text{where } Q(z) = P_1(z^L)F(z)$$

If  $Q(z)$  has all zeros on the unit circle and  $P_0(z)$  has none on the unit circle,  $H(z)$  can be designed as the spectral factor of the  $M^{\text{th}}$  band filter by using Method 1.  $F(z)$  is designed by weighted eigen filter approach by minimizing the stopband energy. Now  $Q(z)$  can be designed to approximate the desired response in the least mean square sense by weighted eigen filter approach by minimizing the stop band energy. The minimization problem can be formulated as given in (29)

$$\varepsilon = \frac{1}{\pi} \sum_{k=1}^{M_s} \sum_{\omega_s(k,1)}^{\omega_s(k,2)} \int |G_{P_0}(e^{j\omega L})| |Q(e^{j\omega})|^2 d\omega \quad (31)$$

where  $P_0(z^L)$  is the spectral factor of  $G_{P_0}(z^L)$ .

The coefficients of  $Q(z)$  and  $P_1(z)$  can be expressed in the vector form as

$$\mathbf{q} = [q(0), q(1), \dots, q(N_q)]^T, \quad \mathbf{p}_1 = [p_1(0), p_1(1), \dots, p_1(N_{p_1})]^T \quad (32)$$

Then

$$\mathbf{q} = \mathbf{A} \mathbf{p}_1 \quad (33)$$

where matrix  $\mathbf{A}$  is defined in (10). If  $Q(z)$  is a linear phase even order filter, (31) can be written as

$$\varepsilon = \mathbf{b}^T \left[ \frac{1}{\pi} \sum_{k=1}^{M_s} \sum_{\omega_s(k,1)}^{\omega_s(k,2)} \int |G_{P_0}(e^{j\omega L})| \mathbf{c}(\omega) \mathbf{c}^T(\omega) d\omega \right] \mathbf{b} \quad (34)$$

where

$$\mathbf{b} = [b_0, b_1, \dots, b_{M_1}]^T, \mathbf{c}(\omega) = [1, \cos(\omega), \dots, \cos(M_1\omega)]^T \quad (35)$$

and  $M_1 = \frac{N_q}{2}$ . The coefficients  $b_n$  are given by

$$b_0 = q(M_1), \quad b_n = 2q(M_1 - n), \quad n \neq 0. \quad (36)$$

If  $P_1(z)$  is a linear phase even order filter, (33) can be written as

$$\mathbf{q} = \mathbf{A}_1 \mathbf{p}_1' \quad (37)$$

where  $\mathbf{A}_1$  is  $(\frac{N_q}{2} + 1) \times (\frac{N_{p_1}}{2} + 1)$  matrix formed from  $\mathbf{A}$  by adding the elements of the last  $N_{p_1}/2$  columns of  $\mathbf{A}$  to that of its first  $N_{p_1}/2$  columns in such a way that  $\mathbf{A}_1(n) = \mathbf{A}(n) + \mathbf{A}(N_{p_1} + 2 - n), 1 \leq n \leq N_{p_1}/2$ ,  $\mathbf{A}(n)$  represents  $n^{\text{th}}$  column of  $\mathbf{A}$  and  $\mathbf{p}_1'$  represents a column vector consisting of the first  $N_{p_1}/2 + 1$  elements of  $\mathbf{p}_1$ . Considering the linear phase property of  $Q(z)$ , (37) can be simplified as

$$\mathbf{q}' = \mathbf{A}_2 \mathbf{p}_1' \quad (38)$$

where  $\mathbf{A}_2$  is  $(\frac{N_q}{2} + 1) \times (\frac{N_{p_1}}{2} + 1)$  matrix formed from  $\mathbf{A}_1$

by adding the elements of the last  $\frac{N_q}{2}$  rows of  $\mathbf{A}_1$  to its first

$\frac{N_q}{2}$  rows in such a way that

$$\mathbf{A}_2(n) = \mathbf{A}_1(n) + \mathbf{A}_1(N_q + 2 - n), 1 \leq n \leq N_q/2, \quad \mathbf{A}_1(n)$$

represents  $n^{\text{th}}$  row of  $\mathbf{A}_1$  and  $\mathbf{q}'$  represents a column vector consisting of the first  $N_q/2 + 1$  elements of  $\mathbf{q}$

Now  $\mathbf{b}$  can be written as

$$\mathbf{b} = \mathbf{A}_3 \mathbf{p}_1' \quad (39)$$

where  $\mathbf{A}_3$  is again  $(\frac{N_q}{2} + 1) \times (\frac{N_{p_1}}{2} + 1)$  matrix obtained by reversing the rows of  $\mathbf{A}_2$ . Substituting (39) in (34) results in

$$\varepsilon = \mathbf{p}_1'^T \mathbf{B} \mathbf{p}_1' \quad (40)$$

where

$$\mathbf{B} = \mathbf{A}_3^T \left[ \frac{1}{\pi} \sum_{k=1}^{M_s} \sum_{\omega_s(k,1)}^{\omega_s(k,2)} \int |G_{P_0}(e^{j\omega L})| \mathbf{c}(\omega) \mathbf{c}^T(\omega) d\omega \right] \mathbf{A}_3 \quad (41)$$

As described in Method 1, the coefficients of  $P_1(z)$  and  $G_{P_0}(z^L)$  are computed in an iterative manner starting with

$G_{P_0}(e^{j\omega L}) = 1$  for all  $\omega$ .  $P_1(z)$  can be computed using weighted eigen filter approach and coefficients of  $G_{P_0}(z)$

can be computed under the constraint that the  $G_H(z)$ , the product filter of  $H(z)$  is an  $M^{\text{th}}$  band filter

When the order of the filter is increased to higher values,  $G_{P_0}(z)$  may contain unit circle zeros and spectral factorization will become impossible. Besides, spectral factorization is possible only if  $G_{P_0}(z)$  is a linear phase filter.

Choice of  $L$  in all the design approaches depends on the value of  $M$  and the input psd. For a given power spectrum, optimum compaction filter could have multiple passbands. So careful selection of  $L$  is required in such a way that the spectral images are well separated. As  $L$  increases, the FIR filter response becomes better. But the image suppressor filter would have sharp response to eliminate the appropriate images and the order of the image suppressor filter increases. In the case of simple psds (monotonically varying), higher values of  $L$  are possible. Even  $L = 2$  is advantageous since it will reduce the number of filter coefficients by approximately 40% in the example considered.

#### 4. SIMULATION RESULTS

We consider the design of a compaction filter for a WSS input  $x(n)$ .  $x(n)$  is a real autoregressive process with psd  $S_{xx}(e^{j\omega})$ . The ideal compaction filter magnitude response must satisfy (14) and (15). We have shown one example each for the different approaches. The performance of the

compaction filter is estimated by calculating the compaction gain.

$$G_{comp} = \frac{\frac{1}{2\pi} \int_0^{2\pi} |H(e^{j\omega})|^2 S_{xx}(e^{j\omega}) d\omega}{\sigma_x^2} \quad (42)$$

**Example 1**

The design of an IFIR compaction filter with  $M=8, N_p = 26, N_f = 16, L = 2$  is done using the algorithm discussed in section 3.1 where  $N_p$  and  $N_f$  are the orders of  $p(n)$  and  $f(n)$  respectively. A linear phase spectral factor with delay equal to half of the filter order is taken. The magnitude squared response of the designed IFIR compaction filter  $H(e^{j\omega})$  along with the input psd and the response of the ideal compaction filter  $D(e^{j\omega})$  are shown in Fig.3(a). It can be seen that the response of the designed filter matches the ideal filter response very closely.

Nyquist constraint is verified for the designed IFIR filter and it is shown in Fig.3(b). The curve is seen to be very close to unity with a variation between 0.979 and 1.036. Since the IFIR filter is a cascade implementation of the model filter and the image suppressor filter, the adjustment of the weighting functions to approximate the brickwall response is much more difficult than for a single filter. Hence Nyquist constraint is satisfied approximately. The compaction gain is calculated for different values of  $N_p$ . A compaction gain of 1.62 is obtained for  $N_p = 34$ , which is close to the ideal value 1.6308.

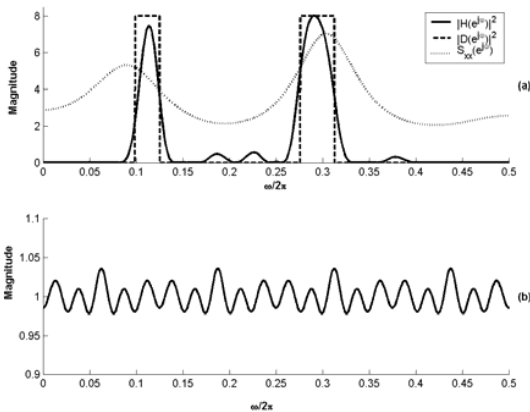


Fig. 3 Input psd  $S_{xx}(e^{j\omega})$ , magnitude squared response of ideal compaction filter,  $|D(e^{j\omega})|^2$  and designed FIR compaction filter  $|H(e^{j\omega})|^2$  for  $M = 8$  (b) Nyquist constraint  $\frac{1}{M} \sum_{k=0}^{M-1} |H(zW_M^k)|^2$

**Example 2**

The IFIR compaction filter is designed using Method 1 described in section 3.2 with  $M=4, L=2$  for the psd shown in Fig 4. Both  $G_p(z)$  and  $G_f(z)$  are designed as Nyquist(2) so that  $H(z)$  is spectral factor of 4<sup>th</sup> band filter. The filter is designed for  $N_f = 11$  and  $N_p = 23$ . The magnitude squared response of the designed IFIR compaction filter  $H(e^{j\omega})$  along with the input

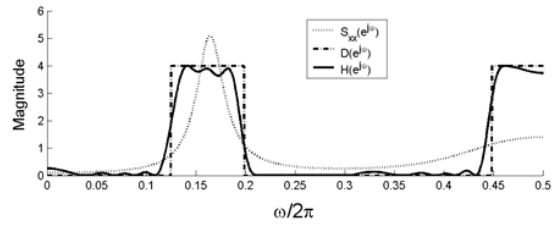


Fig. 4 Input psd  $S_{xx}(e^{j\omega})$ , magnitude squared response of ideal compaction filter,  $|D(e^{j\omega})|^2$  and designed FIR compaction filter  $|H(e^{j\omega})|^2$  for  $M = 4$

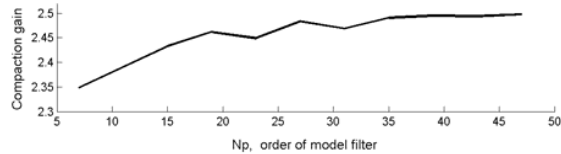


Fig 5 Compaction gain as a function of model filter

psd and the response of the ideal compaction filter  $D(e^{j\omega})$  are shown in Fig.4. As the order of the model filter is increased, the compaction gain improves, moving closer to the ideal value as shown in Fig 5. We get a compaction gain of 2.5 for  $N_p=47$ , which is very close to the ideal value of 2.5594. But since compaction filter is not unique with respect to phase, compaction gain does not increase monotonically with  $N_p$ . We have considered a minimum phase spectral factor in the design.

**Example 3**

The design of compaction filter is done using Method 2 described in section 3.2 for the psd shown in Fig 6. The number of channels is increased to  $M=6$  in this case. Using  $M=4$  results in almost same order for  $Q(z)$  and  $P_0(z^L)$  which means that spectral factorization of higher order polynomial is to be performed. Here  $F(z)$ ,  $P_0(z)$  and  $P_1(z)$  are designed for  $N_f = 6$  and  $N_{p_0} = 5, N_{p_1} = 8$  respectively. Fig.6 shows the magnitude squared response of the designed IFIR compaction

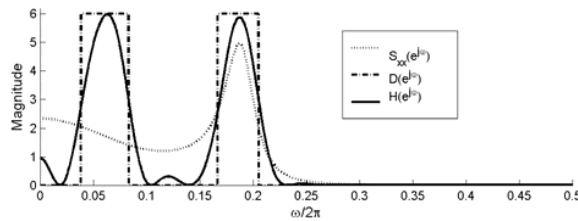


Fig. 6 Input psd  $S_{xx}(e^{j\omega})$ , magnitude squared response of ideal compaction filter,  $|D(e^{j\omega})|^2$  and designed FIR compaction filter  $|H(e^{j\omega})|^2$  for  $M = 6$

filter  $H(e^{j\omega})$  along with the response of the ideal compaction filter. The designed filter closely matches the ideal filter.

The compaction filter can be a multiband filter depending on the input psd. So the choice of higher values of  $L$  will

increase the order of the image suppressor filter. Hence  $L$  more than 3 is found to be incompatible. Since  $L$  depends on input psd and  $M$ , it may be difficult to find out an optimum value of  $L$ .

## V. CONCLUSION

We have designed an IFIR compaction filter using three approaches for different input power spectral densities and the simulation results are comparable with those of FIR compaction filters. As the order of this filter increases, the compaction gain achieved becomes very close to the ideal value. Since eigen filter approach is used, these IFIR design methods are less complex compared to the existing methods and found to be promising since there is appreciable saving in the number of multipliers required for its implementation.

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