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# Cost Based Warranty Optimisation Using Genetic Algorithm

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***Abstract*—**Warranty is a powerful marketing tool for the manufacturer and a good protection for both the manufacturer and the customer. However, warranty always involves additional costs to the manufacturer, which depend on product reliability characteristics and warranty parameters. This paper presents an approach to optimisation of warranty parameters for known product failure distribution to reduce the warranty costs to the manufacturer while retaining the promotional function of the warranty. Combination free replacement and pro-rata warranty policy is chosen as a model and the length of free replacement period and pro-rata policy period are varied, as well as the coefficients that define the pro-rata cost function. Multi-parametric warranty optimisation is done by using genetic algorithm. Obtained results are guideline for the manufacturer to choose the warranty policy that minimises the costs and maximises the profit.

***Keywords*—**costs, genetic algorithm, optimisation, warranty.

## I. Introduction

FROM the customer's point of view warranty has two functions – protective and informative. Protective function means that warranty assures the customer that faulty products will either be repaired or replaced at no cost or at a reduced cost. Warranty performs its informative function by indirectly giving the customer information about the product quality. Warranty also has two functions from the manufacturer's point of view – protective and promotional. Protective function is implied by warranty terms that specify the use of the product and limited coverage or no coverage at all in the case of product misuse. Promotional function is analogous to the informative function from the customer's point of view.

Warranty is one of the key factors in today's customer decision-making process. When choosing among several similar products, the customer will usually buy the product provided with a better warranty. This led to the competition among the manufacturers in offering better warranty to attract more customers. There are many techniques for warranty optimisation [1]–[3]. In this paper, a combination free replacement and pro-rata warranty policy will be optimised in terms of cost reduction for the desired ranges of warranty parameters using a genetic algorithm. To make it possible, warranty costs are expressed using a nonlinear regression model.

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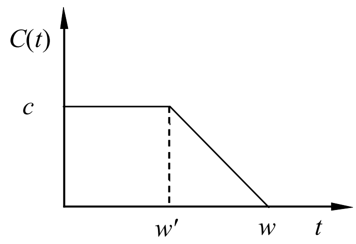
## II. Warranty Cost Calculation

Combination free replacement and pro-rata warranty (FRW/PRW) policy is often used as a compromise between the free replacement warranty (FRW) and the pro-rata warranty (PRW). FRW/PRW policy is comprised of two periods – a period of free replacement followed by a period of pro-rata policy [4]. The manufacturer agrees to replace the product with a new product at no cost to the customer if it fails before $w'$ ($w' < w$) expires, where $w$ is warranty period length and $w'$ is free replacement period length. If a product fails in the time interval from $w'$ to $w$ it is replaced by the manufacturer at a fraction of the replacement cost (pro-rata cost) to the customer. This type of combination warranty has a significant promotional value to the manufacturer and at the same time provides adequate cost control for both the manufacturer and the customer in most cases [5], [6]. FRW/PRW policy is usually offered with nonrepairable products. More about the types of warranty policies can be found in [7] and [8].

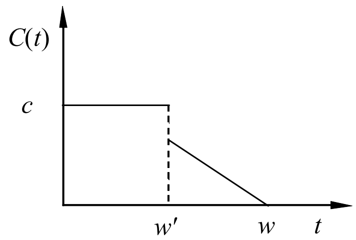
In this paper warranty needs to be optimised for one type of passenger car batteries. According to the collected data from a 16 year long exploitation it is determined that the life of this type of battery follows Weibull distribution with a shape parameter $\beta = 1.63$ and a scale parameter $\eta = 4380$ days. The price per battery unit excluding the warranty cost is $c' = 82$ €. In the analysis done for this paper a pro-rata cost is a linear function of time, and because of that, replacement cost to the manufacturer at time $t$ is calculated using the following equation [5], [6]:

$$C(t) = \begin{cases} c, & 0 \le t < w', \\ kc\left(1 - \delta \dfrac{t - w'}{w - w'}\right), & w' \le t < w, \\ 0, & t \ge w, \end{cases} \tag{1}$$

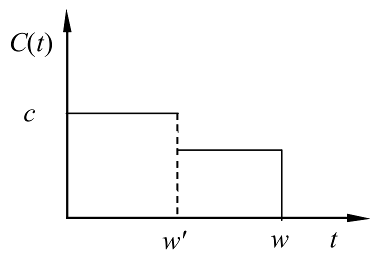
where $c$ is unit price after adding the warranty cost $r$ ($c = c' + r$), $k$ is proportionality coefficient of $c$, and $\delta$ is proportionality coefficient of time of failure in the warranty interval. For $w' = 0$ FRW/PRW policy becomes PRW policy, and for $w' = w$ FRW/PRW policy becomes FRW policy. Fig. 1 illustrates how proportionality coefficients affect warranty policies.



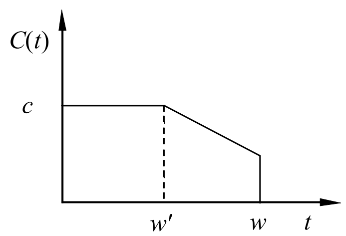

(a) $\delta = 1$, $k = 1$




(b) $\delta = 1$, $0 < k \leq 1$




(c) $\delta = 0$, $0 < k \leq 1$




(d) $0 < \delta \leq 1$, $k = 1$

Fig. 1 FRW/PRW policies with different values of proportionality coefficients

Some assumptions need to be made for the analysis to be done. It is assumed that every failure results in a warranty claim, all warranty claims are valid and all failures are statistically independent. Probability density function for a Weibull distribution is given by

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}, \tag{2}$$

and the cumulative distribution function is defined as

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}. \tag{3}$$

The number of failures for one battery unit by time $t$ has a geometric distribution, so the expected number of failures for one battery unit by time $t$ is

$$E\left[N(t)\right] = \frac{F(t)}{1-F(t)}. \tag{4}$$

The expected number of failures for the whole lot by time $t$ can be calculated by multiplying the expected number of failures for one battery by the product lot size $L$:

$$L \cdot E\left[N(t)\right] = L \cdot \frac{F(t)}{1-F(t)}. \tag{5}$$

The total number of failures for the whole lot in the interval from $t$ to $t+dt$ is

$$L \cdot dE\left[N(t)\right] = L \cdot \frac{f(t)}{\left(1-F(t)\right)^2}, \tag{6}$$

and the expected total cost to the manufacturer for the failures from $t$ to $t+dt$ is

$$d\left(T_C\right) = C(t) \cdot L \cdot \frac{f(t)}{\left(1-F(t)\right)^2}, \tag{7}$$

where $C(t)$ is the replacement cost to the manufacturer at time $t$. Total expected cost to the manufacturer for the whole warranty period is then calculated using the following equation:

$$T_C = L \cdot \left( \int_0^{w'} c \cdot \frac{f(t)}{\left(1-F(t)\right)^2} dt + \int_{w'}^{w} kc\left(1 - \delta\frac{t-w'}{w-w'}\right) \cdot \frac{f(t)}{\left(1-F(t)\right)^2} dt \right). \tag{8}$$

Warranty cost to the manufacturer per unit is obtained by dividing the total cost with the lot size $L$:

$$r = \frac{T_C}{L} = \int_0^{w'} c \cdot \frac{f(t)}{\left(1-F(t)\right)^2} dt + \int_{w'}^{w} kc\left(1 - \delta\frac{t-w'}{w-w'}\right) \cdot \frac{f(t)}{\left(1-F(t)\right)^2} dt. \tag{9}$$

In order to compare the unit price values after adding the warranty cost calculated using these equations with those obtained using the regression model, warranty costs were calculated for the warranty period length $w$ ranging from 1 to 6 years and the free replacement period length $w'$ ranging from 0 to $w$ for three different combinations of proportionality coefficients. Table I shows these calculated values.

TABLE I
UNIT PRICE VALUES IN € AFTER ADDING THE WARRANTY COST

| $w$ [years] | $w'$ [years] 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $k = 1$, $\delta = 1$ | | | | | | | |
| 1 | 82.55 | 83.47 | | | | | |
| 2 | 83.74 | 84.98 | 86.81 | | | | |
| 3 | 85.51 | 87.06 | 89.26 | 92.14 | | | |
| 4 | 87.87 | 89.80 | 92.43 | 95.83 | 100.19 | | |
| 5 | 90.95 | 93.33 | 96.49 | 100.56 | 105.78 | 112.53 | |
| 6 | 94.92 | 97.85 | 101.71 | 106.66 | 113.05 | 121.39 | 132.55 |
| $k = 0.5$, $\delta = 1$ | | | | | | | |
| 1 | 82.27 | 83.47 | | | | | |
| 2 | 83.47 | 84.21 | 86.81 | | | | |
| 3 | 83.72 | 85.23 | 88.02 | 92.14 | | | |
| 4 | 84.83 | 86.52 | 89.53 | 93.95 | 100.19 | | |
| 5 | 86.24 | 88.12 | 91.39 | 96.17 | 102.91 | 112.53 | |
| 6 | 87.99 | 90.09 | 93.67 | 98.87 | 106.23 | 116.79 | 132.55 |
| $k = 1$, $\delta = 0.5$ | | | | | | | |
| 1 | 83.00 | 83.47 | | | | | |
| 2 | 85.25 | 85.88 | 86.89 | | | | |
| 3 | 88.70 | 89.53 | 90.68 | 92.14 | | | |
| 4 | 93.63 | 94.71 | 96.15 | 97.96 | 100.19 | | |
| 5 | 100.60 | 102.03 | 103.89 | 106.21 | 109.05 | 112.53 | |
| 6 | 110.62 | 112.59 | 115.10 | 118.21 | 122.03 | 126.73 | 132.55 |

## III. REGRESSION MODEL AND GENETIC ALGORITHM

The main purpose of this paper is to present the methodology for warranty policy optimisation using genetic algorithm. Genetic algorithm is a method for solving optimisation problems which bases its functioning on the process of natural selection. Population of solutions is modified in every iteration to evolve towards an optimal solution [9]. More about the genetic algorithm can be found in [10].

Fig. 2 shows the proposed methodology flowchart. In order to use the genetic algorithm for the optimisation of warranty policy based on a cost reduction, warranty cost, i.e. unit price value after adding the warranty cost must be expressed in a suitable form. For that purpose, a nonliner regression is chosen as a method. After a trial-and-error process, it was established that the best regression model is defined as

$$r = a_1 \cdot w'^{a_2} + a_3 \cdot k \cdot \left(a_4 + a_5 \cdot \delta\right) \cdot \left(w - w'\right)^{a_6} \cdot \left(w + w'\right)^{a_7} \quad (10)$$

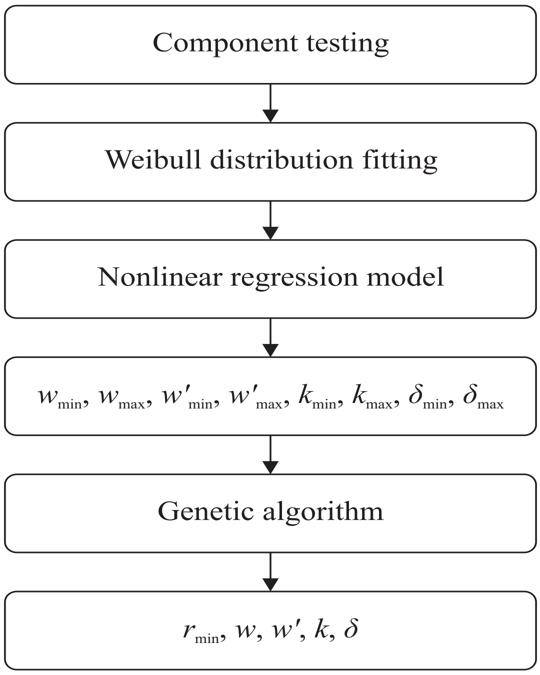



Fig. 2 Methodology flowchart

Performing the nonlinear regression [11] on the set of calculated values shown in Table I give the regression coefficients that finally define the nonlinear regression model:

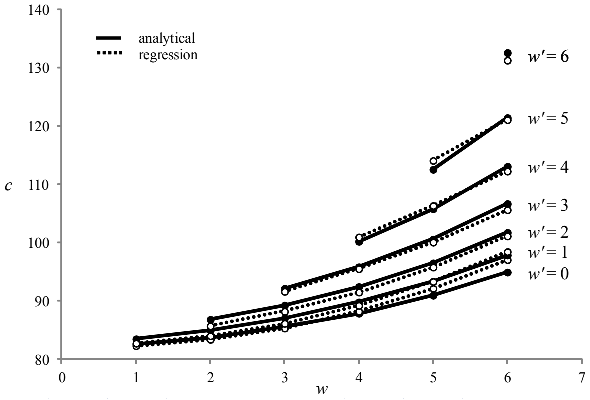
$$r = 0.722065 \cdot w'^{2.356541} + 0.713293 \cdot k \cdot \left(1.164977 - 0.738176 \cdot \delta\right) \cdot \left(w - w'\right)^{0.869450} \cdot \left(w + w'\right)^{1.307407} \quad (11)$$

Unit price values after adding the warranty cost are calculated using (11) for the same combinations of warranty parameters as in Table I. Fig. 3 shows comparison between the unit price values after adding the warranty cost obtained using the analytical equation (9) and those obtained using the nonlinear regression model (11).

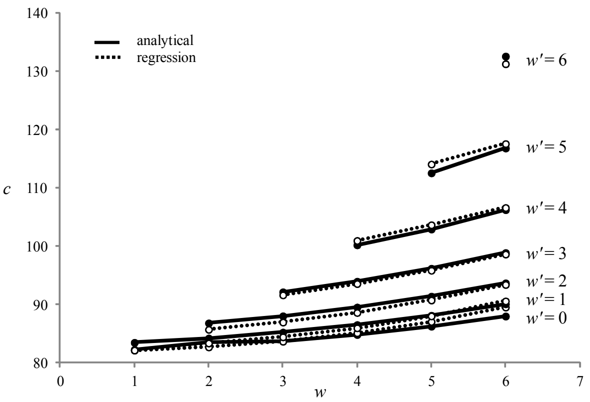
Nonlinear regression model defined by (11) now can be used, according to the flowchart given in Fig. 2, for optimisation using a genetic algorithm in MATLAB's Global Optimization Toolbox. One can choose the desired interval for every parameter that defines the warranty policy, and the genetic algorithm will give the optimal values of warranty parameters – those that produce the lowest warranty cost.

## IV. CONCLUSION

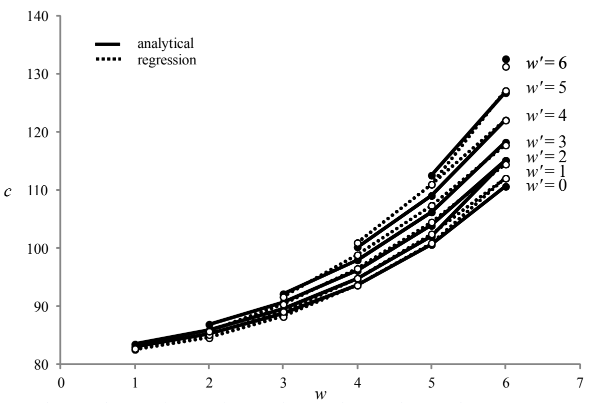
Warranty costs have a great impact on the final price of a product provided with warranty. Manufacturers need to take all the measures they can to reduce these costs. In this paper, FRW/PRW policy is optimised using the genetic algorithm, for which a special nonlinear regression model is prepared. Nonlinear regression model proved to be accurate enough compared to the analytical equations used for the calculation of unit price values after adding the warranty cost. The manufacturer can conduct a market research [12] and afterwards, according to the obtained results, choose the intervals for the warranty parameters. These intervals are the inputs for the genetic algorithm for optimisation of warranty policy in terms of cost to the manufacturer. The outputs from the algorithm are values of warranty parameters that lower the costs to the manufacturer, while retaining the promotional function of the warranty (since parameter intervals are chosen according to the market research).




(a) $k = 1$, $\delta = 1$




(b) $k = 0.5$, $\delta = 1$




(c) $k = 1$, $\delta = 0.5$

Fig. 3 Comparison of analytical unit price values and those obtained using the regression model

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