

# An Analysis of Global Stability of Cohen-Grossberg Neural Networks with Multiple Time Delays

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**Abstract**—This paper presents a new sufficient condition for the existence, uniqueness and global asymptotic stability of the equilibrium point for Cohen-Grossberg neural networks with multiple time delays. The results establish a relationship between the network parameters of the neural system independently of the delay parameters. The results are also compared with the previously reported results in the literature.

**Keywords**—Equilibrium and stability analysis, Cohen-Grossberg Neural Networks, Lyapunov Functionals.

## I. INTRODUCTION

IN recent years, equilibrium and stability properties of different classes of neural networks such as Cohen-Grossberg neural networks, Hopfield-type of neural networks and cellular neural networks have been intensively studied and applied to various engineering problems [1]-[13]. In particular, the existence, uniqueness and global asymptotic stability of the equilibrium point for neural networks proved to be an important property as neural networks with such a convergence dynamics is crucial to solve optimization problems. In recent literature, many researchers have studied the equilibria and stability properties of neural networks and presented various sufficient conditions for the uniqueness and global asymptotic stability of the equilibrium point for neural networks [1]-[13]. On the other hand, it is well known that a significant time delay may occur during the communication between neurons, which may cause a complete change in the dynamical behavior of neural systems. Therefore, determining the affect of the time delays on the equilibrium and stability properties of neural networks is of prime importance. In this paper, by employing more general types of Lyapunov functionals, we will present a new sufficient condition for the uniqueness and global asymptotic stability of the equilibrium point for Cohen-Grossberg neural networks with time delays.

## II. COHEN-GROSSBERG NEURAL NETWORK MODEL AND SOME BASIC CONCEPTS

Cohen-Grossberg neural network model we consider in this paper is assumed to be described by the following set of

differential equations :

$$\begin{aligned} \dot{x}_i(t) = & d_i(x_i(t))[-c_i(x_i(t)) + \sum_{j=1}^n a_{ij}f_j(x_j(t)) \\ & + \sum_{j=1}^n b_{ij}f_j(x_j(t - \tau_{ij})) + u_i] \end{aligned} \quad (1)$$

where  $n$  is the number of the neurons in the network,  $x_i$  denotes the state of the  $i$ th neuron,  $d_i(x_i)$  represents an amplification function, and  $c_i(x_i)$  is a behaved function such that the solution of network model (1) remains bounded. The constants  $a_{ij}$  denote the strengths of the neuron interconnections within the network, the constants  $b_{ij}$  denote the strengths of the neuron interconnections with time delay parameters  $\tau_{ij}$ . Finally, the functions  $f_i(\cdot)$  denote the neuronal activations and the constants  $u_i$  are some external inputs. In system (1),  $\tau_{ij}(t) \geq 0$  represents the delay parameter with  $\tau = \max(\tau_{ij})$ ,  $1 \leq i, j \leq n$ . Accompanying the neural system (1) is an initial condition of the form :  $x_i(t) = \phi_i(t) \in C([- \tau, 0], R)$ , where  $C([- \tau, 0], R)$  denotes the set of all continuous functions from  $[- \tau, 0]$  to  $R$ .

We now give some usual assumptions on the functions  $d_i$ ,  $c_i$  and  $f_i$  :

$A_1$  : The functions  $d_i(x)$ ,  $i = 1, 2, \dots, n$  are continuously bounded, and there exist positive constants  $\mu_i$  and  $\rho_i$  such that  $0 < \mu_i \leq d_i(x) \leq \rho_i$ ,  $\forall x \in R$ .

$A_2$  : The functions  $c_i$  are continuous and there exist constants  $\gamma_i > 0$  such that

$$\frac{c_i(x) - c_i(y)}{x - y} = \frac{|c_i(x) - c_i(y)|}{|x - y|} \geq \gamma_i > 0, \quad i = 1, 2, \dots, n, \quad \forall x, y \in R, x \neq y$$

$A_3$  : There exist some positive constants  $G_i$  such that

$$0 \leq \frac{f_i(x) - f_i(y)}{x - y} \leq G_i, \quad i = 1, 2, \dots, n, \quad \forall x, y \in R, x \neq y$$

We recall some basic vector and matrix norms. For  $x = (x_1, x_2, \dots, x_n)^T$ , the three commonly used vector norms are :  $\|x\|_1 = \sum_{i=1}^n |x_i|$ ,  $\|x\|_2 = \sqrt{\sum_{i=1}^n x_i^2}$ ,  $\|x\|_\infty = \max_{1 \leq i \leq n} |x_i|$ . For any matrix  $A = (a_{ij})_{n \times n}$ ,  $\|A\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}|$ ,  $\|A\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^n |a_{ij}|$ ,  $\|A\|_2 = \sqrt{\lambda_M(A^T A)}$ , where  $\lambda_M(A^T A)$  denotes the maximum eigenvalue of the matrix  $A^T A$ .

This work was supported by the Scientific and Technological Research Council of Turkey, under Project 104E024.

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### III. EXISTENCE AND UNIQUENESS ANALYSIS OF EQUILIBRIUM POINT

In this section we present the following condition that establishes the existence and uniqueness of the equilibrium point for system (1).

*Theorem 1:* Suppose that the assumptions  $A_1$ ,  $A_2$  and  $A_3$  are satisfied. Then, the Cohen-Grossberg neural networks defined by (1) has a unique equilibrium point for each  $u$ , if the following condition holds :

$$\Omega = 2\Gamma G^{-1} - A - A^T - Q > 0$$

where  $\Gamma = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_n)$  and  $G = \text{diag}(G_1, G_2, \dots, G_n)$ , and  $Q = \text{diag}(q_1, q_2, \dots, q_n)$  with

$$q_i = \sum_{j=1}^n (|b_{ij}| + |b_{ji}|), i = 1, 2, \dots, n$$

*Proof:* Let  $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$  denote an equilibrium point of neural network model (1). Then,  $x^*$  satisfies

$$D(x^*)[-C(x^*) + Af(x^*) + Bf(x^*) + u] = 0 \quad (2)$$

Since  $D(x^*)$  is a positive diagonal matrix, from (2), it follows that

$$-C(x^*) + Af(x^*) + Bf(x^*) + u = 0 \quad (3)$$

Let

$$H(x) = -C(x) + Af(x) + Bf(x) + u = 0 \quad (4)$$

where  $H(x) = (h_1(x), h_2(x), \dots, h_n(x))^T$  with

$$h_i(x) = -c_i(x_i) + \sum_{j=1}^n a_{ij}f_j(x_j) + \sum_{j=1}^n b_{ij}f_j(x_j) + u_i, \\ i = 1, 2, \dots, n$$

Since every solution of  $H(x) = 0$  is an equilibrium point of (1), it follows that, for system defined by (1), there exists a unique equilibrium point for every input vector  $u$  if  $H(x)$  is homeomorphism of  $R^n$  (see [5]). In the following, we will prove that  $H(x)$  is a homeomorphism of  $R^n$ .

Let choose two vectors  $x, y \in R^n$  such that  $x \neq y$ . For  $H(x)$  defined by (4), we can write

$$H(x) - H(y) = -(C(x) - C(y)) + A(f(x) - f(y)) \\ + B(f(x) - f(y)) \quad (5)$$

First, consider the case  $x \neq y$  with  $f(x) - f(y) = 0$ ; in this case, we have

$$H(x) - H(y) = -(C(x) - C(y))$$

Under assumption  $A_2$ ,  $x \neq y$  implies that  $C(x) \neq C(y)$ . On the other hand, if  $C(x) \neq C(y)$ , then  $H(x) \neq H(y)$ . Hence,  $x \neq y$  implies that  $H(x) \neq H(y)$ . Now consider the case where  $x - y \neq 0$  and  $f(x) - f(y) \neq 0$ . Multiplying both sides of (5) by  $2(f(x) - f(y))^T$  results in

$$2(f(x) - f(y))^T(H(x) - H(y)) \\ = -2(f(x) - f(y))^T(C(x) - C(y)) \\ + 2(f(x) - f(y))^T A(f(x) - f(y)) \\ + 2(f(x) - f(y))^T B(f(x) - f(y)) \\ = -2(f(x) - f(y))^T(C(x) - C(y)) \\ + (f(x) - f(y))^T(A + A^T)(f(x) - f(y)) \\ + 2(f(x) - f(y))^T B(f(x) - f(y))$$

From the assumptions  $A_2$  and  $A_3$ , we obtain

$$(f(x) - f(y))^T(C(x) - C(y)) \\ \geq (f(x) - f(y))^T \Gamma(x - y) \\ \geq (f(x) - f(y))^T \Gamma G^{-1}(f(x) - f(y))$$

We also note the following inequality

$$2(f(x) - f(y))^T B(f(x) - f(y)) \\ = \sum_{i=1}^n \sum_{j=1}^n 2b_{ij}(f_i(x_i) - f_i(y_i))(f_j(x_j) - f_j(y_j)) \\ \leq \sum_{i=1}^n \sum_{j=1}^n 2|b_{ij}||f_i(x_i) - f_i(y_i)||f_j(x_j) - f_j(y_j)| \\ \leq \sum_{i=1}^n \sum_{j=1}^n |b_{ij}|((f_i(x_i) - f_i(y_i))^2 + (f_j(x_j) - f_j(y_j))^2) \\ = \sum_{i=1}^n \sum_{j=1}^n |b_{ij}|(f_i(x_i) - f_i(y_i))^2 \\ + \sum_{i=1}^n \sum_{j=1}^n |b_{ji}|(f_i(x_i) - f_i(y_i))^2 \\ = \sum_{i=1}^n \sum_{j=1}^n (|b_{ij}| + |b_{ji}|)(f_i(x_i) - f_i(y_i))^2 \\ = \sum_{i=1}^n q_i(f_i(x_i) - f_i(y_i))^2 \\ = (f(x) - f(y))^T Q(f(x) - f(y))$$

Using the above two inequalities in (6) results in

$$2(f(x) - f(y))^T(H(x) - H(y)) \quad (6)$$

$$\leq -2(f(x) - f(y))^T \Gamma G^{-1}(f(x) - f(y)) \\ + (f(x) - f(y))^T(A + A^T)(f(x) - f(y)) \\ + (f(x) - f(y))^T Q(f(x) - f(y)) \\ = -(f(x) - f(y))^T \Omega(f(x) - f(y)) \quad (7)$$

For  $f(x) - f(y) \neq 0$ ,  $\Omega > 0$  implies that

$$2(f(x) - f(y))^T(H(x) - H(y)) < 0$$

form which it can be concluded that  $H(x) \neq H(y)$  when  $f(x) - f(y) \neq 0$ , thus proving that  $H(x) \neq H(y)$  for all  $x \neq y$ .

Now, if we let  $y = 0$  in (7), then we obtain :

$$2(f(x) - f(0))^T(H(x) - H(0)) \\ \leq -(f(x) - f(0))^T \Omega f(x) - f(0)$$

leading to

$$\begin{aligned} & |2(f(x) - f(0))^T(H(x) - H(0))| \\ & > (f(x) - f(0))^T \Omega f(x) - f(0) \end{aligned}$$

from which it follows that

$$\begin{aligned} & 2\|f(x) - f(0)\|_\infty \|H(x) - H(0)\|_1 \\ & > \lambda_m(\Omega) \|f(x) - f(0)\|_2^2 \end{aligned}$$

Considering that  $\|f(x) - f(0)\|_\infty \leq \|f(x) - f(0)\|_2$ ,  $\|H(x) - H(0)\|_1 \leq \|H(x)\|_1 + \|H(0)\|_1$  and  $\|f(x) - f(0)\|_1 \geq \|f(x)\|_1 - \|f(0)\|_1$ , we obtain

$$\begin{aligned} & \|H(x)\|_1 \\ & > \frac{\lambda_m(\Omega) \|f(x)\|_2 - \lambda_m(\Omega) \|f(0)\|_2 - 2\|H(0)\|_1}{2\|P\|_\infty} \end{aligned}$$

Since,  $\|H(0)\|_1$  and  $\|f(0)\|_2$  are finite, then  $\|H(x)\| \rightarrow \infty$  as  $\|f(x)\| \rightarrow \infty$ . (We should point out here that, for unbounded activation functions,  $\|f(x)\| \rightarrow \infty$  if  $\|x\| \rightarrow \infty$ . Therefore, we can conclude that  $\|H(x)\| \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ . As for the bounded activation functions, it is always true that  $\|H(x)\| \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ . [5]) Thus, it follows that the map  $H(x) : R^n \rightarrow R^n$  is homomorphism of  $R^n$ , hence there exists a unique  $x^*$  such  $H(x^*) = 0$  which is a solution of (1). Hence, the proof of the existence and uniqueness of the equilibrium point is now completed. ■

#### IV. STABILITY ANALYSIS OF EQUILIBRIUM POINT

In this section, we will prove that  $\Omega > 0$  also implies the global asymptotic stability of the equilibrium point for neural system (1). To simplify the proofs, we will first shift the equilibrium point of system (1) to the origin. Suppose that  $x^*$  is an equilibrium point of system (1). By using the transformation  $z(t) = x(t) - x^*$ , the equilibrium point  $x^*$  can be shifted to the origin. The neural network model (1) can be rewritten as :

$$\begin{aligned} \dot{z}_i(t) = & \alpha_i(z_i(t))[-\beta_i(z_i(t)) + \sum_{j=1}^n a_{ij}g_j(z_j(t)) \\ & + \sum_{j=1}^n b_{ij}g_j(z_j(t - \tau_{ij}))] \end{aligned} \quad (8)$$

For the transformed system (8), we have

$$\alpha_i(z_i(t)) = d_i(z_i(t) + x_i^*), \quad i = 1, 2, \dots, n$$

$$\beta_i(z_i(t)) = c_i(z_i(t) + x_i^*) - c_i(x_i^*), \quad i = 1, 2, \dots, n$$

$$g_i(z_i(t)) = f_i(z_i(t) + x_i^*) - f_i(x_i^*), \quad i = 1, 2, \dots, n$$

Assumptions  $A_1, A_2, A_3$  respectively imply that

$$0 < \mu_i \leq \alpha_i(z_i(t)) \leq \rho_i, \quad i = 1, 2, \dots, n$$

$$z_i(t)\beta_i(z_i(t)) \geq \gamma_i z_i^2(t), \quad i = 1, 2, \dots, n$$

$$|g_i(z_i(t))| \leq G_i |z_i(t)|, \quad z_i(t)g_i(z_i(t)) \geq 0, \quad i = 1, 2, \dots, n$$

In order to show that  $\Omega > 0$  is also a sufficient condition for global asymptotic stability of the origin of (8), the following positive definite Lyapunov functional will be employed :

$$\begin{aligned} V(z(t)) = & 2n \sum_{i=1}^n \int_0^{z_i(t)} \frac{s}{\alpha_i(s)} ds \\ & + 2\alpha \sum_{i=1}^n \int_0^{z_i(t)} \frac{g_i(s)}{\alpha_i(s)} ds \\ & + \sum_{i=1}^n \sum_{j=1}^n \alpha |b_{ij}| \int_{t-\tau_{ij}}^t g_j^2(z_j(\xi)) d\xi \\ & + \sum_{i=1}^n \sum_{j=1}^n \varepsilon \int_{t-\tau_{ij}}^t g_j^2(z_j(\xi)) d\xi \\ & + \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 \int_{t-\tau_{ij}}^t g_j^2(z_j(\xi)) d\xi \end{aligned}$$

where the  $\alpha$  and  $\varepsilon$  are positive constants to be determined later. The time derivative of the functional along the trajectories of system (8) is obtained as follows

$$\begin{aligned} \dot{V}(z(t)) = & -2 \sum_{i=1}^n n \beta_i(z_i(t)) z_i(t) \\ & + \sum_{i=1}^n \sum_{j=1}^n 2n a_{ij} z_i(t) g_j(z_j(t)) \\ & + \sum_{i=1}^n \sum_{j=1}^n 2n b_{ij} z_i(t) g_j(z_j(t - \tau_{ij})) \\ & - 2\alpha \sum_{i=1}^n \beta_i(z_i(t)) g_i(z_i(t)) \\ & + \alpha \sum_{i=1}^n \sum_{j=1}^n 2a_{ij} g_i(z_i(t)) g_j(z_j(t)) \\ & + \alpha \sum_{i=1}^n \sum_{j=1}^n 2b_{ij} g_i(z_i(t)) g_j(z_j(t - \tau_{ij})) \\ & + \alpha \sum_{i=1}^n \sum_{j=1}^n |b_{ij}| g_j^2(z_j(t)) \\ & - \alpha \sum_{i=1}^n \sum_{j=1}^n |b_{ij}| g_j^2(z_j(t - \tau_{ij})) \\ & + \sum_{i=1}^n \sum_{j=1}^n \varepsilon g_j^2(z_j(t)) - \sum_{i=1}^n \sum_{j=1}^n \varepsilon g_j^2(z_j(t - \tau_{ij})) \\ & + \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 g_j^2(z_j(t)) \\ & - \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 g_j^2(z_j(t - \tau_{ij})) \end{aligned}$$

We have

$$\begin{aligned} & \sum_{i=1}^n \sum_{j=1}^n 2n a_{ij} z_i(t) g_j(z_j(t)) \\ & \leq \sum_{i=1}^n \sum_{j=1}^n \gamma_i z_i^2(t) + \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 a_{ij}^2 g_j^2(z_j(t)) \end{aligned}$$

$$\sum_{i=1}^n \sum_{j=1}^n 2nb_{ij}z_i(t)g_j(z_j(t-\tau_{ij}))$$

$$\leq \sum_{i=1}^n \sum_{j=1}^n \gamma_i z_i^2(t) + \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 g_j^2(z_j(t-\tau_{ij}))$$

$$\alpha \sum_{i=1}^n \sum_{j=1}^n 2b_{ij}g_i(z_i(t))g_j(z_j(t-\tau_{ij}))$$

$$\leq \alpha \sum_{i=1}^n \sum_{j=1}^n |b_{ij}|g_i^2(z_i(t))$$

$$+ \alpha \sum_{i=1}^n \sum_{j=1}^n |b_{ij}|g_j^2(z_j(t-\tau_{ij}))$$

$$-2\alpha \sum_{i=1}^n \beta_i(z_i(t))g_i(z_i(t)) \leq -2\alpha \sum_{i=1}^n \gamma_i z_i(t)g_i(z_i(t))$$

$$\leq -2\alpha \sum_{i=1}^n \gamma_i G_i^{-1} g_i^2(z_i(t))$$

In the light above inequalities,  $\dot{V}(z(t))$  can be written as follows

$$\dot{V}(z(t)) \leq \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 a_{ij}^2 g_j^2(z_j(t))$$

$$+ \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 g_j^2(z_j(t))$$

$$- 2\alpha \sum_{i=1}^n \gamma_i G_i^{-1} g_i^2(z_i(t))$$

$$+ \alpha \sum_{i=1}^n \sum_{j=1}^n 2a_{ij}g_i(z_i(t))g_j(z_j(t))$$

$$+ \alpha \sum_{i=1}^n \sum_{j=1}^n |b_{ij}|g_j^2(z_j(t))$$

$$+ \alpha \sum_{i=1}^n \sum_{j=1}^n |b_{ij}|g_i^2(z_i(t)) + \sum_{i=1}^n \sum_{j=1}^n \varepsilon g_i^2(z_i(t))$$

$$= \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 [a_{ji}^2 + b_{ji}^2] g_i^2(z_i(t))$$

$$+ \sum_{i=1}^n \sum_{j=1}^n \varepsilon g_i^2(z_i(t)) - 2\alpha \sum_{i=1}^n \gamma_i G_i^{-1} g_i^2(z_i(t))$$

$$+ \alpha \sum_{i=1}^n \sum_{j=1}^n 2a_{ij}g_i(z_i(t))g_j(z_j(t))$$

$$+ \alpha \sum_{i=1}^n \sum_{j=1}^n (|b_{ij}| + |b_{ji}|) g_i^2(z_i(t))$$

Let

$$\delta = \max(\frac{1}{\gamma_i} n^2 [a_{ji}^2 + b_{ji}^2])$$

Then, we have

$$\dot{V}(z(t)) \leq \sum_{i=1}^n \sum_{j=1}^n \delta g_i^2(z_i(t)) + \sum_{i=1}^n \sum_{j=1}^n \varepsilon g_i^2(z_i(t))$$

$$- 2\alpha g^T(z(t)) \Gamma G^{-1} g(z(t))$$

$$+ \alpha g^T(z(t)) (A + A^T) g(z(t))$$

$$+ \alpha g^T(z(t)) Q g(z(t))$$

$$= n(\delta + \varepsilon) \|g(z(t))\|_2^2 - \alpha g^T(z(t)) \Omega g(z(t))$$

$$\leq n(\delta + \varepsilon) \|g(z(t))\|_2^2 - \alpha \lambda_m(\Omega) \|g(z(t))\|_2^2$$

$$= -(\alpha \lambda_m(\Omega) - n(\delta + \varepsilon)) \|g(z(t))\|_2^2$$

in which  $\alpha > \frac{n(\delta + \varepsilon)}{\lambda_m(\Omega)}$  implies that  $\dot{V}(z(t))$  is negative definite for all  $g(z(t)) \neq 0$ . ( We know that  $g(z(t)) \neq 0$  implies that  $z(t) \neq 0$ ). Now let  $g(z(t)) = 0$ . In this case  $\dot{V}(z(t))$  satisfies

$$\dot{V}(z(t)) = -2 \sum_{i=1}^n n \beta_i(z_i(t)) z_i(t)$$

$$+ \sum_{i=1}^n \sum_{j=1}^n 2nb_{ij}z_i(t)g_j(z_j(t-\tau_{ij}))$$

$$- \alpha \sum_{i=1}^n \sum_{j=1}^n |b_{ij}|g_j^2(z_j(t-\tau_{ij}))$$

$$- \sum_{i=1}^n \sum_{j=1}^n \varepsilon g_j^2(z_j(t-\tau_{ij}))$$

$$- \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 g_j^2(z_j(t-\tau_{ij}))$$

$$\leq -2 \sum_{i=1}^n n \beta_i(z_i(t)) z_i(t)$$

$$+ \sum_{i=1}^n \sum_{j=1}^n 2nb_{ij}z_i(t)g_j(z_j(t-\tau_{ij}))$$

$$- \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 g_j^2(z_j(t-\tau_{ij}))$$

Since

$$- \sum_{i=1}^n n \beta_i(z_i(t)) z_i(t) + \sum_{i=1}^n \sum_{j=1}^n 2nb_{ij}z_i(t)g_j(z_j(t-\tau_{ij}))$$

$$- \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 g_j^2(z_j(t-\tau_{ij})) \leq 0$$

we obtain

$$\dot{V}(z(t)) \leq - \sum_{i=1}^n n \beta_i(z_i(t)) z_i(t)$$

implying that  $\dot{V}(z(t)) < 0$  for all  $z(t) \neq 0$ . Now consider the case where  $g(z(t)) = z(t) = 0$ . In this case, for  $\dot{V}(z(t))$ , we

have

$$\begin{aligned}\dot{V}(z(t)) = & -\alpha \sum_{i=1}^n \sum_{j=1}^n |b_{ij}| g_j^2(z_j(t - \tau_{ij})) \\ & - \sum_{i=1}^n \sum_{j=1}^n \varepsilon g_j^2(z_j(t - \tau_{ij})) \\ & - \sum_{i=1}^n \sum_{j=1}^n \frac{1}{\gamma_i} n^2 b_{ij}^2 g_j^2(z_j(t - \tau_{ij}))\end{aligned}$$

in which  $\dot{V}(z(t)) < 0$  if there exists at least one nonzero  $g_j(z_j(t - \tau_{ij}))$ . Hence, we can conclude that  $\dot{V}(z(t)) = 0$  if and only if  $g(z(t)) = z(t) = 0$  and  $g_j(z_j(t - \tau_{ij})) = 0$  for all  $i, j$ ,  $\dot{V}(z(t)) < 0$  otherwise. In addition,  $V(z(t))$  is radially unbounded since  $V(z(t)) \rightarrow \infty$  as  $\|z(t)\| \rightarrow \infty$ . Thus, the origin system (8), or equivalently the equilibrium point of system (1) is globally asymptotically stable [14].

Now, we will compare our result with two previously published results, which are restated in the following :

**Theorem 2 (3):** Consider the delayed system (1) and assume that conditions  $(A_1) - (A_2) - (A_3)$  are satisfied. If there exists positive constants  $m_i, i = 1, 2, \dots, n, r_1 \in [0, 1], r_2 \in [0, 1]$ , and following conditions holds:

$$\max_{1 \leq i \leq n} \left\{ \frac{1}{\gamma_i m_i} \left( m_i \sum_{j=1}^n (G_j^{2r_1} |a_{ij}| + G_j^{2r_2} |b_{ij}|) + \sum_{j=1}^n m_j (G_i^{2(1-r_1)} |a_{ji}| + G_i^{2(1-r_2)} |b_{ji}|) \right) \right\} < 2$$

then the equilibrium point  $x^*$  for system (1) is globally asymptotically stable.

**Theorem 3 (4):** Assume that system (1) satisfies the assumptions  $(A_1), (A_2)$  and  $(A_3)$  are satisfied and there exist constants  $p_{ij}, q_{ij}, s_{ij}, t_{ij} \in \mathbb{R}, i, j = 1, 2, \dots, n$ , such that

$$\begin{aligned}\gamma_i - \frac{1}{2} \sum_{j=1}^n [|a_{ij}|^{2-q_{ij}} G_j^{2-p_{ij}} + |a_{ji}|^{q_{ji}} G_i^{p_{ji}} + |b_{ij}|^{2-s_{ij}} G_j^{2-t_{ij}} \\ + |b_{ji}|^{s_{ji}} G_i^{t_{ji}}] > 0\end{aligned}$$

then the equilibrium point  $x^*$  for system (1) is globally asymptotically stable.

We now give the following examples :

**Example 1:** We consider the example where the network parameters are given as follows :

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} b & b \\ b & b \end{bmatrix},$$

$$\Gamma = G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let  $m_i = 1$ . When applying the result of Theorem 2 to this example, stability condition is ensured if only if  $|b| = 0$ .

However, for the same network parameters, our theorem gives the result as  $0 < |b| < 1/2$ .

**Example 2:** Now consider the example where the network parameters are given as follows :

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Let  $G_i = 1$ . Applying the result of Theorem 3 to this example yields conditions  $\gamma_1 > 4$  and  $\gamma_2 > 4$ . For the same network parameters, our theorem gives the conditions  $\gamma_1 > 3$  and  $\gamma_2 > 3$ .

## V. CONCLUSION

This paper presented a new sufficient condition for the existence, uniqueness and global asymptotic stability of Cohen-Grossberg neural networks with time delays. The proposed condition has been derived by using a more general type of Lyapunov functionals. The obtained results have been shown to be considered an alternative result to the previous results derived in the literature.

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