An Improved Lattice Reduction Aided Detection Scheme for MIMO-OFDM System

Jang-Kyun Ahn, Seung-Jun Yu, Eui-Young Lee and Hyoung-Kyu Song

Abstract—This paper proposes an efficient lattice-reduction-aided detection (LRD) scheme to improve the detection performance of MIMO-OFDM system. In this proposed scheme, V candidate symbols are considered at the first layer, and V probable streams are detected with LRD scheme according to the first detected V candidate symbols. Then, the most probable stream is selected through a ML test. Since the proposed scheme can more accurately detect initial symbol and can reduce transmission of error to rest symbols, the proposed scheme shows more improved performance than conventional LRD with very low complexity.

Keywords—Lattice reduction aided detection, MIMO-OFDM, QRD-M, V-BLAST.

I. INTRODUCTION

THE increasing demands of multimedia services and high speed wireless communications lead to a great interest in multiple-input and multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) technique. The MIMO-OFDM architecture, which is a combination of advantages of MIMO and OFDM, is very high capacity and spectral efficiency achieved by simultaneously employing the time, space and frequency domains with multiple transmit/receive antennas.

MIMO-OFDM using spatial division multiplexing(SDM) scheme is regarded as a promising solution to enhance the performance in rich scattering wireless channel. Therefore, several detection schemes have been proposed for detection of MIMO-OFDM system symbols. The vertical Bell Laboratories layered space time (V-BLAST) [1] has demonstrated very high spectral efficiency but it has marginal error performance for linear detection scheme (ZF, MMSE). The ordered decision feedback equalization (DFE) [2] or ordered successive interference cancelation (OSIC) detection scheme has better error performance than linear detection. Because the accurate detection of the first layer improves overall system performance in the ordered DFE or OSIC detection, the performance depends on initial layer detection capability. However, the ordered DFE or OSIC detection has wide gap error performance compared with optimum error performance detection scheme. Among MIMO-OFDM detection schemes, maximum likelihood detection (MLD) scheme has the best error performance for V-BLAST system. However, the complexity of MLD is exponentially increased by the number of transmit antennas

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and constellation level. Therefore, MLD can not use for practical implementation for complexity, although it achieves high transmit data rate due to good error performance. To reduce the MLD complexity, the QR-decomposition M algorithm(QRD-M) scheme which is comparable performance to ML scheme is proposed [3]. Although the QRD-M detection reduces the complexity by selecting M candidates and achieve near ML detection performance, the detection complexity of this scheme is still highly increased by the number of transmit and receive antennas, constellation level and M level [4].

Recently, the lattice-reduction-aided detection(LRD) schemes [5], [6] have been proposed. Although LRD scheme can achieve near MLD performance with low complexity, it is still unable to approach optimal performance.

In this paper, for more accurate detection of transmitted data, an efficient LRD scheme is proposed. From simulation results, the proposed scheme has better error performance than linear and conventional LRD.

II. SYSTEM DESCRIPTION

We consider MIMO-OFDM system with N_T transmit and N_R receive antennas. The OFDM symbol of m-th transmit antenna is represented as $\mathbf{X}_m = [X_m^{(0)}, X_m^{(1)} \ldots, X_m^{(K-1)}]$, where K denotes the number of subcarriers. After a data stream is divided into N_T substreams, OFDM symbols are transmitted over N_T transmit antennas simultaneously. The received signal model on the k-th subcarrier can be written as

$$\mathbf{Y}^{(k)} = \sum_{j=1}^{N_T} \mathbf{H}_j^{(k)} \cdot X_j^{(k)} + \mathbf{N}^{(k)} = \mathbf{H}^{(k)} \cdot \mathbf{X}^{(k)} + \mathbf{N}^{(k)} \quad (1)$$

where j and i are transmit and receive antenna index respectively, $\mathbf{X}^{(k)} = [X_1^{(k)}, X_2^{(k)} \ldots, X_{N_T}^{(k)}]^T$ denotes the $N_T \times 1$ transmit symbol vector, $\mathbf{Y}^{(k)} = [Y_1^{(k)}, Y_2^{(k)}, \ldots, Y_{N_R}^{(k)}]^T$ is the $N_R \times 1$ receive symbol vector and $\mathbf{N}^{(k)} = [N_1^{(k)}, N_2^{(k)}, \ldots, N_{N_R}^{(k)}]^T$ denotes the $N_R \times 1$ complex Gaussian additive noise vector with variance σ_n^2. Moreover, $\mathbf{H}$ is an $N_R \times N_T$ independent and identically distributed (i.i.d) random complex matrix of multipath channel with each element of $H_{i,j}^{(k)}$.

III. CONVENTIONAL LATTICE-REDUCTION-AIDED SCHEME

The channel matrix with large condition number has low performance because it amplifies noise for estimated transmit symbols. If the channel condition number is small, detection ability is increased. Lattice reduction aided detection

TABLE I
LLL ALGORITHM PROCEDURE.

Input : $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{P}$ (default: $\mathbf{P} = \mathbf{I}_{N_T}$)

Output : $\widetilde{\mathbf{Q}}$, $\widetilde{\mathbf{R}}$, $\mathbf{T}$

Initialization : $\widetilde{\mathbf{Q}} = \mathbf{Q}$, $\widetilde{\mathbf{R}} = \mathbf{R}$, $\mathbf{T} = \mathbf{P}$

$k = 2$

while $(k \leq N_T)$

   For $j = k-1:-1:1$

      $\mu = \llbracket \widetilde{\mathbf{R}}(j,k) / \widetilde{\mathbf{R}}(j,j) \rrbracket$

      if $\mu \neq 0$

        $\widetilde{\mathbf{R}}(1:j,k) = \widetilde{\mathbf{R}}(1:j,k) - \mu \widetilde{\mathbf{R}}(1:j,j)$

        $\mathbf{T}(:,k) = \mathbf{T}(:,k) - \mu \mathbf{T}(:,j)$

      end

   end

   if $\delta \widetilde{\mathbf{R}}(k-1,k-1)^2 > \widetilde{\mathbf{R}}(k,k)^2 + \widetilde{\mathbf{R}}(k-1,k)^2$

      $\widetilde{\mathbf{R}}(:,k-1:k) = \widetilde{\mathbf{R}}(:,k:k-1)$, $\mathbf{T}(:,k-1:k) = \mathbf{T}(:,k:k-1)$

      $\alpha = \dfrac{\widetilde{\mathbf{R}}(k-1,k-1)}{\left\| \widetilde{\mathbf{R}}(k-1:k,k-1) \right\|}$, $\beta = \dfrac{\widetilde{\mathbf{R}}(k-1,k-1)}{\left\| \widetilde{\mathbf{R}}(k-1:k,k-1) \right\|}$, $\Theta = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$

      $\widetilde{\mathbf{R}}(k-1:k,k-1:N_T) = \Theta \widetilde{\mathbf{R}}(k-1:k,k-1:N_T)$

      $\widetilde{\mathbf{Q}}(:,k-1:k) = \widetilde{\mathbf{Q}}(:,k-1:k)\Theta^T$

      $k = \max(k-1,2)$

   else

      $k = k+1$

   end

end

(LRD) scheme uses the reduced condition number of channel to estimate transmitted symbols. It uses Lenstra-Lenstra-Lovasz(LLL) method to reduce the channel condition number.

A. LLL (Lendstra-Lenstra-Lovasz) algorithm

In this paper, LLL algorithm performs with real-value channel matrix and received symbols are as follows

$$\mathbf{H} = \begin{bmatrix} \text{Re}(\mathbf{H}) & -\text{Im}(\mathbf{H}) \\ \text{Im}(\mathbf{H}) & \text{Re}(\mathbf{H}) \end{bmatrix},$$

$$\mathbf{X} = \begin{bmatrix} \text{Re}(\mathbf{X}) \\ \text{Im}(\mathbf{X}) \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} \text{Re}(\mathbf{Y}) \\ \text{Im}(\mathbf{Y}) \end{bmatrix}. \quad (2)$$

LLL algorithm is based on QR decomposition of channel matrix. LLL algorithm is shown in Table 1. $\llbracket \cdot \rrbracket$ is the rounding operation. The parameter δ with $1/4 < \delta < 1$ is used to achieve a faster convergence. A new channel matrix $\widetilde{\mathbf{H}}$ with QR decomposition is as follows

$$\widetilde{\mathbf{H}} = \widetilde{\mathbf{Q}}\widetilde{\mathbf{R}} = \mathbf{HT}, \quad (3)$$

where $\mathbf{T}$ with determinant ± 1 is unimodular matrix. $\widetilde{\mathbf{H}}$ with low condition number has more orthogonal matrix than original channel matrix $\mathbf{H}$. The original channel matrix $\mathbf{H}$ and new channel matrix $\widetilde{\mathbf{H}}$ have same lattice. Consequently, a new channel matrix $\widetilde{\mathbf{H}}$ may achieve a smaller condition number and better performance results than $\mathbf{H}$.

B. LRD (Lattice-Reduction-Aided detection)

The conventional LRD uses linear detection scheme. Linear scheme with $\mathbf{H}$ gives low diversity order but LRD scheme achieves the maximum receive diversity. A new matrix $\widetilde{\mathbf{H}}$ is given as Eqn. (3). By using this relation in Eqn. (1), it is rewritten as follows

$$\mathbf{Y} = \mathbf{HTT}^{-1}\mathbf{X} + \mathbf{N} = \widetilde{\mathbf{H}}\mathbf{T}^{-1}\mathbf{X} + \mathbf{N} = \widetilde{\mathbf{H}}\mathbf{Z} + \mathbf{N}, \quad (4)$$

where $\mathbf{Z} = \mathbf{T}^{-1}\mathbf{X}$. We multiply the vector $\mathbf{Y}$ by the Moore-Penrose pseudo-inverse of the channel matrix $\widetilde{\mathbf{H}}^+$ of the reduced channel matrix $\widetilde{\mathbf{H}}$.

$$\overline{\mathbf{Z}} = \widetilde{\mathbf{H}}^+\mathbf{Y} = \mathbf{T}^{-1}\mathbf{X} + \overline{\mathbf{N}} = \mathbf{Z} + \overline{\mathbf{N}}, \quad (5)$$

where $\overline{\mathbf{N}} = \widetilde{\mathbf{H}}^+\mathbf{N}$. In Eqn. (5), $\widetilde{\mathbf{H}}^+$ is a Moore-Penrose pseudo-inverse matrix. The zero-forcing (ZF) $\widetilde{\mathbf{H}}^+$ matrix is determined as

$$\tilde{\mathbf{H}}^+_{ZF} = (\widetilde{\mathbf{H}}^H \widetilde{\mathbf{H}})^{-1} \widetilde{\mathbf{H}}^H, \quad (6)$$

and the minimum mean square error (MMSE) $\widetilde{\mathbf{H}}^+$ matrix is

$$\tilde{\mathbf{H}}^+_{MMSE} = (\widetilde{\mathbf{H}}^H \widetilde{\mathbf{H}} + \sigma_n^2 \mathbf{I})^{-1} \widetilde{\mathbf{H}}^H. \quad (7)$$

where $\mathbf{I}$ is an identity matrix and $(\cdot)^H$ is the conjugate transpose operation. The estimated symbols $\mathbf{Z}$ is as follows

$$\widehat{\mathbf{Z}} = Q(\overline{\mathbf{Z}}) = Q(\widetilde{\mathbf{H}}^+\mathbf{Y}). \quad (8)$$

Here, the quantization $Q(\cdot)$ corresponds to a rounding operation because the symbols in the lattice are integer elements. The estimated symbols are finally transformed from the original basis as follows

$$\widehat{\mathbf{X}} = \mathbf{T}\widehat{\mathbf{Z}} = \mathbf{TT}^{-1}\mathbf{X}. \quad (9)$$

To use lattice theory, the original constellation points consist of symbols in $\mathbb{Z}_\mathbb{C}$, where $\mathbb{Z}_\mathbb{C}$ is set of integers. However, general L-QAM (where $\text{Re}\{x\} \in \{-\sqrt{L}+1, \cdots, \sqrt{L}-1\}$ and $\text{Im}\{x\} \in \{-\sqrt{L}+1, \cdots, \sqrt{L}-1\}$) constellations neither consist of continuous integer nor contain the origin and thus it is necessary to scale and shift the original constellation [7]. For example, it is assumed that the shifted and scaled constellation symbols are transmitted in noiseless channel. The received signal vector is

$$\mathbf{Y}' = \mathbf{HX}' = \mathbf{H}\frac{1}{2}[\mathbf{X} + \mathbf{l}] = \frac{1}{2}\mathbf{Y} + \frac{1}{2}\mathbf{Hl}, \quad (10)$$

where $\mathbf{X}'$ is shifted and scaled for transmit symbols. Moreover, $\mathbf{X}$ is original symbols in $\mathbb{Z}_\mathbb{C}$. $\mathbf{l}$ is $2N_T \times 1$ vectors with $[1, \ldots, 1]^T$. It multiply the Moore-Penrose pseudo-inverse matrix of $\tilde{\mathbf{H}}$ in Eqn. (10) as follows

$$\mathbf{Z} = \frac{1}{2}\mathbf{H}^+[\mathbf{HT}(\mathbf{T}^{-1}\mathbf{X}) + \mathbf{HT}(\mathbf{T}^{-1}\mathbf{l})] = \mathbf{T}^{-1}\mathbf{X}'. \quad (11)$$

To transfer continuous integer, we multiply $\mathbf{T}$ at $\mathbf{Z}$ and then transmitted symbols $\mathbf{X}$ is as follows

$$\overline{\mathbf{X}} = 2\mathbf{T}Q(\mathbf{Z}) - \mathbf{l}. \quad (12)$$

IV. PROPOSED DETECTION SCHEME

In this section, we propose improved performance detection scheme for MIMO-OFDM system. The basic idea is that V probable substream is detected at the first layer and then rest layer is detected with LRD scheme, because the performance of system considerably depends on detection capability of the first layer. Among the decoded substreams, the most probable stream is selected by likelihood test. The whole algorithm is described as follows.

The QR decomposition of channel matrix $\mathbf{H}$ is executed: $\mathbf{H} = \mathbf{QR}$, where $\mathbf{R}$ is an upper triangular matrix and $\mathbf{Q}$ is an orthonormal matrix satisfied with $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$. By multiplying $\mathbf{Q}^H$, the $N_T \times 1$ output vector can be expressed as

$$\mathbf{W} = \mathbf{Q}^H\mathbf{Y} = \mathbf{Q}^H\mathbf{HX} + \mathbf{Q}^H\mathbf{N} = \mathbf{RX} + \tilde{\mathbf{N}}, \quad (13)$$

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N_T} \end{bmatrix} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N_T} \\ 0 & r_{2,2} & \cdots & r_{2,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{N_T,N_T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_T} \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_{N_T} \end{bmatrix}.$$

In the first layer, we calculate metric values for the first symbol as follows

$$\left\| w_{N_T} - r_{N_T,N_T} \cdot \widehat{x}_{candi} \right\|^2 \quad (14)$$

where $\widehat{x}_{candi}$ denotes all possible constellation symbol and saves the metric value in memory. We calculate repeatedly metric values as many as constellation size of modulated signal. Therefore, C times metric calculations are performed, where C is constellation size. Then, metric values are ordered from the lowest to the largest and only the number of $V(V \in C)$ symbols, which have the smallest metric values, are retained as follows

$$\widehat{\mathbf{X}}_{candi} = [x_1^{(1)}, \cdots x_1^{(v)}, \cdots x_1^{(V)}], \quad (15)$$

In the rest layer, we use LRD scheme. Before we detect the rest symbols, we revise Eqn. (13) as follows

$$\begin{bmatrix} \widehat{w}_1 \\ \widehat{w}_2 \\ \vdots \\ \widehat{w}_M \end{bmatrix} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,M} \\ 0 & r_{2,2} & \cdots & r_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{M,M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_M \end{bmatrix}, \quad (16)$$

where M is $N_T - 1$. $\widehat{w}_k$ is as follows

$$\widehat{w}_k = w_k - r_{N_T,N_T} \cdot x_{N_T} (\text{where } 1 \leq k \leq N_T - 1). \quad (17)$$

We detect rest symbols to use LRD scheme. To detect rest symbols, LRD scheme executes V times. V stream is $\widehat{\mathbf{x}} = [\widehat{\mathbf{x}}_k^{(1)}, \cdots, \widehat{\mathbf{x}}_k^{(v)}, \ldots \widehat{\mathbf{x}}_k^{(V)}]$.

Finally, we perform the likelihood test using V stream. The likelihood test can be expressed as

$$P(\mathbf{Y}|\widehat{\mathbf{X}}^{(v)}) = [(\mathbf{Y} - \mathbf{H}\widehat{\mathbf{X}}^{(v)})^H (\mathbf{Y} - \mathbf{H}\widehat{\mathbf{X}}^{(v)})]. \quad (18)$$

Eqn. (18) is equivalent to minimum Euclidean distance. Thus, we estimate transmitted symbols as following method

$$\widehat{\mathbf{X}}_{final} = \arg \min_{\widehat{\mathbf{x}}^{(v)} \in \widehat{\mathbf{x}}} \left\| \mathbf{Y} - \mathbf{H}\widehat{\mathbf{X}}^{(v)} \right\|. \quad (19)$$
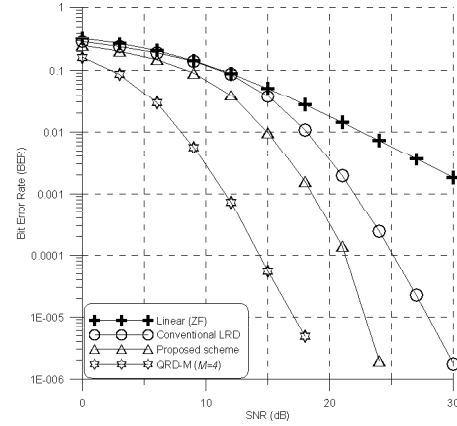

Fig. 1. BER performance of QRD-M, linear, conventional LRD and proposed detection scheme with QPSK modulation.
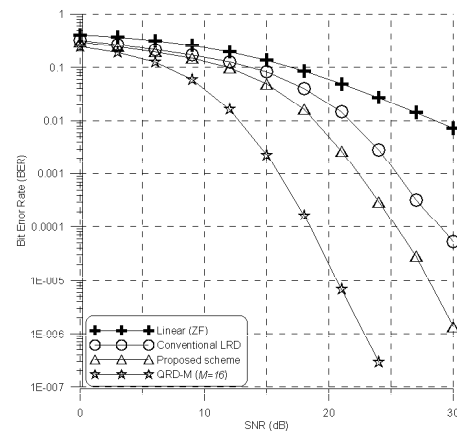

Fig. 2. BER performance of QRD-M, linear, conventional LRD and proposed detection scheme with 16-QAM modulation.

Therefore, proposed detection scheme with low complexity is approaching ML performance.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed scheme compared with the QRD-M detection scheme. We consider MIMO-OFDM system with $N_T = N_R = 4$ and set the number of subcarriers to 64. Moreover, the number of paths is 7. We assume that the channel is frequency flat fading during one OFDM symbol period. Moreover, we suppose that the channel state information (CSI) is known to receiver perfectly. In this paper, we use the LRD detection scheme with ZF of new channel state as Eqn. (5).

Fig. 1 shows the BER performance of proposed scheme with $V = 4$. The BER performance of QRD-M with $M = 4$ is also shown in Fig. 1. The QRD-M performance is compared to MLD(Maximum likelihood detection) scheme. As expected, the proposed scheme has better performance than linear and conventional LRD scheme. Linear detection scheme and conventional LRD have low error performance due to incorrect symbol at the first layer. However, the proposed scheme has better error performance due to adoption of V candidate

symbols at the first layer. The proposed detection scheme has more accurate symbol at the first layer. Moreover, when ordered channel condition is used, we acquire comparable error performance of QRD-M ($M = 4$).

Fig. 2 shows that the proposed scheme with $V = 16$ is better performance than linear and conventional LRD scheme with 16-QAM modulation. Compared with conventional LRD, performance gain of the proposed detection scheme has better performance about 5 dB at 10^{-3} .

VI. CONCLUSION

The performance of MIMO-OFDM system with DFE, OSIC and LRD scheme is limited by the first detected symbol due to error propagation. For this problem, we propose improved detection scheme which can detect more accurately has lower complexity than QRD-M or ML detection scheme. Moreover, the proposed detection scheme uses LRD instead of linear and conventional LRD scheme to improve performance. Simulation results show that the performance is improved by using V candidates symbols at the first layer. The proposed scheme has low complexity compared with ML, QRD-M scheme but has comparable performance at high SNR.

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