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The Study of Increasing Environmental Temperature on the Dynamical Behaviour of a Prey-Predator System: A Model

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Abstract—It is well recognized that the green house gases such as Chlorofluoro Carbon (CFC), CH_4, CO_2 etc. are responsible directly or indirectly for the increase in the average global temperature of the Earth. The presence of CFC is responsible for the depletion of ozone concentration in the atmosphere due to which the heat accompanied with the sun rays are less absorbed causing increase in the atmospheric temperature of the Earth. The gases like CH_4 and CO_2 are also responsible for the increase in the atmospheric temperature. The increase in the temperature level directly or indirectly affects the dynamics of interacting species systems. Therefore, in this paper a mathematical model is proposed and analysed using stability theory to asses the effects of increasing temperature due to greenhouse gases on the survival or extinction of populations in a prey-predator system. A threshold value in terms of a stress parameter is obtained which determines the extinction or existence of populations in the underlying system.

Keywords—Equilibria, Green house gases, Model, Populations, Stability.

I. Introduction

THE atmospheric concentration of greenhouse gases such as carbondioxide, chlorofluoro carbon (CFC), methane and nitrous oxide is increasing due to rapid industrialization, extensive use of automobiles and burning of fossil fuels, construction of power plants and other anthropogenic activities. The excess of CFC depletes the ozone layer in the atmosphere and therefore the heat accompanied with the sun rays are less absorbed due to which the atmospheric temperature of the earth is increasing. The greenhouse gases like CH_4 and CO_2 are directly responsible for the increase in the atmospheric temperature. In recent years researchers, [2], [3] have predicted that the greenhouse gases will dramatically change global weather pattern in the next century and temperature of the Earth will rise in the years to come. Now, it is evident that the average global temperature of the Earth is rising due to increasing greenhouse gases. [4] provide an intriguing example of how one of the fundamental food-web properties that is connectance-changes with increasing temperature. In this paper a theoretical model based on assumptions from metabolic theory and foraging biology has been studied and it has been shown that the increasing temperature would have large effects on connectance with further consequences on the stability of interacting species systems. [5], [6] focus on how climatic warming affects the metabolic rate of organisms, that is, the power required to sustain them and how these changes in metabolism scale up to ecosystem processes. It has been shown in the papers of [7] and [8] that the increasing temperature changes the processes at different levels of biological organization. [9] has shown that species higher in the food web like top predators tend to be more sensitive to temperature change. [10] has found that top predators moving towards cooler climates may trigger trophic cascades and coextinctions may also occur. Thus, it is noticed that the increase in temperature level may directly or indirectly effect the dynamics of interacting species systems. Therefore, it is essential to assess mathematically the effects of increasing CO_2, CFC and CH_4 on populations in order to take necessary measures to avoid any adverse impact on an ecosystem. For understanding the consequences of the greenhouse effects on ecosystem an investigation of the interspecific interactions within biotic communities is required. A very few models to study temperature dependent interacting species systems exist [11], [12], [13], [14], [15], [16], [17], [18], [19], [20].

In view of the above, therefore in this paper, a mathematical model has been proposed and analyzed to study the effects of increasing temperature due to greenhouse gases on the survival or extinction of the populations in a prey-predator system. In the model it is assumed that the temperature increases directly due to greenhouse gases CH_4 and CO_2 and indirectly due to the greenhouse gas CFC (Chlorofluoro carbon) because of the depletion of the ozone concentration in the atmosphere by CFC. In the model it is further assumed that the rise in temperature negatively effects the intrinsic growth rate of the prey and adversely effects the prey-predator interaction phenomenon.

II. Mathematical Model

Let N_1 denote the density of a prey population which is growing logistically and N_2 denote the density of a predator population. C denotes the concentration of CFC (Chlorofluoro carbon). C_1 denotes the combined concentration of CH_4 and CO_2. Z denotes ozone concentration. We consider here that T is elevated temperature or average increased temperature of the surrounding environment where the species live.

For a predator let its searching time per unit prey is d_1. Hence, searching time of the predator population for prey density N_1 is d_1N_1. If the handling time spent per unit prey by a predator is d_2 then, the handling time for prey density N_1 is d_2N_1.

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It is assumed in the model construction that the searching time is adversely affected by the increasing temperature of the environment and therefore d_1 is taken to be $d_1/(1+B(T-T_0))$. Similarly, it is also assumed in the model formulation that the handling time is also adversely affected by the increasing temperature of the environment and therefore, d_2 is considered to be $d_2/(1+B(T-T_0))$. Thus, the total searching and handling time by a predator for prey density N_1 is given by

$$\frac{d_1N_1}{1+B(T-T_0)}+\frac{d_2N_1}{1+B(T-T_0)}=\frac{(d_1+d_2)N_1}{1+B(T-T_0)}$$
$$=\frac{a_1N_1}{1+B(T-T_0)} \quad (1)$$

where, $a_1=d_1+d_2$.

From the expression (1), we note that when the environment is at the normal temperature; T_0, that is, at $T=T_0$, the predator behaves naturally and there is no change in their searching and handling time. We also notice from (1) that the predation rate will only be affected when temperature T exceeds T_0. With the above notations and assumptions, the mathematical model of the system under consideration is given by the following system of nonlinear differential equations:

$$\frac{dN_1}{dt}=r_1(T)N_1-\frac{a_1N_1N_2}{1+B(T-T_0)}-\frac{r_{10}{N_1}^2}{K_{10}}, \quad (2)$$

$$\frac{dN_2}{dt}=-r_{20}N_2+\frac{a_2N_1N_2}{1+B(T-T_0)}, \quad (3)$$

$$\frac{dC}{dt}=P-\frac{C}{\tau}-\beta CZ, \quad (4)$$

$$\frac{dC_1}{dt}=I_0-\frac{C_1}{\tau_1}, \quad (5)$$

$$\frac{dZ}{dt}=Q_0-\alpha_2Z-\beta ZC, \quad (6)$$

$$\frac{dT}{dt}=h(C_1-C_{10})+\frac{K_1}{K_2+Z}-\alpha_1(T-T_0), \quad (7)$$

with the initial conditions as:
$N_1(0)>0$, $N_2(0)>0$, $C(0)\geq 0$, $C_1(0)\geq 0$, $Z(0)>0$, $T(0)>0$

In the present analysis we assume the following form of $r_1(T)$:

$$r_1(T)=r_{10}-r_{11}(T-T_0),\ r_1(T)>0\ \forall T,\ r_1(T_0)=r_{10} \quad (8)$$

and $a_2=\gamma a_1$ where γ is conversion coefficient. It may be noted here that at the normal temperature T_0, the growth rate of prey population is r_{10} which is its intrinsic growth rate. The system parameters are defined as follows:

r_{20} is natural death rate of predator population. B is a constant which measures the stress of temperature on both searching and handling time. I_0 is input rate of C_1. C_{10} is the natural atmospheric concentration of C_1. P is input rate of C. τ is average atmospheric residence time of CFC. τ_1 is average atmospheric residence time of C_1(CH_4 and CO_2 taken together). β is the depletion rate of ozone due to CFC. h is proportionality constant. Q_0 is the natural formation rate of ozone concentration in the atmosphere. T_0 is average normal temperature of earth surface of the area occupied by the populations under study. $r_1(T)$ is growth rate of population N_1. K_{10} is carrying capacity of prey population N_1. α_1 is coefficient of surface heat transfer and α_2 is natural depletion rate of ozone concentration. Here, all the parameters K_1, K_2, r_{10}, K_{10}, r_{11}, B, a_1, a_2, I_0, Q_0, β, α_1, α_2, h and r_{20} are taken to be positive constants.

III. Boundedness and Dynamical Behaviour

In this section we will establish that the solutions of the model given by the set of equations (2) to (7) with equation (8) are bounded in ${R^6}_+$. The boundedness of solutions is given by the following lemma.

Lemma 3.1: *All the solutions of the model will lie in the region* $V_1=\{(N_1,N_2,C,C_1,Z,T)\in {R^6}_+ : 0<N_1\leq K_{10}, 0<N_2\leq r_{10}K_{10}/\eta, 0<C\leq C_M, 0<C_1\leq C_{1M}, 0<Z_m\leq Z\leq Z_M, 0<T_m\leq T\leq T_M\}$, *as* $t\to\infty$, *for all positive initial values* $(N_1(0),N_2(0),C(0),C_1(0),Z(0),T(0))\in {R^6}_+$, *where,* $C_M=P\tau$, $C_{1M}=(I_0+Q_0)/\eta$, $\eta=min(1/\tau_1,\alpha_2)$, $T_M=\frac{hI_0\tau_1}{\alpha_1}+\frac{K_1}{\alpha_1(K_2+Z_m)}+T_0$, $T_m=\frac{\frac{K_1}{K_2+Z_M}-hC_{10}}{\alpha_1}>0$ *if* $\frac{K_1}{K_2+Z_M}>hC_{10}$
$Z_M=Q_0/\alpha_2$ *and* $Z_m=Q_0/(\alpha_2+\beta P\tau)$.
(for proof see Appendix A)

We now find all the feasible equilibria of the system (2) - (7).The system of equations (2) - (7) has three feasible equilibria $E_i(i=1,2,3)$ as given below:

1. $E_1({N_1}^*,{N_2}^*,C^*,{C_1}^*,Z^*,T^*)$,
where, ${N_1}^*=0$, ${N_2}^*=0$,

$$C^*=\frac{P\tau}{1+\beta\tau Z^*}, \quad (9)$$

$${C_1}^*=I_0\tau_1, \quad (10)$$

$$Z^*=\frac{-a_2+\sqrt{{a_2}^2-4a_1a_3}}{2a_1}, \quad (11)$$

$a_1=\alpha_2\beta\tau$,
$a_2=\alpha_2+\beta\tau(P-Q_0)$,
$a_3=-Q_0$,

$$T^*=\frac{1}{\alpha_1}\left[h({C_1}^*-C_{10})+\frac{K_1}{K_2+Z^*}+\alpha_1T_0\right]. \quad (12)$$

The equilibrium E_1 exists if $I_0\tau_1>C_{10}$.

2. $E_2({N_1}^*,{N_2}^*,C^*,{C_1}^*,Z^*,T^*)$,
where,

$${N_1}^*=\frac{r_1(T^*)K_{10}}{r_{10}},$$

${N_2}^*=0$ and
C^*, ${C_1}^*$,Z^*, T^* are given by (9) - (12) respectively.
The equilibrium E_2 exists if $I_0\tau_1>C_{10}$ and $r_1(T^*)>0$.

3. $E_3({N_1}^*,{N_2}^*,C^*,{C_1}^*,Z^*,T^*)$,
where,

$${N_1}^*=\frac{r_{20}}{\gamma a_1}(1+B(T^*-T_0)),$$

$${N_2}^*=\frac{1+B(T^*-T_0)}{K_{10}\gamma{a_1}^2}[r_1(T^*)K_{10}\gamma a_1-r_{10}r_{20}(1+B(T^*-T_0))]$$

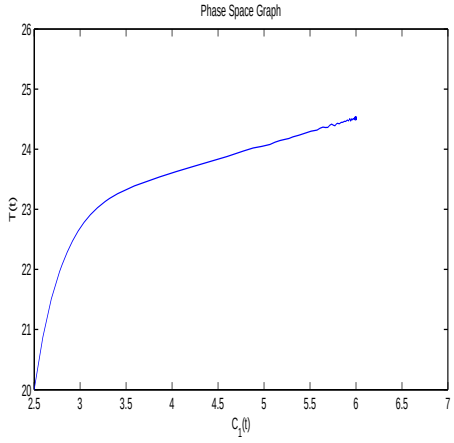

Fig. 1. Phase space diagram for concentration of greenhouse gases $C_1(t)$ and temperature $T(t)$.

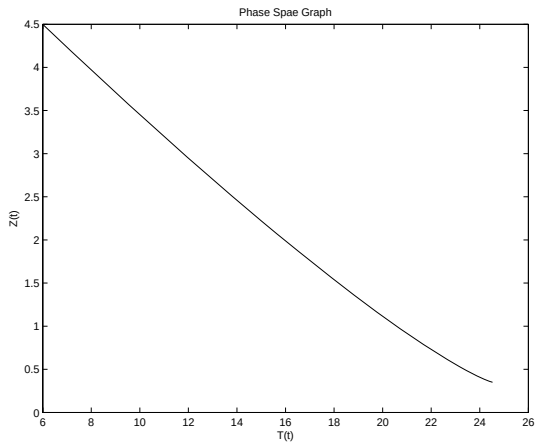

Fig. 2. Phase space diagram for temperature $T(t)$ and ozone concentration $Z(t)$.

and
C^*, $C_1{}^*$,Z^*, T^* are given by (9) - (12) respectively.
The equilibrium E_3 exists if $I_0\tau_1 > C_{10}$, $T^* > T_0$ and

$$\frac{r_{10}}{K_{10}} > \frac{\gamma a_1 r_{11}(T^* - T_0)}{\gamma a_1 K_{10} - r_{20}(1 + B(T^* - T_0))} \tag{13}$$

Remark: From the equilibrium value it is noted that the environmental temperature increases on account of increasing CH_4 and CO_2 concentrations (see Fig.1) and decreasing ozone concentration in the atmosphere (see Fig.2). Further, it may be noted that the equilibrium ozone concentration decreases due to the increase in the equilibrium concentration of chlorofluoro-carbon (see Fig.3).

Now we discuss the dynamical behaviour of the model by conducting the local and global stability analysis of the equilibria of the model.

A. Local Stability

The characteristic equation associated with the variational matrix about equilibrium E_1 is given by

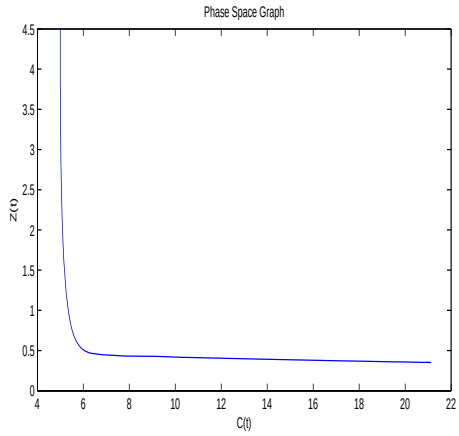

Fig. 3. Phase space diagram for concentration of chlorofluoro carbon $C(t)$ and ozone concentration $Z(t)$.

$$(J_1-\lambda)(J_2-\lambda)(J_5-\lambda)(J_8-\lambda)\{(J_7-\lambda)(J_3-\lambda)-J_4J_6\} = 0, \tag{14}$$

where, $J_1 = r_1(T^*)$, $J_2 = -r_{20}$, $J_3 = -1/\tau - \beta Z^*$,
$J_4 = -\beta C^*$, $J_5 = -1/\tau_1$, $J_6 = -\beta Z^*$,
$J_7 = -(\alpha_2 + \beta C^*)$, $J_8 = -\alpha_1$
From the nature of the roots of the characteristic equation (14) we observe that the equilibrium point E_1 is locally unstable provided $r_1(T^*) > 0$.
Remark: If $r_1(T^*) < 0$, then E_1 is locally asymptotically stable and obviously both prey and predator populations would die out eventually.
The characteristic equation related to the equilibrium point E_2 is obtained as

$$(G_2-\lambda)(G_5-\lambda)(G_1-\lambda)(G_8-\lambda)\{(G_3-\lambda)(G_7-\lambda)-G_4G_6\} = 0, \tag{15}$$

where, $G_1 = r_1(T^*) - 2r_{10}{N_1}^*/K_{10} = -r_1(T^*)$,
$G_2 = -r_{20} + a_2{N_1}^*/(1 + B(T^* - T_0))$, $G_3 = -1/\tau - \beta Z^*$,
$G_4 = -\beta C^*$, $G_5 = -1/\tau_1$, $G_6 = -\beta Z^*$,
$G_7 = -(\alpha_2 + \beta C^*)$, $G_8 = -\alpha_1$
From the characterstic equation (15) we find that the equilibrium point E_2 is linearly asymptotically stable under the condition given by:

$$\frac{r_{10}}{K_{10}} < \frac{\gamma a_1 r_{11}(T^* - T_0)}{\gamma a_1 K_{10} - r_{20}(1 + B(T^* - T_0))} \; and \; T^* > T_0 \tag{16}$$

The characteristic equation associated with the variational matrix about equilibrium E_3 is given by

$$(P_7 - \lambda)(\alpha_1 + \lambda)\{P_6P_8 - (P_5 - \lambda)(P_9 - \lambda)\} \times\{(P_1 - \lambda)(P_4 - \lambda) - P_2P_3\} = 0, \tag{17}$$

where, $P_1 = r_1(T^*) - \frac{a_1{N_2}^*}{1+B(T^*-T_0)} - \frac{2r_{10}{N_1}^*}{K_{10}} = -\frac{r_{10}r_{20}}{K_{10}\gamma a_1}(1+B(T^* - T_0))$,
$P_2 = -\frac{a_1{N_1}^*}{1+B(T^*-T_0)}$, $P_3 = \frac{\gamma a_1{N_2}^*}{1+B(T^*-T_0)}$,
$P_4 = -r_{20} + \frac{\gamma a_1{N_1}^*}{1+B(T^*-T_0)}$, $P_5 = -\frac{1}{\tau} - \beta Z^*$, $P_6 = -\beta C^*$,
$P_7 = -1/\tau_1$, $P_8 = -\beta Z^*$, $P_9 = -(\alpha_2 + \beta C^*)$

From the nature of the roots of characteristic equation (17) we find that the equilibrium point E_3 is linearly asymptotically stable provided $T^* > T_0$.
Further, from the above analysis it is noted that E_2 is linearly stable only when E_3 does not exist and E_3 is linearly stable only if E_2 is unstable.
It is observed from the analysis that the stability conditions are depending upon the equilibrium temperature level and average normal temperature.

B. Global Stability

Next, we discuss the global stability of the interior equilibrium point E_3.
Theorem 3.1: *The box V_1 is a compact positively invariant set in space* (N_1, N_2, C, C_1, Z, T).
Proof: Consider the system given by Eqs. (2)-(7). To prove the theorem, we consider the point $X' = ({N_1}', {N_2}', C', {C_1}', Z', T')$ out side the box V_1, with ${N_1}' > K_{10}, {N_2}' > r_{10}K_{10}/\eta, C' > P\tau, {C_1}' > (I_0 + Q_0)/\eta, Z' > Q_0/\alpha_2$ and $T' > D/\alpha_1$ and take the box V_1 in the phase space (N_1, N_2, C, C_1, Z, T) with one vertex located at the origin and the other at X'. Now, let us compute the angle that the flow makes with each one of the faces of V_1 not lying on the coordinate planes. Consider the planes $\Pi_{N_1} : N_1 = {N_1}', \Pi_{N_2} : N_2 = {N_2}', \Pi_C : C = C', \Pi_{C_1} : C_1 = {C_1}', \Pi_Z : Z = Z'$ and $\Pi_T : T = T'$ and let $n_{N_1}, n_{N_2}, n_C, n_{C_1}, n_Z$ and n_T are outward unit normal vectors (with respect to box V_1) respectively to each plane. Then

$$n_{N_1}\frac{dX}{dt}|_{\pi_{N_1}} = N_1'\left(r_1(T') - \frac{r_{10}N_1'}{K_{10}} - \frac{a_1N_2'}{1+B(T'-T_0}\right)$$

then we get

$$n_{N_1}\frac{dX}{dt}|_{\pi_{N_1}} \leq N_1'\left(-r_{11}T' - \frac{a_1N_2'}{1+B(T'-T_0)}\right)$$

hence,

$$n_{N_1}\frac{dX}{dt}|_{\pi_{N_1}} \leq 0$$

Similarly we can show that

$$n_{N_2}\frac{dX}{dt}|_{\pi_{N_2}} \leq 0, n_C\frac{dX}{dt}|_{\pi_C} \leq 0, n_{C_1}\frac{dX}{dt}|_{\pi_{C_1}} \leq 0,$$
$$n_Z\frac{dX}{dt}|_{\pi_Z} \leq 0, n_T\frac{dX}{dt}|_{\pi_T} \leq 0$$

where,
$dX/dt = (dN_1/dt, dN_2/dt, dC/dt, dC_1/dt, dZ/dt, dT/dt)$.
Thus, the flow along the normals to each of the plane is again moving towards the box. Clearly we can say that box V_1 is compact positively invariant box. This completes the proof of the theorem 3.1. Now it is clear by the above theorem that the trajectories of the system cannot cross V_1 once they enter inside. It is also observed that the interior equilibrium E_3 lies inside V_1. Moreover, E_3 is only attractor inside V_1, which is established in the following theorem.
Theorem 3.2: The equilibrium E_3 is non-linearly asymptotically stable with respect to solution initiating in the interior of V_1 if the following inequalities hold:

$$A_1r_{10}\left(\frac{\gamma a_1N_M}{1+B(T_m-T_0)} + r_{20}\right) > \frac{K_{10}{a_1}^2(1-\gamma{N_2}^*A_1)^2}{(1+B(T_m-T_0))^2} \tag{18}$$

and

$$2A_4A_2(\alpha_2+\beta C^*)\left(\frac{1}{\tau}+\beta Z_m\right) > \beta^2(A_4C^*+A_2Z_m)^2 \tag{19}$$

(for proof see Appendix B)

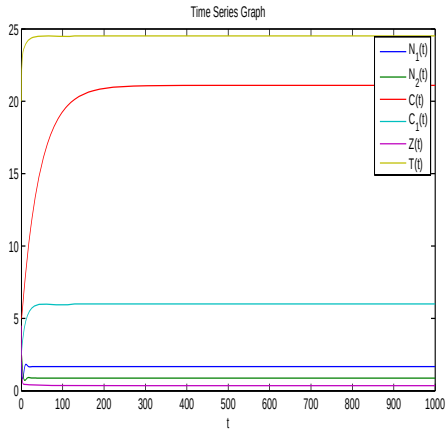

Fig. 4. Behavior of the trajectories of model with respect to time when $B < B_0$.

IV. Numerical Example

For the model, consider the following values of parameters-
$r_{10} = 0.9$, $r_{20} = 0.5$, $K_{10} = 3.0$, $r_{11} = 0.001$, $a_1 = 0.4$, $\gamma = 0.75$, $P = 0.5$,
$T_0 = 22$, $\tau = 60.0$, $\tau_1 = 10.0$, $\beta = 0.02$, $I_0 = 0.6$, $Q_0 = 0.5$, $C_{10} = 0.5$,
$h = 2.5$, $\alpha_1 = 1.2$, $\alpha_2 = 1.0$, $K_1 = 0.5$, $K_2 = 1.5$.
For the above set of values of parameters and $B = 0.001 < B_0 = 0.287$, we obtain the following value of interior equilibrium point $E_3({N_1}^*, {N_2}^*, C^*, {C_1}^*, Z^*, T^*)$
${N_1}^* = 1.67$, ${N_2}^* = 0.87$, $C^* = 21.09$, ${C_1}^* = 6.0$, $Z^* = 0.35$, $T^* = 24.52$
It is noted that for the above set of parametric values, the stability conditions (13), (18) and (19) are satisfied. Hence, ${E_3}^*$ is globally asymptotically stable (see Fig. 4).
For the above parametric values, when $B = 0.288 > B_0 = 0.287$ is considered then the conditions for the existence of the interior equilibrium point E_3 is violated and in this case the equilibrium point E_2 exists with the following equilibrium values:
${N_1}^* = 2.83$, ${N_2}^* = 0.00$, $C^* = 21.09$, ${C_1}^* = 6.0$, $Z^* = 0.35$, $T^* = 24.52$
The linear stability condition given by (16) for E_2 is satisfied for the above set of parametric values with $B = 0.0238$ and the stability behaviour of the model for $B > B_0$ is shown in Fig. 5.

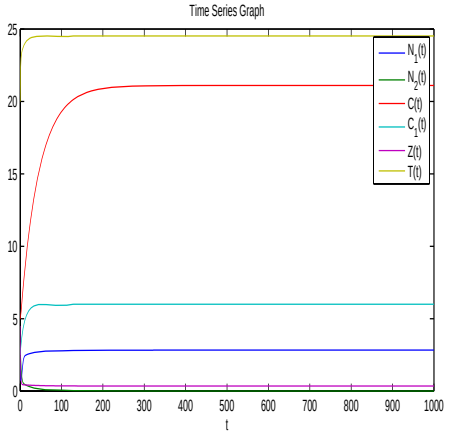

Fig. 5. Behavior of the trajectories of model with respect to time when $B > B_0$.

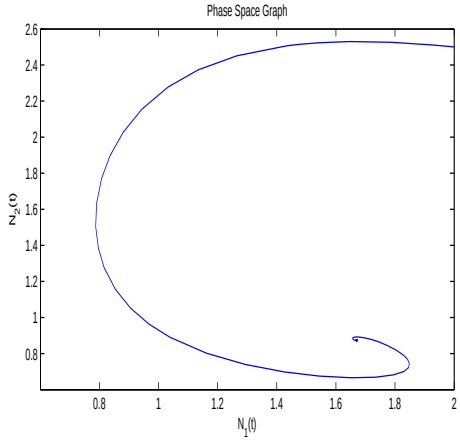

Fig. 6. Phase space diagram for prey population $N_1(t)$ and predator population $N_2(t)$ when $B < B_0$.

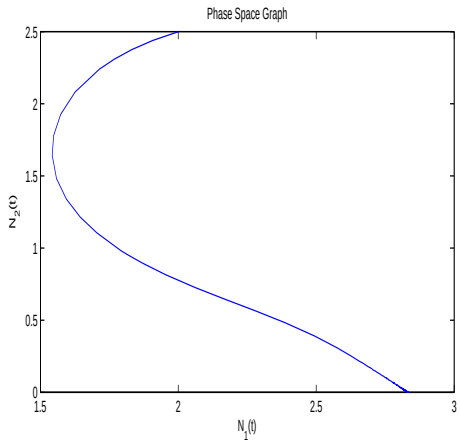

Fig. 7. Phase space diagram for prey population $N_1(t)$ and predator population $N_2(t)$ when $B > B_0$.

V. Conclusion

From the linear stability analysis of the equilibrium point E_2 it is concluded that the population with density N_2 would tend to extinction and population with density N_1 would survive but at lower equilibrium value due to the decrease in its growth rate on account of elevated temperature. The non-trivial positive equilibrium point E_3 exists only when the equilibrium point E_2 is unstable. Hence, from the linear as well as non-linear stability analysis of the non-trivial positive equilibrium E_3 it is concluded that the prey and predator populations would co-exist if the parameter B which measures the stress of temperature is less than its threshold value B_0 (see Figs. 4 and 6). However, it may be noted here that the prey and predator populations would co-exist even if the threshold value B_0 is taken to be zero. Further, it is shown that if the value of the parameter B is more than its threshold value B_0 then the predator population tend to extinction (see Figs. 5 and 7). Numerical example has been supplemented to validate the analytical results. The graphs of all the variables have been plotted with respect to time and from these graphs the stability behaviour is illustrated (see Figs. 4 and 5).

Appendix A
Proof of Lemma 3.1

Proof: From Eqs. (2) and (3) we get,

$$\frac{d(N_1+N_2)}{dt} \leq (r_{10} - r_{11}(T-T_0))N_1 - r_{20}N_2$$

$$-\frac{N_1N_2}{1+B(T-T_0)}(a_1-a_2)$$

$$\leq r_{10}K_{10} - r_{11}(T_m - T_0)N_1 - r_{20}N_2 - \frac{N_1N_2}{1+B(T-T_0)}(a_1-a_2)$$

$$\leq r_{10}K_{10} - \eta(N_1+N_2)$$

if $a_2 < a_1$, i.e. $\gamma < 1$.
Where $\eta = min(r_{11}(T_m - T_0), r_{20})$ and it is assumed that $T_m > T_0$.

Then by the usual comparison theorem we get as $t \to \infty$:

$$(N_1+N_2) \leq \frac{r_{10}K_{10}}{\eta}$$

and hence,

$$N_2 \leq \frac{r_{10}K_{10}}{\eta}$$

From Eq. (7) and using the solution of C_1 in eq.(7) we get,

$$\frac{dT}{dt} \leq D + hBe^{-t/\tau_1} - \alpha_1 T$$

where, $A = \frac{K_1}{K_2+Z_m} + \alpha_1 T_0$ and $D = hI_0\tau_1 + A$
Then by the usual comparison theorem we get as $t \to \infty$:

$$T \leq \frac{D}{\alpha_1}$$

i.e.

$$T \leq \frac{hI_0\tau_1}{\alpha_1} + \frac{K_1}{\alpha_1(K_2+Z_m)} + T_0 = T_M$$

Again from Eq. (7) we get ,

$$\frac{dT}{dt} \geq \left(\frac{K_1}{K_2+Z_M} - hC_{10}\right) - \alpha_1 T$$

Then by the usual comparison theorem we get as $t \to \infty$:

$$T \geq \frac{\frac{K_1}{K_2+Z_M} - hC_{10}}{\alpha_1} = T_m$$

and $T_m > 0$ if $\frac{K_1}{K_2+Z_M} > hC_{10}$
Similarly from Eqs. (4) and (6), we get as $t \to \infty$:

$$C \leq P\tau = C_M,\ Z \leq \frac{Q_0}{\alpha_2} = Z_M$$

Again from Eq. (6) we get

$$\frac{dZ}{dt} \geq Q_0 - \alpha_2 Z - \beta Z P \tau$$

By the usual comparison theorem we get as $t \to \infty$:

$$Z \geq \frac{Q_0}{\alpha_2 + \beta P \tau} = Z_m$$

By adding Eqs. (5) and (6), we get

$$\frac{d(C_1+Z)}{dt} \leq I_0 + Q_0 - \eta(C_1+Z)$$

where, $\eta = min(\frac{1}{\tau_1}, \alpha_2)$ and by usual comparison theorem we get as $t \to \infty$:

$$(C_1+Z) \leq \frac{I_0+Q_0}{\eta}$$

and hence,

$$C_1 \leq \frac{I_0+Q_0}{\eta} = C_{1M}$$

This completes the proof of the lemma 3.1.

Appendix B
Proof of Theorem 3.2

Proof: Taking the perturbations about the equilibrium value as follows:

$$N_1 = {N_1}^* + u_1(t), N_2 = {N_2}^* + u_2(t), C = C^* + v_1(t),$$
$$C_1 = {C_1}^* + w_1(t), Z = Z^* + x(t), T = T^* + t_1(t).$$

the non-linearised system of equations from (2) to (7) about equilibrium point E_3 is given by

$$\frac{du_1}{dt} = ({N_1}^*+u_1)\{-r_{11}t_1 - \frac{a_1 {N_2}^* B t_1}{(1+B(T^*-T_0))(1+B(T-T_0))} - \frac{r_{10}u_1}{K_{10}} - \frac{a_1 u_2}{1+B(T-T_0)}\} \quad (20)$$

$$\frac{du_2}{dt} = -r_{20}u_2 - \frac{B t_1 \gamma a_1 {N_1}^* {N_2}^*}{(1+B(T-T_0))(1+B(T^*-T_0))} + \frac{\gamma a_1}{1+B(T-T_0)}(N_1 u_2 + {N_2}^* u_1) \quad (21)$$

$$\frac{dv_1}{dt} = -\frac{v_1}{\tau} - \beta C^* x - \beta(Z^*+x)v_1 \quad (22)$$

$$\frac{dw_1}{dt} = -\frac{w_1}{\tau_1} \quad (23)$$

$$\frac{dx}{dt} = -\alpha_2 x - \beta C^* x - \beta(Z^*+x)v_1 \quad (24)$$

$$\frac{dt_1}{dt} = \frac{-K_1}{(K_2+Z^*)(K_2+Z^*+x)} + hw_1 - \alpha_1 t_1 \quad (25)$$

Consider,

$$G(t) = \left[u_1 - {N_1}^* log(1+\frac{u_1}{{N_1}^*})\right] + \frac{1}{2}A_1 {u_2}^2 + \frac{1}{2}A_2 {v_1}^2 + \frac{1}{2}A_3 {w_1}^2 + \frac{1}{2}A_4 x^2 + \frac{1}{2}A_5 {t_1}^2$$

where, $A_i (i = 1 \text{ to } 5)$ are arbitrary positive constants.
The time derivative of $G(t)$ is given by

$$\frac{dG}{dt} = \frac{u_1}{{N_1}^*+u_1}\frac{du_1}{dt} + A_1 u_2 \frac{du_2}{dt} + A_2 v_1 \frac{dv_1}{dt} + A_3 w_1 \frac{dw_1}{dt} + A_4 x_1 \frac{dx_1}{dt} + A_5 t_1 \frac{dt_1}{dt}$$

Now, using the system of equations (20)-(25) in dG/dt in the region V_1, we get

$$\begin{aligned}\frac{dG}{dt} \leq -\{&\frac{1}{2}\frac{r_{10}}{K_{10}}{u_1}^2 + \frac{a_1}{1+B(T_m-T_0)}(1-\gamma {N_2}^* A_1)u_1 u_2 \\ &+ \frac{A_1}{2}\left(\frac{\gamma a_1 N_M}{1+B(T_m-T_0)} + r_{20}\right){u_2}^2 + \frac{1}{2}\frac{r_{10}}{K_{10}}{u_1}^2 \\ &+ \left(r_{11} + \frac{a_1 {N_2}^* B}{(1+B(T^*-T_0))(1+B(T_m-T_0))}\right)u_1 t_1 \\ &+ \frac{A_5}{4}\alpha_1 {t_1}^2 + \frac{A_1}{2}\left(\frac{\gamma a_1 N_M}{1+B(T_m-T_0)} + r_{20}\right){u_2}^2 \\ &+ \frac{A_1 B \gamma a_1 {N_1}^* {N_2}^*}{(1+B(T^*-T_0))(1+B(T_m-T_0))}u_2 t_1 + \frac{A_5}{4}\alpha_1 {t_1}^2 \\ &+ \frac{A_4}{2}(\alpha_2 + \beta C^*)x^2 + \beta(A_4 C^* + A_2 Z_m)x v_1 \\ &+ A_2(\frac{1}{\tau} + \beta Z_m){v_1}^2 + \frac{A_4}{2}(\alpha_2+\beta C^*)x^2 \\ &+ \frac{A_5 K_1}{(K_2+Z^*)(K_2+Z_M)}x t_1 + \frac{A_5}{4}\alpha_1 {t_1}^2 \\ &+ \frac{A_3}{\tau}{w_1}^2 - A_5 h w_1 t_1 + \frac{A_5}{4}\alpha_1 {t_1}^2\}\end{aligned}$$

Using the sylvester's criteria in the right hand side of the above expression and then choosing A_1, A_3, A_4, A_5 as follow

$$A_5 > 2\frac{K_{10}}{\alpha_1 r_{10}}\left[r_{11} + \frac{a_1 {N_2}^* B}{(1+B(T^*-T_0))(1+B(T_m-T_0))}\right]^2,$$

$$A_1\left[\frac{B\gamma a_1 {N_1}^* {N_2}^*}{(1+B(T^*-T_0))(1+B(T_m-T_0))}\right]^2 < \frac{A_5}{2}\left[r_{20} + \frac{\gamma a_1 N_M}{1+B(T_m-T_0)}\right]\alpha_1$$

and

$$A_4(\alpha_2 + \beta C^*) > 2A_5 \left[\frac{K_1}{(K_2 + Z)(K_2 + Z_m)} \right]^2$$

it may be shown that dG/dt is negative definite if the conditions (18) and (19) are satisfied. Thus, it is proved that E_3 is globally (non-linearly) asymptotically stable in the region V_1.

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