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Stability of Functionally Graded Beams with Piezoelectric Layers Based on the First Order Shear Deformation Theory

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***Abstract*—**Stability of functionally graded beams with piezoelectric layers subjected to axial compressive load that is simply supported at both ends is studied in this paper. The displacement field of beam is assumed based on first order shear deformation beam theory. Applying the Hamilton's principle, the governing equation is established. The influences of applied voltage, dimensionless geometrical parameter, functionally graded index and piezoelectric thickness on the critical buckling load of beam are presented. To investigate the accuracy of the present analysis, a compression study is carried out with a known data.

***Keywords*—**Stability- Functionally graded beam- First order shear deformation theory -Piezoelectric layer.

I. INTRODUCTION

STRUCTURAL stability is considered to be one of the most important engineering issues in the design and application of slender structures. Buckling and postbuckling are the two main types of structural instability, they often govern the failure of structures under static or dynamic compressive loading conditions, thus, have been investigated by several researchers in the past decades. Dynamic buckling and postbuckling phenomena are directly dependent on the dynamic loading velocity characteristics and the duration of the impacting load, as well as on the material's inherent properties and the structure's geometry [1–3]. The buckling behavior of composite beams and plates with piezoelectric patches bonded to their surfaces or with piezoelectric layers embedded has been the subject of a number of investigations [4–6].

Piezoelectric materials have been used in the past few years in a variety of applications ranging from active control to noise suppression. Their lightweight, relatively low cost, small size and good frequency response make them an attractive alternative to conventional point actuators commonly used. In all these applications, piezoelectric actuators are used to enhance the performance of a structural system by inducing a favorable structural deformation. Detailed models on the iteration between piezoelectric sensors or actuators with the structure to which they are bonded or embedded have been developed [7–9]. The use of finite element method in the analysis of piezoelectric coupled structures has been studied [10–13].

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Crawley and de Luis [14] developed the analytical model for the static and dynamic response of a beam structure with segmented piezoelectric actuators either bonded or embedded in a laminated composite.

LaPeter and Cudney [15] proposed an analytic model for the segmented piezoelectric actuators bonded on a beam or a plate, and found the equivalent forcing functions of the actuators. The piezoelectric bimorph column structures were used as sensing elements.

Dobrucki and Pruchnicki [16] presented an analysis theory of an axisymmetric piezoelectric bimorph. They also described a sensing theory for using the axisymmetric piezoelectric bimorph. Chandrashekhara and Bhatia [17] developed a finite element model for the active buckling control of laminated composite plates with surface bonded or embedded piezoelectric sensors that are either continuous or segmented. The dynamic buckling behavior of the laminated plate subjected to a linearly increasing compression load is investigated in their work. Chase and Bhashyam [18] derived optimal design equations to actively stabilize laminated plates loaded in excess of the critical buckling load using a large number of sensors and actuators. Such work finds application in aircraft wing skins.

To the author's knowledge, there is no analytical solution available in the open literatures for stability of functionally graded Engesser-Timoshenko beams with piezoelectric layers. In the present work, the stability of a functionally graded Engesser-Timoshenko beam with piezoelectric actuators subjected to axial compressive loads is studied. Appling the Hamilton's principle, the equilibrium equations of beam are derived and solved. The effects of the applied voltage, dimensionless geometrical parameter and functionally graded index on the critical buckling load of beam are presented. To investigate the accuracy of the present analysis, a compression study is carried out with a known data.

II. FORMULATION

The formulation that is presented here is based on the assumptions of Engesser-Timoshenko beam theory. Based on this theory, the displacement field can be written as [20]:

$$u(x,z) = z\phi(x)$$
$$w(x,z) = w_0(x,z) \quad (1)$$

In view of the displacement field given in Eqs. (1), the strain displacement relations are given by [20]:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = z\frac{d\phi}{dx}$$
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \phi + \frac{dw}{dx} \tag{2}$$

Consider a functionally graded beam with piezoelectric actuators and rectangular cross-section as shown in Fig. 1. The thickness, length, and width of the beam are denoted, respectively, by $h, L,$ and $b.$ Also, h_T and h_B are the thickness of top and bottom of piezoelectric actuators, respectively. The $x-y$ plane coincides with the midplane of the beam and the $z-$axis located along the thickness direction.

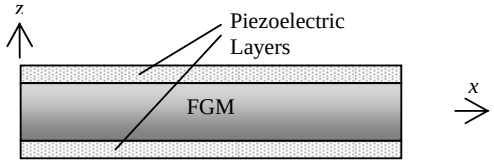

Fig. 1 Schematic of the problem studied.

The Young's modulus E is assumed to vary as a power form of the thickness coordinate variable z $(-h/2 \le z \le h/2)$ as follow [19]:

$$E(z) = (E_c - E_m)V + E_m, \qquad V = \left(\frac{2z+h}{2h}\right)^k \tag{3}$$

where k is the power law index and the subscripts m and c refer to the metal and ceramic constituents, respectively. The constitutive relations for functionally graded Engesser-Timoshenko beam with piezoelectric layers are given by [21]:

$$\sigma_{xx} = Q_{11}(z)\varepsilon_{xx} - e_{31}E_z$$
$$\sigma_{xz} = Q_{55}(z)\gamma_{xz} - e_{15}E_x \tag{4}$$

where

$$E_i = \frac{V}{h_i} \tag{5}$$

where σ_{xx}, σ_{xz} , $\mathrm{Q}_{11}(z)$ and $\mathrm{Q}_{55}(z)$ are the normal ,shear stresses and plane stress-reduced stiffnesses and e_{31}, e_{15} are piezoelectric elastic stiffnesses respectively. Also, u and w are the displacement components in the $x-$ and $z-$ directions, respectively.

The potential energy can be expressed as [20]:

$$U = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz})\,\mathrm{d}v \tag{6}$$

Substituting Eqs. (2)-(4) into Eq. (6) and neglecting the higher-order terms, we obtain

$$U = \frac{1}{2}\int_v [(Q_{11}(z)\left(z\frac{d\phi}{dx}\right) - e_{31}E_z)(z\frac{d\phi}{dx}) + (Q_{55}(z)(\phi + \frac{dw}{dx}) - e_{15}E_x)(\phi + \frac{dw}{dx})]dv \tag{7}$$

The width of beam is assumed to be constant, which is obtained by integrating along y over $v.$ Then Eq. (7) becomes

$$U = \frac{b}{2}\int_0^L [D\left(\frac{d\phi}{dx}\right)^2 + \frac{A}{2(1+\nu)}(\phi^2 + (\frac{dw}{dx})^2 + 2\phi\frac{dw}{dx})]dx - \frac{b}{2}\int_0^L \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} (ze_{31}E_z\frac{d\phi}{dx} + e_{15}E_x\varphi + e_{15}E_x\frac{dw}{dx})dzdx \tag{8}$$

where

$$A = \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} Q_{55}(z)\mathrm{d}z$$
$$D = \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} z^2 Q_{11}(z)\mathrm{d}z \tag{9}$$

where A and D are the shear rigidity and flexural rigidity respectively. Note that, no residual stresses due to the piezoelectric actuator are considered in the present study and the extensional displacement is neglected. Thus, the potential energy can be written as

$$U = \frac{b}{2}\int_\circ^L [D\ (\frac{d\phi}{dx})^2 + A(\phi^2 + (\frac{dw}{dx})^2 + 2\phi\frac{dw}{dx} - e_{31}(h_T V_T + h_B V_B)\frac{d\phi}{dx} - e_{15}(V_T + V_B)(\phi + \frac{dw}{dx})]dx \tag{10}$$

where V_T and V_B are the applied voltages on the top and bottom actuators respectively. The beam is subjected to the axial compressive loads, P as shown in Fig. 2.

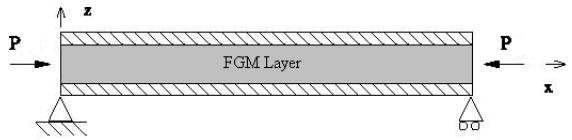

Fig. 2 Simply supported beam under periodic loads.

The work done by the axial compressive load can be expressed as [20]:

$$W = \frac{1}{2}\int_0^L P\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x \tag{11}$$

We apply the Hamilton's principle to derive the equilibrium equations of beam, that is [21]:

$$\delta\int_0^t (T - U + W)\,\mathrm{d}t = 0 \tag{12}$$

Substitution from Eqs. (10) and (11) into Eq. (12) leads to the following equilibrium equations of the functionally graded Engesser-Timoshenko beam with piezoelectric layers

$$\begin{aligned}(P - bA)\frac{d^2w}{dx^2} + bA\left(\frac{d\phi}{dx}\right) = 0\\ A\left(\phi + \frac{dw}{dx}\right) + 2e_{15}V_T + 2D\left(\frac{d^2\phi}{dx^2}\right) = 0\end{aligned} \tag{13}$$

The boundary conditions for the pin-ended Timoshenko column are given by:

$$w = \frac{d^2w}{dx^2} = \frac{d\phi}{dx}, \qquad at \quad x = 0 \quad and \quad x = L \tag{14}$$

Substituting Eq. (14) into (13) and by equating power-law index to zero and neglecting the piezoelectric effect, the critical Engesser-Timoshenko buckling load of a homogeneous beam will be derived, that is:

$$p_{cr} = \frac{\left(\frac{\pi}{L}\right)^2 \frac{bh^3 Q_{11}}{12}}{1 + \left(\frac{L}{\pi}\right)^2 \frac{12Q_{55}}{bh^2 Q_{11}}} \tag{15}$$

The above equation has been reported by Wang and Reddy [20].

III. NUMERICAL RESULTS

The mechanical buckling behaviors of simply supported functionally graded Engesser-Timoshenko beams with piezoelectric actuators are studied in this paper. It is assumed that both the top and bottom piezoelectric layers have the same thickness; $h_T = h_B$ and the same voltages are applied to both actuators. The material properties of the beam are listed in Table 1.

TABLE I MATERIAL PROPERTIES

Property	Piezoelectric layer	FGM layer	
		Stainless steel	Nickel
Young's modulus E (GPa)	63	221.04	223.95
Poisson's ratio ν	0.3	0.3	0.3
Length L (m)	0.3	0.3	0.3
Thickness h (m)	0.00005	0.01	0.01
Density ρ (Kgm^{-3})	7600	8166	8900
Piezoelectric constant e_{31}, e_{15} (Cm^{-2})	17.6	-	-

The Poisson's ratio is chosen to be 0.3 for both materials. The critical buckling loads for Bernoulli-Euler beam (BEB) and Engesser-Timoshenko beam (ETB) evaluated considering of $h_a / h = 0.1$, $b / h = 1$, $L = 1$, $V = 10$ v and several values of dimensionless geometrical parameter h / L are shown in Fig. 3. It is seen that the critical buckling loads for Engesser-Timoshenko beam are generally lower than corresponding values of Bernoulli-Euler[22] beam. Fig. 4. demonstrates the buckling loads for functionally graded Engesser-Timoshenko beam. It is seen that the critical buckling loads for Engesser-Timoshenko beam increased with an increase of the ratio h / L and decreased with an increase of power-law index of constituent volume fraction. The variation of critical buckling loads for Engesser-Timoshenko beam versus h / L for different applied voltage is shown in Fig. 5.

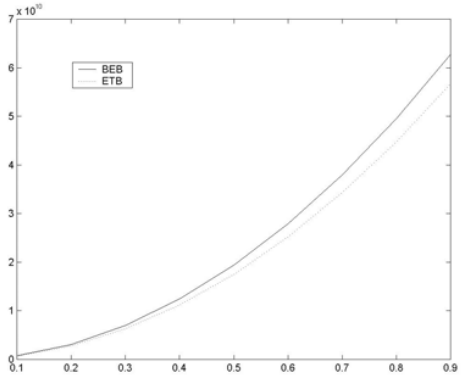

Fig. 3. Comparison of the Critical Buckling Load of FG Beam with Piezoelectric Actuators Versus h / L .

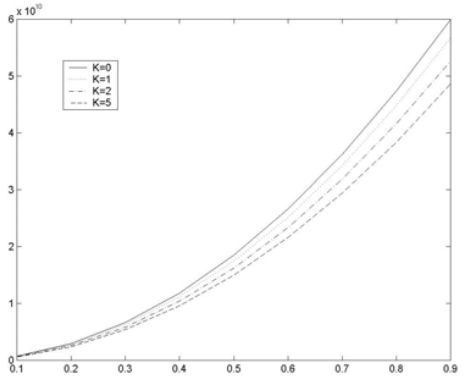

Fig. 4. Critical Buckling Load of FG Beam with Piezoelectric Actuators Versus h/L for $V = 10$ v.

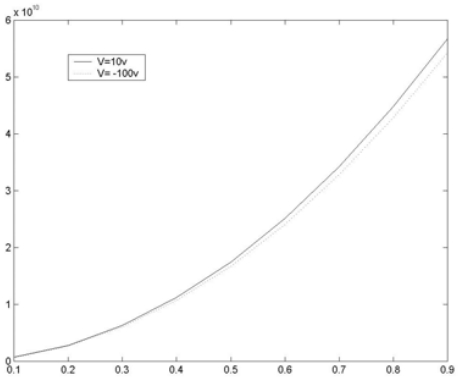

Fig. 5. Effect of Applied Voltage on the Critical Buckling Load of FG Beam with Piezoelectric Actuators.

IV. CONCLUSION

The stability of a functionally graded Engesser-Timoshenko beam with piezoelectric actuators subjected to axial compressive loads is studied. It is conclude that:

1- The piezoelectric actuators induce tensile piezoelectric force produced by applying negative voltages that significantly affect the stability of the functionally graded Engesser-Timoshenko beam with piezoelectric actuators.
2- The critical buckling loads of FG Engesser-Timoshenko beam are generally lower than corresponding values for the homogeneous Engesser-Timoshenko beam.
3- The critical buckling loads of FG Engesser-Timoshenko beam under axial compressive load generally increases with the increase of relative thickness h/L.
4- The accuracy of Engesser-Timoshenko beam theory is more than Bernoulli-Euler beam theory.

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