Identification of Nonlinear Systems Using Radial Basis Function Neural Network

C. Pislaru, A. Shebani

Abstract—This paper uses the radial basis function neural network (RBFNN) for system identification of nonlinear systems. Five nonlinear systems are used to examine the activity of RBFNN in system modeling of nonlinear systems; the five nonlinear systems are dual tank system, single tank system, DC motor system, and two academic models. The feed forward method is considered in this work for modelling the non-linear dynamic models, where the K-Means clustering algorithm used in this paper to select the centers of radial basis function network, because it is reliable, offers fast convergence and can handle large data sets. The least mean square method is used to adjust the weights to the output layer, and Euclidean distance method used to measure the width of the Gaussian function.

Keywords—System identification, Nonlinear system, Neural networks, RBF neural network.

I. INTRODUCTION

THE concept of a neural network was originally conceived as an attempt to model the biophysiology of the brain. An artificial neural network (ANN) offers a potential solution for problems which require complex data analysis and promise to form the future basis of an improved alternative to current engineering practice. The neural networks have many applications in various fields of study, including modeling and control of linear and nonlinear systems. Neural networks have been developed in different ways, where various algorithms and methods have been applied, such as backpropagation (BP) rule and radial basis function (RBF) [1]–[3]. Artificial neural networks (ANNs) are often used for applications where it is difficult to state explicit rules. Often it seems easier to describe a problem and its solution by giving examples; if sufficient data is available a neural network can be trained. There is a wide range of application domains where ANNs are being used, including classification, compression, noise reduction, optimization, prediction, and recognition. Neural networks with their remarkable ability to derive meaning from complicated or imprecise data can be used to extract patterns and detect trends that are too complex to be noticed by either humans or other computer techniques. A trained neural network can be thought of as an "expert" in the category of information it has been given to analyze. This expert can then be used to provide projections given new situations of interest. Identification of nonlinear systems based on RBFNN was done by many researchers but without mention about the main three parameters in RBFNN, and how they selected these

parameters (centers, width and weights). Other researchers used the genetic algorithm to improve the RBFNN performance; this will be a more complicated identification process [4]–[15].

II. ARTIFICIAL NEURAL NETWORKS (ANNs)

A. Simple Perceptron

The simple perceptron shown in Fig. 1, it is a single processing unit with an input vector $X = (x_0, x_1, x_2 \ldots x_n)$. This vector has n elements and so is called n-dimensional vector.

Each element has its own weights usually represented by the n-dimensional weight vector, e.g. $W = (w_0, w_1, w_2 \ldots w_n)$.
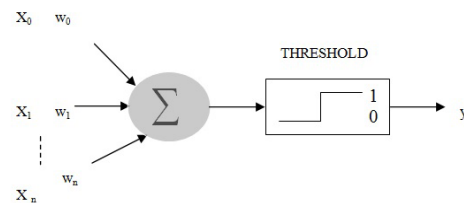

Fig. 1 Simple perceptron diagram

The output y can be described by the following equation:

$$y = \sum_{i=0}^{n} W_i X_i \tag{1}$$

where W_i are the weights and X_i are the inputs [2], [16]–[19].

B. Multilayer Perceptron

Multilayer perceptron, are constructed from multiple layers of elements, neurons or nodes, such as in Fig. 2, it consists of units that constitute the input layer, an output layer and a number of intermediate layers (hidden layers).
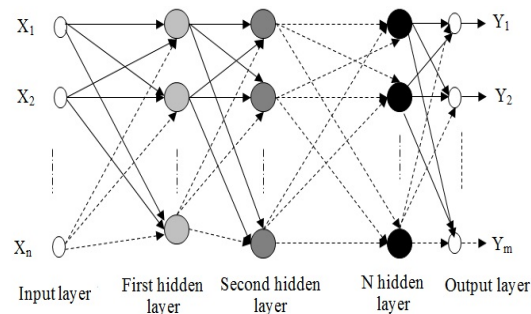

Fig. 2 Multilayer perceptrons diagram

The network requires a set of data as inputs to the input layer. The outputs of the input layer are then fed as weighted

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inputs to the first hidden layer. The outputs from the first hidden layer are fed as weighted inputs to the second hidden layer and so on. This process continues until the output layer is reached [1], [2], [13], [16], [19].

C. Radial Basis Function Neural Network (RBFNN)

Feed-forward layered neural networks have increasingly been used in many areas such as modeling and control of non-linear systems. One example of a feed forward neural network is the Radial Basis Function (RBF) network. The RBF network can be regarded as a special three layer network, including input, hidden and output layers. Full explanations of the connections of these layers together with the activation function are given in the next sections. The performance of the RBF depends on the proper selection of three important parameters (centers, widths and weights).

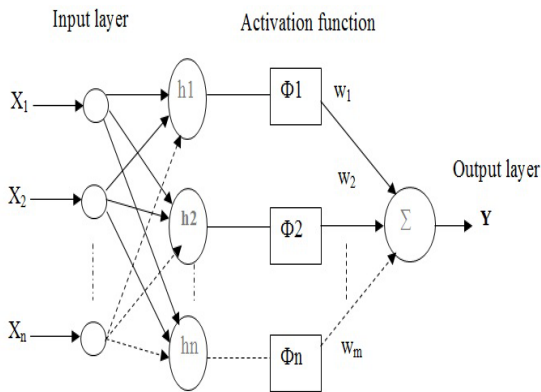The RBF neural network has a feed forward structure consisting of three layers as shown in Fig. 3.

Fig. 3 Radial basis function structure

The output vector y is given by:

$$y = \sum_{j=1}^{N} W_j \Phi_j \qquad (2)$$

where (W_j) is the weight of the (j^{th}) node and (Φ_j) is the activation function.

The hidden node calculates the distance between the center and the RBFNN input vector, and then passes the result through the nonlinear activation function (Φ) to the output layer.

The Gaussian activation function can be written as:

$$\Phi_j(x) = exp\frac{-\sum_{j=1}^{n}(X_i - c_j)^2}{\sigma_j^2} \qquad (3)$$

$$i = 1, 2, 3... n, j = 1, 2, 3...m$$

where (σ_j) is the width of the Gaussian function, (m) is the number of centers and (n) the dimension of the input space. The major requirement is that the function must tend to zero quite rapidly as the distance increases between the input X and center C.

The Radial Basis Function Network consists of three important parameters, centers, width and weights. The values of these parameters are generally unknown and may be found during the learning process of the network. The major problem which therefore remains is one of how to select an appropriate set of RBF centers. To overcome this problem, the network requires some strategy for selecting the adequate set of centers, hence clustering algorithm have been used extensively [2], [13], [16], [19].

The K-Means clustering algorithm is used in this paper for selecting the centers, because of its simplicity and ability, to produce good results. The K means algorithm will do the following three steps until convergence such as in Fig. 5, iterate until stable (no object move group):
1) Determine the centroid coordinate
2) Determine the distance of each object to the centroids
3) Group the object based on minimum distance (find the closest centroid).

The most common method used to select the width is the Euclidean distance measure. This method is used in this paper because it is simple to calculate and more reliable. The shortest distance between vector X and vector C is the Euclidean distance which is defined as:

$$E_{dist} = \sqrt{\sum_{i=1}^{n} Xi - cj} \qquad (4)$$

where n the vector dimension, and Edist is the Euclidean distance [1], [16].

In this paper, the adaptive weights have been performed using least square algorithm:

$$W_j(n+1) = W_j(n) + \mu\left(y_d(t) - y(t)\right)g_j(n) \qquad (5)$$

where μ is the learning factor ($0 < \mu \leq 1$), it is a positive gain factor term that controls the adaptation rate of the algorithm, y and y_d are the actual output and the desired output respectively, and t is the current time [2], [16].

The training process of RBFNN is consists of the following steps:
1) Apply the input vector (x) from the training data set to the input layer.
2) Compute the output of the hidden layer.
3) Compute the RBF network output vector and compare it with the desired output, and then adjust the weight vector to reduce the difference.
4) Repeat steps 1 to 3 for each vector in the training set.
5) Repeat steps 1 to 4 until the error tends to zero.

The structure for achieving the forward modeling is shown in Fig. 4, where the RBF network is used to represent the forward dynamics of the system; the system is placed in parallel with the neural network model, and at each instant of time (t) the past (m) inputs and the past (n) outputs of the system are fed into the RBF neural network [14]–[16].
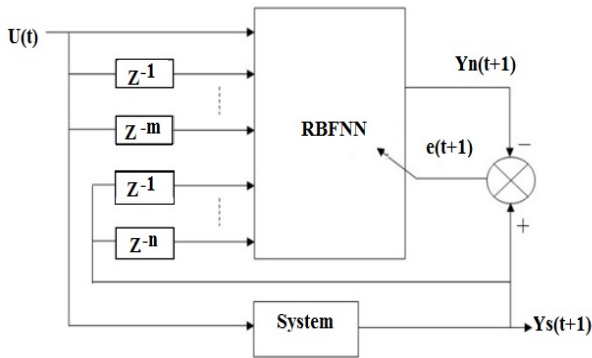
Fig. 4 Series-parallel identification schemes

The system is governed by the following nonlinear difference equation:

$$Ys(t + 1) = f[Ys(t), \ldots, Ys(t - n + 1), U(t), U(t + 1), \ldots, U(t - m + 1)] \quad (6)$$

where: $Ys(.)$ is the plant output, and $U(.)$ is the input sequences.

The neural based identification model is assumed to have the same structure as the plant and is given by:

$$\widehat{Ym}(t + 1) = \widehat{f}[Ys(t), \ldots, Ys(t - n + 1), U(t), U(t + 1), ., U(t - m + 1) \quad (7)$$

where $\widehat{f}(.)$ represents the non-linear input-output map of the network. Simulation Results

Simulations have been performed using the RBF neural network. Various non-linear plants were tested and very good results were obtained. Five different plants were used as examples to show the properties of the radial basis function neural network (RBFNN). Fig. 5 is a block diagram of a series-parallel model which is used in this paper to system identification of nonlinear systems. The output of the plant and the network are compared and the resulting error is used to update the network parameters the weights.
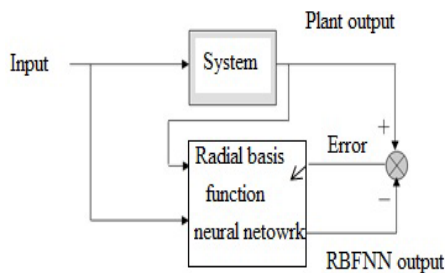

Fig. 5 Series-parallel identification schemes

A. Dual Tank System

The system in consideration consists of two interconnected water tanks linked together by an orifice. The water is fed into the system using an electrical water pump, the pump switched ON to move the water level in tank one to a certain level. As a result, the water level in tank two will rise with a certain rate until it reached a steady state level. The scheme of the system is given in Fig. 6. The inlet water quantity is denoted by q_i, the

outlet quantities are q_1 and q_2 respectively. The water levels are denoted by L_1 and L_2. The cross sections of a dual tank are A_1 and A_2. The cross sections of the tubes are denoted by R_1 and R_2.
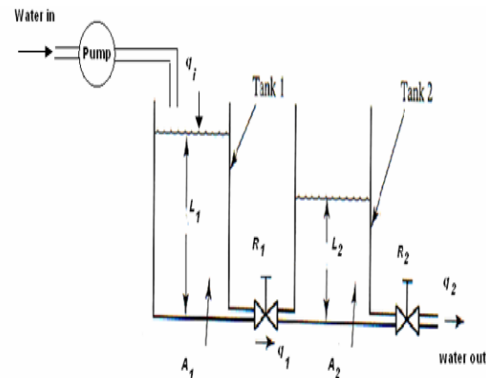

Fig. 6 Dual tank system

The following differential equations describing the process dynamics of the dual tank [20]–[23]:

$$q_1 = \frac{(L_1 - L_2)}{R_1} \quad (8)$$

$$q_2 = \frac{(L_2)}{R_2} \quad (9)$$

$$\frac{dL_1}{dt} = \frac{(q_i - q_1)}{A_1} \quad (10)$$

$$\frac{dL_2}{dt} = \frac{(q_1 - q_2)}{A_2} \quad (11)$$

where: the system output to be (L_2), it is the height of the fluid in the tank2; and the system input to be (q_i), it isthe flow into the tank1.

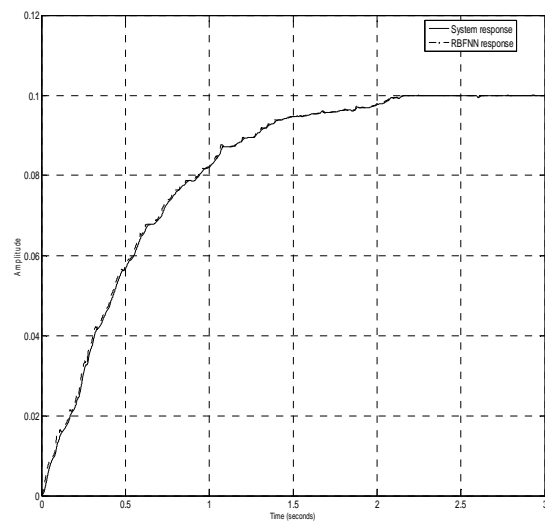The dual tank system response and RBFNN response are shown in Fig. 7.

Fig. 7 System output and RBFNN output

B. Single Tank System

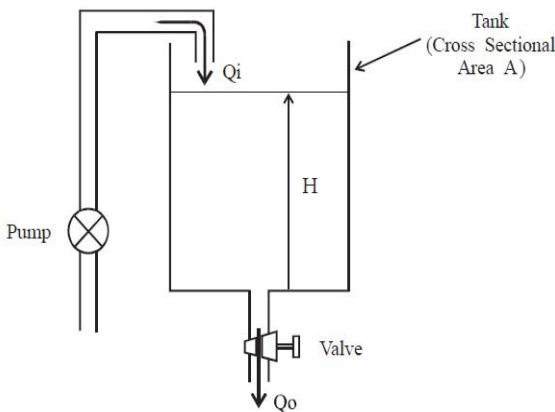The system in consideration is a single water tank such as in Fig. 8.

Fig. 8 Single tank system

The transfer function of this system is:

$$Q_i - Q_0 = A\frac{dH}{dt} \qquad (12)$$

where the system output to be (H), it is the height of the fluid in the tank; and the system input to be (Q_i), it is the flow into the tank [24].

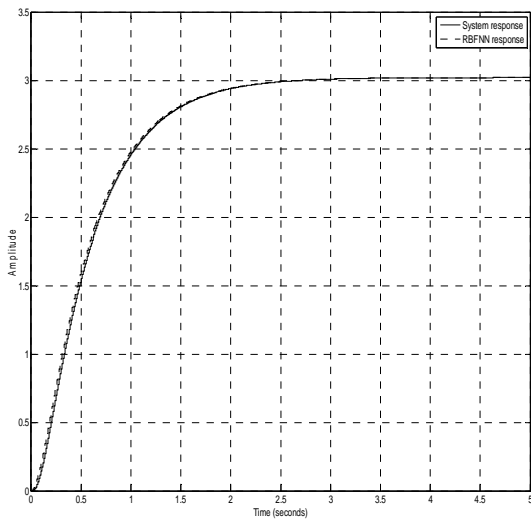The single tank system response and RBFNN are illustrated in Fig. 9.

Fig. 9 System output and RBFNN output

C. DC Motor System

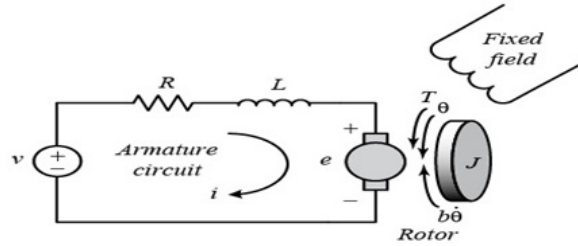The plant considered for identification is a DC motor system such as in Fig. 10.

Fig. 10 DC motor system

The transfer function of the DC motor system is:

$$\frac{\dot{\theta}(s)}{V(s)} = \frac{K}{(J\,S+b)(L\,S+R)+K^2}\left(\frac{rad/sec}{v}\right) \qquad (13)$$

where $\dot{\theta}$ is the rotational speed (output) and V is the armature voltage (input) [25].

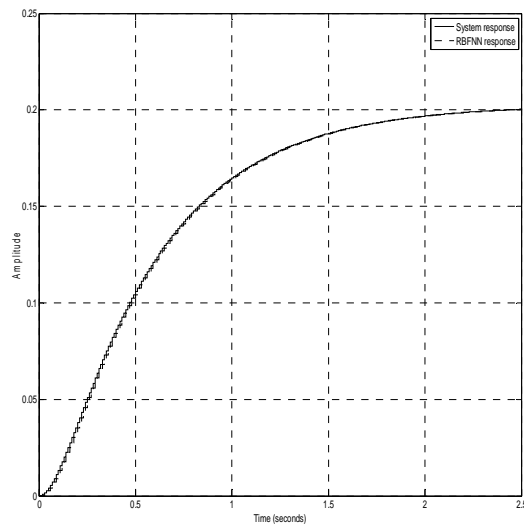The DC motor system response and RBFNN response are depicted in Fig. 11.

Fig. 11 System output and RBFNN output

D. Nonlinear System (Academic Model)

The plant considered for identification is a non-linear model governed by the difference equation:

$$Y_P(t) = 0.8\,Y_P(t-1) + f[u(t-1)] \qquad (14)$$

where the unknown non-linear function has the form:

$$f[u] = (u-0.8)u(u+0.5) \qquad (15)$$

where: u and $Y_P(t)$ are the input and output to the system, respectively [16].

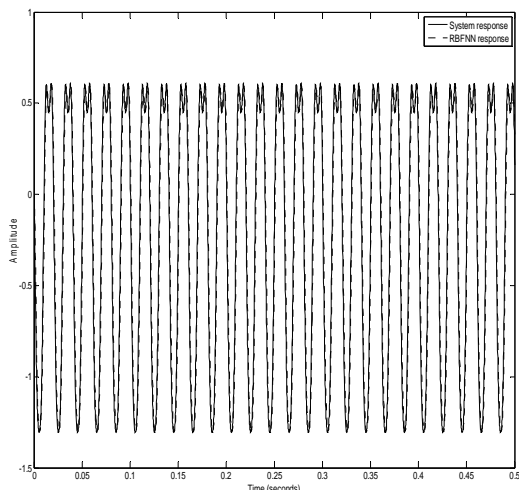The nonlinear system response and RBFNN response are depicted in Fig. 12.

Fig. 12 System output and RBFNN output

E. Nonlinear System (Academic Model)

The plant considered for identification is a non-linear model governed by the equation:

$$(1 - 1.425\, Z^{-1} + 0.496\, Z^{-2})Y(t) = (-0.102\, Z^{-2} + 0.173\, Z^{-3})g_u(t) \quad (16)$$

where the unknown non-linear function has the form

$$g_u(t) = 2\, u(t) - 0.25\, u^2(t) \quad (17)$$

The $u(t)$ and $Y(t)$ are the input and output to the system, respectively [16], [21].

The nonlinear system response and RBFNN response for are depicted in Fig. 13.
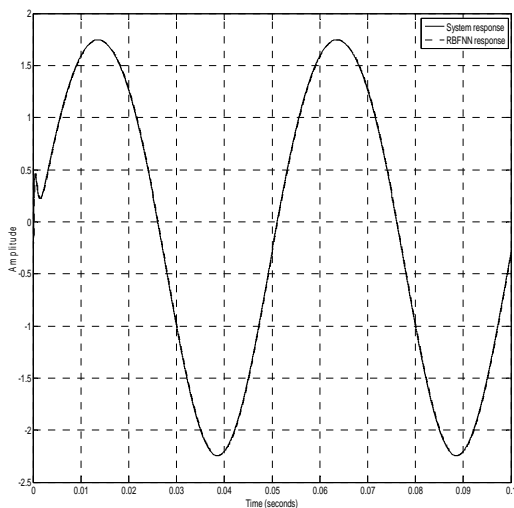

Fig. 13 System output and RBFNN output

III. CONCLUSION

Simulation results show that the RBF neural network exhibits good performance and ability in modelling dynamic systems. It also found that the response of the desired signal

was heavily dependent on the value of the learning factor, centers, width and weights of RBF. Also, the simulation results show that the radial basis function neural network is a powerful tool for nonlinear system modelling; it is an accurate method for modeling the nonlinear systems. One of the main advantages of RBFNN is that the ability of quick modifying in the modelling during the change of the dynamics of the process.

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