

# Multiple Model and Neural based Adaptive Multi-loop PID Controller for a CSTR Process

R.Vinodha S. Abraham Lincoln and J. Prakash

**Abstract**—Multi-loop (De-centralized) Proportional-Integral-Derivative (PID) controllers have been used extensively in process industries due to their simple structure for control of multivariable processes. The objective of this work is to design multiple-model adaptive multi-loop PID strategy (Multiple Model Adaptive-PID) and neural network based multi-loop PID strategy (Neural Net Adaptive-PID) for the control of multivariable system. The first method combines the output of multiple linear PID controllers, each describing process dynamics at a specific level of operation. The global output is an interpolation of the individual multi-loop PID controller outputs weighted based on the current value of the measured process variable. In the second method, neural network is used to calculate the PID controller parameters based on the scheduling variable that corresponds to major shift in the process dynamics. The proposed control schemes are simple in structure with less computational complexity. The effectiveness of the proposed control schemes have been demonstrated on the CSTR process, which exhibits dynamic non-linearity.

**Keywords**—Multiple-model Adaptive PID controller, Multivariable process, CSTR process.

## I. INTRODUCTION

**P**ROPORTIONAL-Integral-Derivative (PID) controllers have been used extensively in the chemical industries since they are simple, are often effective and represent the basic building blocks available in many process control systems. For control application, multi-loop PID controllers are often preferred to the multivariable approach at regulatory level. Many plants have older or "legacy" control systems that do not possess the capabilities to support the implementation of complex multivariable controllers. In that case, it may be quite simple and easy to implement the multi-loop PID control algorithm, as compared to multivariable control schemes.

In the paper [4], a predictive control strategy based on neuro-fuzzy (NF) model of the plant is applied to continuous stirred tank reactor (CSTR) which is a highly nonlinear process. In this article, and the way in which the NF model can be used to predict the behavior of the CSTR process over a certain prediction horizon are described, and some comments about the optimization procedure are made. An optimizer algorithm based on evolutionary programming technique (EP) uses the predicted outputs and determines the input sequence in a time window. The present optimized input is applied to the plant, and the prediction time window shifts for another phase of plant output and input estimation.

In the paper [5], an adaptive neural network controller for the control of nonlinear dynamical system is proposed.

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This approach is adaptive in structure, and unlike standard adaptive controllers, uses no explicit model of the process in the design. Traditional neural networks are not practical in adaptive environments because of the large number of weights normally associated with them. In this proposed structure, the controller network has very few connection weights and hence is well suited for real-time implementation.

In the work suggested by Danielle and Cooper [6] a multiple-model adaptive strategy is followed to maintain the performance of the controller over a wide range of operating levels for processes that are stationary in time, but nonlinear with respect to operating levels. The technique involves designing and combining multiple linear DMC controllers. Each controller has their own step response model that describes the process dynamics at a specific level of operation. Then finally, global controller output forwarded to the final control element is obtained by interpolation between the individual controller outputs based on the values of the measured process variables.

Even though powerful artificial intelligence based control schemes have been implemented for such processes, we intend to design and compare the performance of different Adaptive schemes namely Multiple model based Adaptive multi-loop PID [MM Adap-PID] control scheme and neural network based multi-loop PID [NN Adap-PID] control scheme, favoring less computational complexity in existing PID controller to suit industrial requirements.

The organization of the paper is as follows. Section II discusses about the Multiple-model Adaptive Multi-loop controller design and Neural network Adaptive controller design, CSTR process is explained in section III and adaptive strategies followed in multi-loop PID control of CSTR process are discussed in section IV and V. Simulation studies are discussed in section VI and the conclusion drawn from the simulation studies in section VII.

## II. CONTROLLER DESIGN PROCEDURE

### A. Design of Multiple Model Adaptive [MM Adap-PID] Controller

The multiple-model based control system consists of a family of controllers (Local Linear Controllers) and a scheduler. At each sampling instant the scheduler will assign weights for each controllers and the weighted sum of the outputs will be applied as input to the plant. The scheduler will make its decision on the basis of a number of different variables such as state variables, process inputs, measured disturbances and auxiliary variables. The design procedure is discussed with respect to a 2x2 multivariable process.

(i) Selection of operating points to cover the entire process operating range: Let  $y_j$  be the selected  $n$  operating points for all  $j = 1$  to  $n$ .

(ii) Selection of scheduling variable and weight calculation: For each operating point  $y_j$ , for all  $j = 1$  to  $n$ , the weights are calculated based on  $y_{meas}$ , which is the actual value of the measured process variable at the current sampling instant, using the algorithm as

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(If  $y_{meas} \leq y_1$ ), then
For  $i=1$  to  $n$ 
If ( $i=1$ ), then  $W_i=1$ 
If ( $i \neq 1$ ), then  $W_i=0$ 
end
For  $j=1$  to  $n-1$ 
If  $y_j \leq y_{meas} \leq y_{j+1}$ , then
For  $i = n$  to  $1$ 
```

$$\text{If } (i = j + 1), \text{ then } W_i = \frac{y_{meas} - y_j}{y_{j+1} - y_j}$$

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    If ( $i=j$ ), then  $W_i=1-W_{i+1}$ 
    If ( $i \neq j$ ), then  $W_i=0$ 
end
end
For  $i=1$  to  $n$ 
If ( $i=n$ ), then  $W_i=1$ 
If ( $i \neq n$ ), then  $W_i=0$ 
end
```

The weighting factors are in the range of  $[0 \ 1]$ .

(iii) Calculation of adaptive controller output change:

The adaptive controller output change for the two control variables namely  $\Delta u_{adapt}$  and  $\Delta v_{adapt}$  respectively is a weighted average of each linear controller output change  $\Delta u_i$  and  $\Delta v_i$  as given in equation (1).

$$\Delta u_{adapt} = \sum_{i=1}^n W_i \Delta u_i; \Delta v_{adapt} = \sum_{i=1}^n W_i \Delta v_i \quad (1)$$

In equation (1),  $W_i$  is a weighting factor and  $n$  represents number of linear PID controllers used to control the nonlinear system.

(iv) Calculation of local controller output:

The change in controller outputs at each operating point is the standard PID control in velocity form algorithm and is given in equation (2).

$$\begin{aligned} \Delta u_i(k) &= k_{c1,i}(e_1(k) - e_1(k-1)) + \frac{k_{c1,i}T}{T_{r1,i}}e_1(k) \\ &\quad + \frac{k_{c1,i}T_{d1,i}}{T}(e_1(k) - 2e_1(k-1) + e_1(k-2)) \\ \Delta v_i(k) &= k_{c2,i}(e_2(k) - e_2(k-1)) + \frac{k_{c2,i}T}{T_{r2,i}}e_2(k) \\ &\quad + \frac{k_{c2,i}T_{d2,i}}{T}(e_2(k) - 2e_2(k-1) + e_2(k-2)) \end{aligned} \quad (2)$$

In equation (2),  $e_1$  and  $e_2$  are error in loop1 and loop2 respectively,  $T$  is the sampling time,  $k_{c1,i}$ ,  $k_{c2,i}$ ,  $T_{r1,i}$ ,  $T_{r2,i}$

and  $T_{d1,i}$ ,  $T_{d2,i}$  are the proportional gains, integral time values and derivative time values of the  $i^{th}$  multiloop PID controller determined using standard optimal PID tuning methods.

(v) Calculation of global controller output:

The global multi-loop PID controller output for the two variables is calculated as

$$u(k) = u(k-1) + \Delta u_{adapt}$$

$$v(k) = v(k-1) + \Delta v_{adapt} \quad (3)$$

### B. Design of Neural Network Adaptive [NN Adap-PID] Controller

The Neural network Adaptive PID controller consists of a set of neurons linked through suitable activation functions to assign proper weights between the net. The net is trained with suitable scheduling variable as input and controller parameters as the output. At each sampling instant based on the value of the scheduling variable, controller parameters are calculated.

### III. CONTINUOUS STIRRED TANK REACTOR (CSTR)

The first principles model of the continuous stirred tank system and the operating point data (Refer Table.I) as specified in the Pottman and Seborg paper has been used in the simulation studies [3]. Highly non-linear CSTR process is very common in chemical and petrochemical plants. In the process considered for simulation study (as shown in Fig.1), an irreversible, exothermic reaction  $A \rightarrow B$  occurs in constant volume reactor that is cooled by a single coolant stream.

The CSTR system has two state variables, namely the reactor temperature and the reactor concentration. The process is modeled by the following equations:

$$\frac{dC_A(t)}{dt} = \frac{q(t)}{V}(C_{A0}(t) - C_A(t)) - k_0 C_A(t) \exp\left(\frac{-E}{RT(t)}\right) \quad (4)$$

$$\begin{aligned} \frac{dT(t)}{dt} &= \frac{q(t)}{V}(T_0(t) - T(t)) - \frac{(-\Delta H)k_0 C_A(t)}{\rho C_p} * \exp\left(\frac{-E}{RT(t)}\right) \\ &\quad + \frac{\rho_c C_{pc}}{\rho C_p V} * q_c(t) * (1 - \exp\left(\frac{-hA}{\rho C_p q_c(t)}\right)) * (T_{c0}(t) - T(t)) \end{aligned} \quad (5)$$

The state  $x(t)$  and input  $u(t)$  vectors are given by  $x(t)=[C_A; T]$  and  $u(t)=[q; q_c]$ . The continuous linear state space model is obtained by linearizing the differential equations (4 and 5) around the nominal operating point  $C_{AS}$  and  $T_S$ . The objective of the proposed work is to control the essential CSTR variables namely the concentration and Temperature.

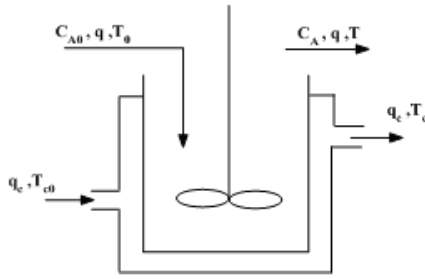


Fig. 1. CSTR Process

TABLE I  
STEADY STATE OPERATING DATA

Process variable	Normal operating condition
Measured product concentration ( $C_A$ )	0.0989 mol/ l
Reactor temperature (T)	438.7763 K
Coolant flow rate ( $q_c$ )	103 l/ min
Process flow rate (q)	100.0 l/ min
Feed concentration ( $C_{A0}$ )	1 mol/ l
Feed temperature ( $T_0$ )	350.0 K
Inlet coolant temperature ( $T_{c0}$ )	350.0 K
CSTR volume (V)	100 l
Heat transfer term (hA)	$7 \times 10^5$ cal/ (min.k)
Reaction rate constant( $k_0$ )	$7.2 \times 10^{10}$ min <sup>-1</sup>
Activation energy term (E/R)	$1 \times 10^4$ K
Heat of reaction ( $-\Delta H$ )	$-2 \times 10^5$ cal/ mol
Liquid density ( $\rho, \rho_c$ )	$1 \times 10^3$ g/l
Specific heats( $C_p, C_{pc}$ )	1 cal/ (g.k)

TABLE II  
OPERATING POINTS

operating points	q (lpm)	$q_c$ (lpm)	$C_A$ (mol/l)	T (K)
Operating point 1	102	97	0.0762	444.7
Operating point 2	100	103	0.0989	438.77
Operating point 3	98	109	0.1275	433

#### IV. MM ADAP-PID CONTROLLER FOR CSTR PROCESS

In this subsection, design of multi-loop PID controllers on the basis of linear models developed at different operating points and the method to combine the multi-loop PID controller outputs to yield a global controller output has been outlined. In this work we have intended to interpolate three multi-loop PID controllers. The transfer function matrix at selected operating point is as follows.

At operating level 1

$$G(s) = \begin{pmatrix} \frac{0.0092s-0.0241}{s^2+5.8359s+16.9521} & \frac{0.0448}{s^2+5.8359s+16.9521} \\ \frac{-0.947s-10.0538}{s^2+5.8359s+16.9521} & \frac{-0.9413s-12.5760}{s^2+5.8359s+16.9521} \end{pmatrix}$$

At operating level 2

$$G(s) = \begin{pmatrix} \frac{0.0090s-0.0244}{s^2+2.7792s+11.1625} & \frac{0.0412}{s^2+2.7792s+11.1625} \\ \frac{-0.8878s-7.4215}{s^2+2.7792s+11.1625} & \frac{-0.8878s-8.8960}{s^2+2.7792s+11.1625} \end{pmatrix}$$

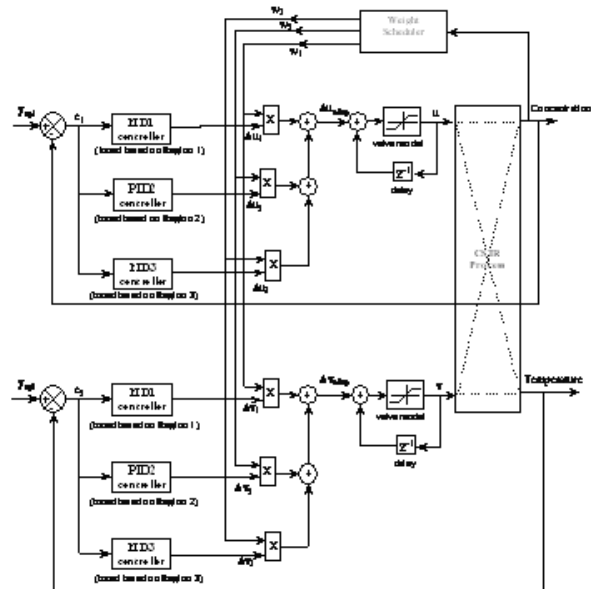


Fig. 2. MM Adap-PID Control Scheme for CSTR Process

At operating level 3

$$G(s) = \begin{pmatrix} \frac{0.0087s-0.0236}{s^2+0.628s+6.9612} & \frac{0.0375}{s^2+0.628s+6.9612} \\ \frac{-0.83s-5.3028}{s^2+0.628s+6.9612} & \frac{-0.82s-6.3148}{s^2+0.628s+6.9612} \end{pmatrix}$$

The weights are calculated with measured concentration using the algorithm described in section II. In this work, reactor concentration is controlled based on coolant flow rate and temperature is controlled by feed flow rate as directed by RGA matrix by Bristol in [2]. The reactor concentration versus coolant flow rate is in the form

$$G_{1,i}(s) = \frac{k_{p1,i}}{s^2 + 2\delta_i\omega_{n,i}s + 1} \quad \forall i = 1 \text{ to } 3 \quad (6)$$

The IMC based PID tuning procedure proposed in [1] and [2] will yield the following controller parameters:

$$K_{c1,i} = \frac{2\delta_i}{\omega_{n,i}K_{p1,i}\lambda_1}; Tr_{1,i} = \frac{2\delta_i}{\omega_{n,i}}; T_{d1,i} = \frac{1}{2\delta_i\omega_{n,i}} \quad (7)$$

The temperature versus feed flow rate is in the form

$$G_{2,i}(s) = \frac{k_{p2,i}(-\beta_i s + 1)}{s^2 + 2\delta_i\omega_{n,i}s + 1} \quad \forall i = 1 \text{ to } 3 \quad (8)$$

The IMC based PID tuning procedure will yield the following controller parameters:

$$K_{c2,i} = \frac{2\delta_i}{\omega_{n,i}K_{p2,i}(\beta_i + \lambda_2)}; Tr_{2,i} = \frac{2\delta_i}{\omega_{n,i}}; T_{d2,i} = \frac{1}{2\delta_i\omega_{n,i}} \quad (9)$$

The PID controller's parameters at three different operating points have been reported in Table III. It should be noted that the controller gain has been found to be function of the filter time constant lamda ( $\lambda_1$  and  $\lambda_2$ ). The lamda values chosen here are  $\lambda_1 = 3$  and  $\lambda_2 = 14$ .

TABLE III  
PID CONTROLLER'S PARAMETERS AT THREE DIFFERENT  
OPERATING POINTS

operating points	Region1 At $q=102$ $q_c=97$ $C_A=0.0762$ $T=444.7$	Region2 At $q=100$ $q_c=103$ $C_A=0.0989$ $T=438.7763$	Region3 At $q=98$ $q_c=109$ $C_A=0.1275$ $T=433$
Damping factor( $\delta$ )	0.7087	0.416	0.119
frequency (rad/sec)( $\omega_n$ )	100	103	0.0989
$k_{c1}$	$\frac{132}{\lambda_1}$	$\frac{67.32}{\lambda_1}$	$\frac{16.7}{\lambda_1}$
$k_{c2}$	$\frac{0.5804}{0.0943+\lambda_2}$	$\frac{0.3746}{0.1196+\lambda_2}$	$\frac{0.1184}{0.1093+\lambda_2}$
$T_{r1,i}=T_{r2,i}=T_i$	0.3443	0.249	0.09
$T_{d1,i}=T_{d2,i}=T_d$	0.171	0.35	1.5925

TABLE IV  
[NN ADAP-PID] CONTROLLER FOR CSTR PROCESS

Neural Network Training Parameter	Specification
Number of Input neuron in the Input layer	4-Controller gain $K_{c1}$ , $K_{c2}$ , Integral Time $T_i$ and derivative time $T_d$
Number of Hidden neuron in the Hidden layer	10
Initial weight selection	Nguyen-Widrow criterion
Activation function	HiddenLayer: Tansigmoidal
Training Algorithm	Levenberg-Marquardt optimization algorithm
Objective Function	Mean square error:1.24258e-007 Number of epoch: 500

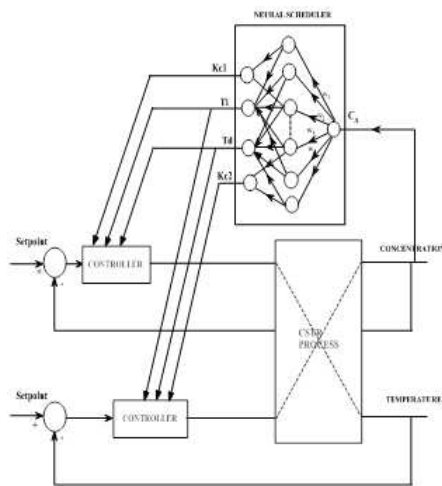


Fig. 3. NN Adap-PID Control Scheme for CSTR Process

## V. [NN ADAP-PID] CONTROLLER FOR CSTR PROCESS

The dynamic feed forward Back Propagation neural network is used. The network is trained with process parameters namely  $k_{p1}$ ,  $k_{p2}$ ,  $T_i$  and  $T_d$  for wide concentration range  $C_A$ . The structure of the feed-forward neural network and its training parameters are reported in Table IV.

The training of the feed-forward neural network has been achieved using the commands available in the neural network toolbox of MATLAB. After training, the neural network model is directly tested on non-linear process and performance is validated. The neural network shown in Fig.3 accepts the scheduling variable ( $C_A$ ) as input and computes the controller parameters namely controller gain  $k_{c1}$ , integral time  $T_i$  and derivative time  $T_d$  to adapt loop1 controller. Similarly the controller parameters namely controller gain  $K_{c2}$ , integral time  $T_i$  and derivative time  $T_d$  to adapt loop2 controller are also calculated at every instant based on the scheduling variable. Since the denominator of the characteristic equation is same for both the loops as evident from the transfer function models given in section IV, integral time  $T_i$  and derivative time  $T_d$  are same for the loops.

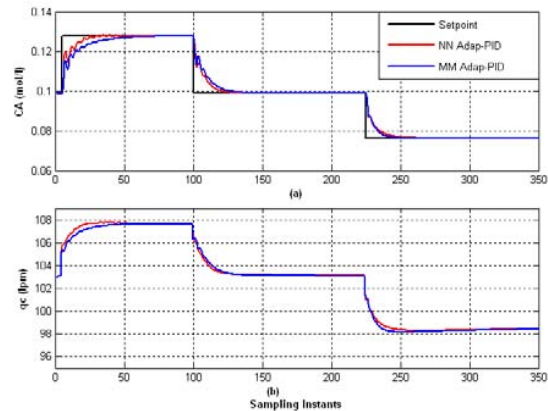


Fig. 4. Servo response of CSTR Process

## VI. SIMULATION RESULTS

In all the simulation runs, the process is simulated using the nonlinear first principles model given in equation(4) and (5) and the true state variables (Concentration and Temperature) are computed by solving the nonlinear differential equations using differential equation solver in Matlab 7.0. The entire simulation has been performed with the following initial conditions:  $C_A = 0.0989$  mol/lit;  $Q_c = 103$  lit/min;  $q = 100$  lit/min;  $T = 438.7763^\circ\text{K}$ .

### A. Servo Response

In order to assess the tracking capability of designed controllers, setpoint variations in concentration as given in Fig. 4 and setpoint variations in temperature as given in Fig. 5 have been introduced. From the responses it can be inferred that, both the controllers designed for the CSTR process are able to maintain the variables concentration and temperature at the desired setpoints. The variation in controller outputs is presented in Fig.4 and Fig.5. The observations of the simulation studies are as follows.

As the setpoint namely concentration is changed from 0.0989 mol/l to 0.1275 mol/l, 0.1275 mol/l to 0.0989 mol/l and 0.0989 mol/l to 0.0762 mol/l, both MMAdap-PID and NNAdap-PID tracks the setpoint variations. NNAdap-PID settles faster when setpoint transition is from 0.0989 mol/l

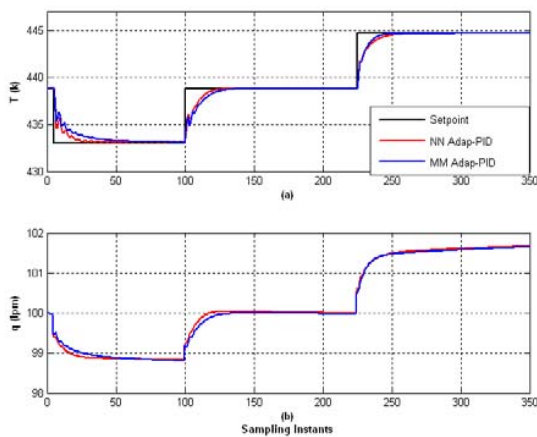


Fig. 5. Servo response of CSTR Process

to 0.1275 mol/l and 0.1275 mol/l to 0.0989 mol/l. MM Adap-PID settles faster when setpoint transition is from 0.0989 mol/l to 0.0762 mol/l [Refer Fig.4]. Similarly as the setpoint namely temperature is changed from 438.77 °K to 433 °K, 433 °K to 438.77 °K and 438.77 °K to 444.7 °K, both MMAdap-PID and NNAdap-PID tracks the setpoint variations. NNAdap-PID settles faster when setpoint transition is from 438.77 °K to 433°K and 433 °K to 438.77 °K. MM Adap-PID settles faster when setpoint transition is from 438.77 °K to 444.7 °K [Refer Fig.5]. Further, it is evident that the manipulated variables variation namely the coolant flow rate and feed flow rate are found to be smooth in both NN Adap-PID and MMAdap-PID controllers [Refer Fig.4 and Fig.5]. The ISE values are given in Table V.

### B. Servo Regulatory Response

The disturbance rejection capabilities of the controllers have been analyzed in the presence of step change in the inlet temperature. A step change in the inlet temperature of magnitude 1°K (from 350°K to 349°K) has been introduced at 150th sampling instant and maintained up to 300th sampling instant. At 150th sampling instant [Refer Fig.6 and Fig.7], it can be inferred that both the controllers (MMAdap-PID, NNAdap-PID) are able to reject the disturbance quickly and bring the process variables (concentration and temperature) back to their respective setpoints. This part of the simulation demonstrates that both the controllers are able to reject the disturbance at nominal operating point.

The ISE values are given in Table VI. From the ISE values, the NN Adap-PID is better than MM Adap-PID controller for servo regulatory operation. With the disturbance being present, a step change in the setpoint has been introduced at 225th sampling instant and it can be noted that both the controllers are able to maintain the controlled variables at their setpoints. At 300th sampling instant the disturbance has been removed and both controllers are able to maintain the controlled variables at their setpoints. This part of the simulation demonstrates that both the controllers are able to

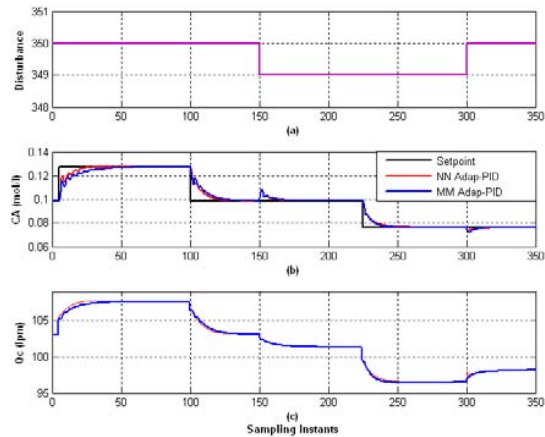


Fig. 6. Servo regulatory response of CSTR Process

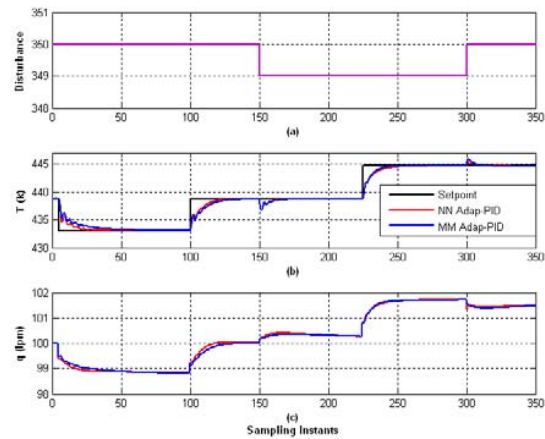


Fig. 7. Servo regulatory response of CSTR Process

TABLE V  
ISE VALUES - SERVO RESPONSE

Sampling Intervals	MM Adap-PID		NN Adap-PID	
	Loop1	Loop2	Loop1	Loop2
1:10	$8.26924e^{-4}$	33.807	$8.2443e^{-4}$	32.549
11:100	0.0054	202.615	0.0033	107.097
101:225	0.0057	244.141	0.0042	178.098
226:350	0.0019	132.218	0.0020	141.973

TABLE VI  
ISE VALUES - SERVO REGULATORY RESPONSE

Sampling Intervals	MM Adap-PID		NN Adap-PID	
	Loop1	Loop2	Loop1	Loop2
1:10	$8.26924e^{-4}$	33.807	$8.2443e^{-4}$	32.549
11:100	0.0054	202.615	0.0033	107.097
101:225	0.0084	346.756	0.0066	270.514
226:350	0.0022	149.28	0.0021	139.800

reject the disturbance at shifted operating point.

## VII. CONCLUSION

In this paper the authors have proposed two simple control schemes to control the variables concentration and temperature of the CSTR process. The two controllers designed use the same control law and differ only in the calculation of controller parameters. From the extensive simulation studies, it can be concluded that the proposed controllers have good setpoint tracking, disturbance rejection at nominal and shifted operating points. Further, it can be concluded that the servo regulatory performance of NN Adap-PID is found to be better than MM Adap-PID.

## REFERENCES

- [1] M. Morari and E. Zafiriou, *Robust Process Control*, Upper Saddle River, NJ: Prentice Hall, 1989.
- [2] B. Wayne Bequette, *Process Control, Modeling, Design and Simulation*, India, Prentice Hall, 2004.
- [3] M. Pottmann and D.E. Seborg, *Identification of Non-linear Process Using Reciprocal Multi quadratic Functions*, Journal of Process Control, vol 2, pp.189-203, 1992.
- [4] M. Jalili Kharaajoo, *Predictive Control of a Continuous Stirred Tank Reactor based on Neuro-fuzzy Model of the Process*, SICE Annual Conference in Fukui, vol 57, pp.3005-3011, Aug 2003.
- [5] Venugopal G. Krishnapura and Arthur Jutan, *A Neural Adaptive Controller*, Chemical Engineering Science, vol 55, pp.3803-3812, 2000.
- [6] D. Danielle and D. Cooper, *A Practical Multiple Model Adaptive Strategy for Multivariable Model Predictive Control*, Control Engineering Practice, vol 11, pp.649-664, 2003.