

Diagnosing the Cause and its Timing of Changes in Multivariate Process Mean Vector from Quality Control Charts using Artificial Neural Network

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Abstract—Quality control charts are very effective in detecting out of control signals but when a control chart signals an out of control condition of the process mean, searching for a special cause in the vicinity of the signal time would not always lead to prompt identification of the source(s) of the out of control condition as the change point in the process parameter(s) is usually different from the signal time. It is very important to manufacturer to determine at what point and which parameters in the past caused the signal. Early warning of process change would expedite the search for the special causes and enhance quality at lower cost. In this paper the quality variables under investigation are assumed to follow a multivariate normal distribution with known means and variance-covariance matrix and the process means after one step change remain at the new level until the special cause is being identified and removed, also it is supposed that only one variable could be changed at the same time. This research applies artificial neural network (ANN) to identify the time the change occurred and the parameter which caused the change or shift. The performance of the approach was assessed through a computer simulation experiment. The results show that neural network performs effectively and equally well for the whole shift magnitude which has been considered.

Keywords—Artificial neural network, change point estimation, monte carlo simulation, multivariate exponentially weighted moving average

I. INTRODUCTION

It has been proven that quality control charts are very effective in detecting out of control signals. If a control chart signals a change in the process parameter, examining the process for special causes only at the time of the signal may be ineffective. Identifying the time of the parameter change will substantially assist the signal diagnostics procedure since it makes the search for special causes more efficient and corrective measures can be implemented sooner. According to the following literature review, it is well known that when a control chart signals an out of control condition, searching for a special cause in the vicinity of the signal time would not always lead to prompt identification of the source(s) of the out of control condition as the change point in the process parameter(s) is usually different from the signal time.

In univariate environment several researchers including Samuel et al. [1], Samuel and Pignatiello [1],[2], Hawkins and Qiu [3], Perry et al. [4],[5], Ghazanfari et al. [6], and Noorossana et al. [7] have investigated change point estimation in the presence of different change types such as step, linear trend, and monotonic changes.

But, when several characteristics of a manufactured component are to be monitored simultaneously identifying the change point by itself would not effectively lead to the source of disturbance. In other words, in multivariate environment, effective root cause analysis requires not only the identification of the change point but also the knowledge on the variable(s) leading to the change in the process parameters. Nedumaran et al. [8] referred to or discussed? issue of change point identification for χ^2 control chart, when several quality characteristics are to be monitored simultaneously. They used maximum likelihood estimator to estimate a step change shift in a mean vector when observations follow a multivariate normal distribution. Several authors including Montgomery [9], Wade and Woodall [10], Hawkins [11], Hayter and Tusi [12], Kourti and MacGregor [13], Nottingham et al. [14], Niaki and Abbasi [15], Guh [16], and Hawng [17],[18],[19] have investigated issues related to the diagnostic analysis in multivariate environment. Bersimis et al. [20] provides a comprehensive literature review on the multivariate control charts along with different diagnostic analyses to identify variable(s) associated with the out of control condition.

This paper focuses on *MEWMA* charts and how to identify out of control signals. It describes how neural network may be used to identify the step change point in the process mean vector. Next section describes *MEWMA* procedure and section three provides ANN architecture to detect the change point, and identify which parameters have caused the change and in both parts the performance of the approach is assessed by using Monte Carlo simulation, finally the conclusions are provided.

II. THE MEWMA PROCEDURE

The *MEWMA* chart was introduced by Lowry, Woodall, Champ and Rigdon [21]. They suppose that we observe X_1, X_2, \dots in the univariate case i.e. when $p = 1$. The univariate EWMA chart is based on these values:

$$z_i = \tau X_i + (1 - \tau) z_{i-1} \quad (1)$$

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$i = 1, 2, \dots$, where $Z_0 = \mu_0 = 0$ and $0 < r \leq 1$. If $X_1, X_2, \dots, X_i, \dots$ are iid $N(0, \sigma^2)$ random variables, then the mean of Z_i is 0 and the variance is

$$\sigma_{z_i}^2 = \frac{r}{2-r} [1-(1-r)^{2i}] \sigma^2 \quad i = 1, 2, \dots \quad (2)$$

Thus, when the control value of the mean is 0, the control limits of the EWMA chart are often set at $\pm L\sigma_{z_i}$, here L and r are the parameters of the chart. In the multivariate case, according to Lowry, Woodall, Champ and Rigdon [21] a natural extension is to define the vectors of EWMA's,

$$Z_i = R X_i + (1 - R) Z_{i-1} \quad i = 1, 2, \dots \quad (3)$$

where $Z_0 = 0$ and $R = \text{diag}(r_1, r_2, \dots, r_p)$, $0 < r \leq 1$, $j = 1, 2, \dots, p$. The MEWMA chart gives an out of control signal as soon as:

$$T_i^2 = \mathbf{z}'_i \Sigma_{z_i}^{-1} \mathbf{z}_i > L \quad (4)$$

Where $L (>0)$ is chosen to achieve a specified in control ARL and Σ_{z_i} is the covariance matrix of Z_i . If there is no a priori reason to weight past observations differently for the p quality characteristics being monitored, then $r_1 = r_2 = \dots = r_p = r$. If the variables being monitored are not of equal importance and the desired ARL performance is such that the ARL should not be a function of λ , then the method of Hawkins [11] is recommended. Another possibility, proposed by Lowry, Woodall, Champ and Rigdon [21] is to use a different matrix in (3) to calculate the quadratic form of the MEWMA chart.

If $r_1 = r_2 = \dots = r_p = r$, then the MEWMA vectors can be written as:

$$Z_i = r X_i + (1 - r) Z_{i-1}, \quad i = 1, 2, \dots \quad (5)$$

$$\Sigma_{z_i}^2 = \frac{r}{2-r} [1-(1-r)^{2i}] \Sigma^2 \quad (6)$$

Analogous to the situation in the univariate case, the MEWMA chart is equivalent to Hotelling's χ^2 chart if $r=1$. As MacGregor and Harris [22] pointed out for the univariate case, using the exact variance of the EWMA statistic leads to a natural fast initial response for the EWMA chart. Thus initial out of control conditions are detected more quickly. This is also true for the MEWMA chart. Because, however, it may be more likely that the process will stay in control for a while and then shifts out of control, we will assume for a chart design and in our ARL comparisons that the asymptotic (as $i \rightarrow \infty$) covariance matrix, that is:

$$\Sigma_{z_i}^2 = \frac{r}{2-r} \Sigma^2 \quad (7)$$

used to calculate the MEWMA statistic in (6) unless otherwise indicated.

A. Multivariate Process Model

Suppose that a multivariate process is monitored by means of a MEWMA control chart on p important quality characteristics. Let $\mathbf{X}_{ij} = (X_{ij1}, X_{ij2}, \dots, X_{ijp})$ be a p vector which represents the p characteristics on the j th observation ($j = 1, 2, \dots, n$) in the i th subgroup of size n . Suppose further that

When the process is in control, the \mathbf{X}_{ij} 's are independent and identically distributed and follow a p variate normal distribution with mean vector μ_0 and covariance matrix Σ_0 . that \mathbf{X}_{ij} 's are iid $N_p(\mu_0, \Sigma_0)$, when the process is in control. Then

$$\bar{X}_i = \frac{1}{n} \sum_{j=1}^n x_{ij} \quad (8)$$

When the i th subgroup is observed, the statistic (4) is used. We will assume that, when the multivariate process means changes, there has been a step change from its in control value of $\mu = \mu_0$ to an unknown value $\mu = \mu_1$ where $\mu_0 \neq \mu_1$. If the statistic of MEWMA exceeds L , we may conclude that the step-change in the process mean has occurred at some unknown time τ where $0 \leq \tau \leq T-1$. Hence, it follows that the subgroup averages X_1, X_2, \dots, X_τ came from the in control process and the subgroup averages $X_{\tau+1}, X_{\tau+2}, \dots, X_T$ from the out of control process. We further assume that there is no change in the covariance structure and that the process means remain at the new level μ_1 until the special cause has been identified and removed. This is the same method used by Nedumaran et al. [8] for χ^2 control chart that is used here for MEWMA control chart.

III. IDENTIFYING THE PARAMETER WHICH CAUSE THE SIGNAL BY ARTIFICIAL NEURAL NETWORK

Artificial neural network is an approach to information processing that does not require algorithm or rule development. The three essential features of a neural computing network are the computing units, the connections between the computing units, and the training algorithm used to find values of the network parameters. The application of neural networks involves selecting feature vectors, establishing the network architecture choosing the activation function and training. The selection of feature vector in a crucial training affects the neural network. The feature vector must be able to help classifying shifts. Although many classical approaches and diagnostic analyses have been proposed in the literature for process control and disturbance interpretation in multivariate settings, relatively few studies have discussed the issue of signal detection and disturbance interpretation simultaneously. Guh [16] proposes a neural network based model that is capable of monitoring and performing diagnostic analyses in a bivariate process simultaneously. A step change in the mean vector and a known constant variance-covariance matrix are the two major assumptions of the model. Furthermore, Hwang [17],[18],[19] introduces a neural network model for detecting out of control conditions and simultaneously diagnosing the source of a step shift in the mean of a bivariate normal process. Moreover, Guh and Shiue [23] propose a decision tree-based model which is capable of detecting mean shift in a P variate normal process and at the same time helps to identify the variable(s) responsible for the out of control condition. Although different authors including Nishina [24], Nedumaran et al. [8], Sullivan

and Woodall [25], Li et al. [26], Hawkins and Zamba [27] have proposed procedures for identifying change point when a step change occurs in the mean of a multivariate process, but no one has pointed simultaneous identification of a change point and diagnostic analysis in multivariate settings, except Atashgar and Noorosana [28] whose research propose a model based on neural networks which detect an out of control condition, and estimate the linear change point in the process mean and performing diagnostic analysis to identify the variable (s) responsible for the out of control condition. They assume that the vector of observations follow a bivariate normal distribution with known variance-covariance matrix and the covariance matrix is unaffected by the disturbance.

To the best of our knowledge, no one has addressed simultaneous use of multivariate control chart and neural networks for identification of a step change point. In the previous research of Ahmadzadeh [29] multivariate control chart and neural network is used simultaneously and the result is compared with maximum likelihood estimation (*MLE*) approach which was used only for detecting the change point. In this paper we assume that the vector of observations follow a five normal variable distribution with known variance-covariance matrix and the covariance matrix is unaffected by the disturbance. We consider simultaneous use of multivariate control chart and neural networks for detection and identification of a step change point also diagnostic analysis regarding identification of variable which has caused the disturbance, in multivariate settings. Then two phases are considered. At the first stage we will find the change point and at the next phase we diagnose the variable which has caused the change, in both phases we use *MEWMA* control chart for detecting out of control condition

A. Phase One: Identifying Change Point

The neural network adopted in this study is a three-layer, fully connected, feed-forward network with a back propagation training algorithm that has been successfully applied to various classification problems. The neural network architecture including an input layer with n nodes, a hidden layer with 40 nodes, and an output layer with n nodes. (n = size of observation until *MEWMA* gives out the control signal). The activation function is important in the training stage. The one most extensively applied in back propagation algorithm is the tangent-sigmoid transfer function with output values in the interval $[-1, 1]$. Cheng [30] claimed that the tangent sigmoid transfer function effectively detects process changes in every direction. The tangent sigmoid function is used herein as the activation function in the hidden layer while linear function is for the output layer. The input vectors are presented to the network and propagated forward to yield the output. While Training, the weights and biases are iteratively modified according to the difference between the target and generated outputs. The Mean Square Error (*MSE*) associated with the output layer is propagated backward through the network, by modifying the weights. The popularity of the

gradient descent search method is based more on its simplicity rather than its search power. The training is terminated when the *MSE* of the difference of two successive iterations is within a predetermined tolerance 0.001 or when 100 epochs had been performed. When the *MEWMA* control chart gives signals for a sample which is out of control, the sample data are collected as input data to the trained network. In this stage it is assumed that the generating data has five variable normal distribution with mean $(0,0,0,0,0)$, unit variances and a correlation coefficient of 0.5. The values of $(X_1, X_2, X_3, X_4, X_5)$ are the observations, the values $(Z_1, Z_2, Z_3, Z_4, Z_5)$ correspond to the *MEWMA* vector in (3) with $r = 0.1$, and the values of T^2 were obtained using (2). The values of $L = 14.74$ were obtained using simulation to provide in control ARL's of 200, and we train and test the network with T^2 estimator of *MEWMA*. In the training stage Monte Carlo simulation is used, for example generation of 100 five variate normal values with mean μ_0 for first class, and creating shift magnitude of λ at observation 100 and generating again 100 five variate normal data with mean μ_1 as the second class. To start the training stage some transformation has been done on the raw data. First we calculate the mean of five consequent data to smooth data and consider it as X_i ($i=1..200$) then we put 10 consequent X_i as input vector. Output data for the first class are 0 and for the second class are 1. This work has been done hundred times for each shift magnitude all over the training phase. Seven shifts of magnitude $\lambda = 0.25, 0.5, 1, 1.5, 2, 2.5$ and 3.0 were considered. As soon as the input vector is passed through the trained network, the output activation at each output is examined against a pre specified decision interval to yield a transference output 0 or 1. The performance of the proposed network was tested using Monte Carlo simulation. The observations were assumed to come from a $N_p(\mu_0, \sigma_0)$ distribution when the process is in control. Although only one process dimension, namely $p = 5$ was considered there is no difference regarding result for multi dimensional parameters. One hundred subgroups of size $n = 1$ were generated randomly from the in-control distribution. If the *MEWMA* statistic for any of these subgroups exceeded L , all data from that subgroup were discarded and replaced with new ones. The new *MEWMA* statistic was then recomputed and compared with L . This procedure was repeated, as required, until 100 subgroups from the in control process had *MEWMA* statistics that did not exceed L . Thus, the *MEWMA* control chart did not issue any false alarms. Starting with subgroup 101, the simulated process mean was changed from μ_0 to μ_1 by introducing a shift of magnitude λ within the in control mean where subgroups were then generated from the out of control process until a subgroup's *MEWMA* statistic exceeded its L , that is, until the control chart issued a genuine alarm signal. The change point estimation was then calculated following that genuine alarm signal, using the aforementioned method. This set of data is considered as input vector to the network to test the network. The output of the network

concludes a vector of 0 and 1, if our network has a good training it must include a series of 0 and 1, that is shown where 0 is changed to 1 is, where the change or shift in mean occurs. This procedure was then replicated 10,000 times, and the average of those 10,000 change point estimates its standard error. Seven shifts of magnitude $\lambda = 0.25, 0.5, 1, 1.5, 2, 2.5$ and 3.0 are considered. The results of this simulation study are presented in the following Tables. In Table I we show the time at which the *MEWMA* control chart is expected to issue a genuine signal of a change in the process mean. Also results show that NN_1 averages are in fact close to the actual change point of $t = 100$ for all shift magnitudes and for all dimensions considered. Thus, on average, our proposed network change point estimate is close to the actual time of change regardless of the values of the shift magnitude and process dimension. For example Table I shows, when $p = 5$ and $\lambda = 1$, the *MEWMA* control chart on average will signal the change in the process mean on subgroup 108.68 when the actual change occurred after the 100th subgroup. That is, the control chart on average signals the change 8.68 subgroups after the actual change. However NN_1 will on average detect the change in the process mean on subgroup 101.88, it means 1.88 subgroup after the actual time of the change neural network can detect the change. Thus, seeking special causes at the time of the signal might be futile. Instead if they should search for special causes around the time it happened to appropriately conclude, the special cause. Results show that the neural network change point estimates around the actual change point will not depend on the shift magnitude and for all shift magnitude is approximately up to two subgroups after the actual change point. Results show that *MEWMA* will signal the shift magnitudes greater than one and will detect all shift magnitudes.

B. Phase Two: Identifying the Parameter Which Has Caused The Signal

This phase considers several assumptions. First it assumes no change in the covariance structure and the process means when change remains at the new level of μ_1 during the period of identifying the change point Not new, repeated. Further it assume that only one variable mean is changed in the study period. The neural network adopted in this phase is a multi layer perceptrons (*MLP*) with two-layer, fully connected, feed-forward network with a back propagation training algorithm. The neural network architecture includes an input layer with $N*N*100 = 5*500$ nodes, a first hidden layer with 20 nodes, second hidden layer with 5 nodes to cover 5 variables. The output layer has $N*N*100$ nodes, $N =$ the number of variables. The activation function for the first layer is linear and for the second layer is tangent-sigmoid. Again training is terminated when the *MSE* of the difference of two successive iterations is within a predetermined tolerance 0.001 or when 100 epochs had been performed.

• Step 1: Training

This step assumes that the change point (*CP*) is available from the previous phase. So the distance between *CP* and the

signal of *MEWMA* control chart is known. It ensures that only the mean of one variable has been changed. The training stage considers a matrix with the dimension of $N*(N*100)$ as an input and another matrix with the

TABLE I

EXPECTED TIME OF A SIGNAL WITH $MEWMA=\tau$, AVERAGE OF CHANGE POINT ESTIMATES BY NN_1 , STANDARD ERROR FOR τ , $P=5$, $\tau=100$

λ	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
τ	172.63	124.25	112.80	108.68	105.30	103.86	103.08	102.5
NN_1	101.91	101.92	101.91	101.88	101.18	100.79	100.76	100.8
Std	0.38	0.30	0.18	0.11	0.04	0.03	0.02	0.02

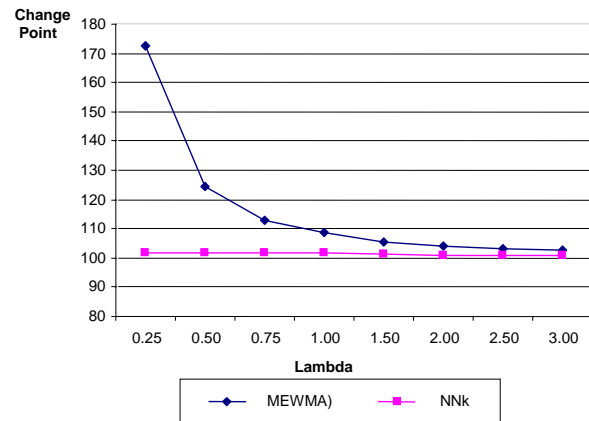


Fig. 1 Expected time of a signal with *MEWMA*, Average of change point estimates By NN_1 10000 time simulation for $\tau = 100$ and $p = 5$

Same dimensions as the out put N is the number of variables (parameters), set here at $N=5$. Then we assume a matrix of $5*100$ for each variable, so that, for first variable it means we have a matrix of 5 rows and 100 columns, in which the first array (variable mean's) changed as an input matrix, and we put the first array of the output matrix equal to 1 in the training stage and the another rows equal to 0, also for second variable changed as an input matrix, we put the second array of the output matrix equal to 1 and the another rows equal to 0, and so on until the fifth variable. It means, in the training stage our network can learn and identify which variables (parameters) has been changed.

• Step 2: Testing

This step builds upon the results of the first phase namely the *CP*. We transfer the fault data to the train network of the first step. Because in the training step our network learn to identify which variable mean's has been changed then when face with the fault data it can diagnose which variable mean's has been changed (we have the mean of the unchanged data). We do this for 10,000 times to decrease the error and find

mean error of our experiment. We can do it for every dimension of variables.

Results show that the NN will identify the variable (parameter) for which the mean has changed causing the signal in $MEWMA$ control chart. Also diagnosing the variable is done independent of the magnitude of the shift. NN_1 , in first phase, will detect the change point very soon after it happens, for all shift magnitudes. NN_2 in phase two determines which variable has caused the change. Table II shows that for $\lambda=1$ the design NN_2 identifies the first variable's mean alteration with probability of 97.5% and the diagnostic error for the 2nd and 5th variables are 2% and 0.5% respectively. Also for $\lambda=3$ the diagnosis is 100% correct which shows that the net work performs very well. In all the following Tables P is number of variables, τ is time of the change point. Changing mean of the first to five variables the result is the same and the NN_2 identifies the variables causing the change. Table III to Table XI demonstrate this point.

IV. CONCLUSIONS

In this paper two approaches were proposed for identifying the time of a step change and identifying the variable of causing the change in a multivariate production process mean vector. First phase is designing a neural network, NN_1 , to identify the time of the change point and phase two applies another neural network, NN_2 , to diagnose the variable whose mean has changed. The described methods show how the $MEWMA$ control charts can be used in conjunction with NN . When a $MEWMA$ control chart signals a change in the process mean, a search for special causes responsible for the change can be made to determine when exactly the actual change has taken place. This is especially true when there is only a small change in the process mean since the average run length can be quite large you did not show that this is covered well. The simulation studies showed that given a change in the process mean vector, the network in first phase performed effectively in detecting the actual change point and in phase two the network performed very well in detecting the actual variable responsible for the change for all shifts magnitude. Also results show that the network in phase one will detect the time of a change point for all shift magnitudes up to two subgroups after the actual change point. The network in phase two diagnoses the responsible variable(s) for the change with probability of at least 96% and max error in detecting other variable is about 2% in this example. Ultimately, the results show that two proposed networks performs effectively well in detecting the shifts.

TABLE II

RESULT OF DESIGN NN_2 WHEN FIRST VARIABLE MEAN CHANGED FOR $P=5$, $\tau=100$

λ	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
1 st	97%	99%	100%	97.50%	100%	98%	100%	100%
2 nd	0	0	0	2%	0	1%	0	0
3 rd	1%	0	0	0	0	0	0	0
4 th	2%	0	0	0	0	0	0	0
5 th	0	1%	0	0.5%	0	1%	0	0

TABLE III

RESULT OF DESIGN NN_2 WHEN SECOND VARIABLE MEAN CHANGED FOR $P=5$, $\tau=100$

λ	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
1 st	0	0	1%	0	1%	0	1%	0
2 nd	100%	100%	99%	98%	98%	100%	99%	100%
3 rd	0	0	0	0	0	0	0	0
4 th	0	0	0	0	1%	0	0	0
5 th	0	0	0	2%	0	0	0	0

TABLE IV

RESULT OF DESIGN NN_2 WHEN THIRD VARIABLE MEAN CHANGED FOR $P=5$, $\tau=100$

λ	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
1 st	0	0	1%	0	1%	2%	0	0
2 nd	0	0	0	2%	0	1%	0	0
3 rd	100%	100%	99%	97%	98%	96%	100%	99%
4 th	0	0	0	0	1%	1%	0	1%
5 th	0	0	0	1%	0	1%	0	0

TABLE V

RESULT OF DESIGN NN_2 WHEN FORTH VARIABLE MEAN CHANGED FOR $P=5$, $\tau=100$

λ	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
1 st	1%	0	0	1%	0	2%	0	0
2 nd	0	0	0	2%	0	1%	0	0
3 rd	1%	0	0	0	0	0	0	0
4 th	98%	99%	100%	96.5%	100%	96%	100%	100%
5 th	0	1%	0	0.5%	0	1%	0	0

TABLE VI

RESULT OF DESIGN NN_2 WHEN FIFTH VARIABLE MEAN CHANGED FOR $P=5$,
 $\tau=100$

λ	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
1 st	0	0	0	0	0	1%	0	0
2 nd	1%	0	1%	1%	0	0	1%	0
3 rd	1%	0	0	1%	0	0	0	0
4 th	2%	0	0	0	0	1%	0	0
5 th	96%	100%	99%	98%	100%	98%	99%	100%

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