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Stability Optimization of Functionally Graded Pipes Conveying Fluid

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Abstract**—**This paper presents an exact analytical model for optimizing stability of thin-walled, composite, functionally graded pipes conveying fluid. The critical flow velocity at which divergence occurs is maximized for a specified total structural mass in order to ensure the economic feasibility of the attained optimum designs. The composition of the material of construction is optimized by defining the spatial distribution of volume fractions of the material constituents using piecewise variations along the pipe length. The major aim is to tailor the material distribution in the axial direction so as to avoid the occurrence of divergence instability without the penalty of increasing structural mass. Three types of boundary conditions have been examined; namely, Hinged-Hinged, Clamped-Hinged and Clamped-Clamped pipelines. The resulting optimization problem has been formulated as a nonlinear mathematical programming problem solved by invoking the MatLab optimization toolbox routines, which implement constrained function minimization routine named “fmincon” interacting with the associated eigenvalue problem routines. In fact, the proposed mathematical models have succeeded in maximizing the critical flow velocity without mass penalty and producing efficient and economic designs having enhanced stability characteristics as compared with the baseline designs.

Keywords**—**Functionally graded materials, pipe flow, optimum design, fluid- structure interaction

I. INTRODUCTION

THE concept of functionally graded materials *(FGMs*), in which the properties vary spatially within a structure, was originated in Japan in 1984 during the space project, in the form of proposed thermal barrier material capable of withstanding high temperature gradients. *FGMs* may be defined as advanced composite materials that fabricated to have graded variation of the relative volume fractions of the constituent materials [1]. *FGMs* can be promising in several applications such as, spacecraft heat shields, high performance structural elements, heat exchangers, oil and hydraulic pipelines, etc. A basic introduction to the mechanics of an elastic pipe containing flowing fluid may be found in many published literatures. Paidoussis and Issid [2] introduced the fundamental governing differential equations of flexible pipes containing flowing fluid. Both dynamics and stability were dealt with where the flow velocity can be either constant or with small harmonic component. It was shown that the system could be subjected to both divergence and flutter instabilities at higher flow velocities. The bending motion of a simply supported pipeline conveying fluid was investigated in [3], where a power series solution was used to solve the governing differential equations. The study showed that the ratio of fluid mass to total mass could have a considerable effect on the natural vibration characteristics of the system. Other recent work dealt with the analysis of pipes made of advanced composite materials can be found in the literature. Zou et al [4] presented a state-variable model developed for the analysis of fluid-induced vibration of composite pipeline systems. The effect of fluid Poisson’s ratio, the ratio of pipe radius to pipe wall thickness, laminate lay-up, fluid velocity and pressure were all considered in the analysis. Rabeih et al [5] studied the effect of composite material parameters on the natural frequencies and critical flow velocities of pipes conveying fluid with different configuration. A finite element model was derived based on Timoshenko beam theory of a generally orthotropic material pipe and results showed that the critical flow velocity is greatly affected by the composite material properties. The dynamic characteristics of fluid-conveying functionally graded materials cylindrical shells were investigated by Sheng and Wang [6]. A power-law was implemented to model the grading of material properties across the shell thickness and the analysis was performed using modal superposition and Newmark’s direct time integration method. Concerning system optimization, Tanaka et al. [7] applied variational principles combined with finite elements to maximize the critical flow velocity through a cantilevered pipeline having a constant structural mass. The pipe inner diameter was kept constant, while the wall thickness distribution was determined through the optimization process. Another work considering maximization of the fundamental bending frequency of a uniform cantilevered pipe for a fixed fluid velocity was given by Sallstrom [8]. The chosen design variables comprised the location and values of lumped masses, springs or dampers connected to the pipe. An optimum design problem to find the minimal structural mass at fixed critical flow speed was addressed in [9], with the finite element method applied to solve the associated linear equation of motion. Maalawi and Ziada [10] presented new methodologies for maximizing the critical flow velocity (divergence velocity) through flexible pipelines for a specified total mass. Optimum solutions were restricted to the case of simply supported pipes with the design variables taken only to be the wall thickness and length of each module composing the pipeline. All of the numerical examples treated in these studies have involved only single mode divergence cases, which is termed as unimodal optimization in which the lowest eigenvalue is well separated from the higher ones. Multimodal structural optimization was treated by Bendsoe et al. [11], in which a bound formulation using Lagrange multipliers was implemented. The true optimality conditions were derived for the problem with multiple eigenvalue constraints [12], but without any solution algorithm proposed. A more accurate method for calculating Lagrange multipliers was presented in [13], where the actual

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modality of the problem can be determined. Simplified modeling with the application of the penalty function method and Powell's multi-dimensional search technique to find the constrained optimum solutions was employed in [14]. Such standard non-gradient methods avoid the singularities in calculating the eigenvalue derivatives with respect to the design variables due to the multiplicity of the objective function. More recently, Librescu and Maalawi [15] introduced the underlying concepts of using material grading in optimizing subsonic wings against torsional instability, where both continuous and piecewise structural models were successfully implemented. Other recent work by Maalawi [16], [17] considered buckling optimization of functionally graded columns and cylindrical shells.In this paper, further investigations of the static instability phenomenon are considered by presenting a more spacious optimization model and extending the analysis to cover both effects of material, thickness grading and type of support boundary conditions. The model incorporates the effect of changing the volume fractions of the constituent materials for maximizing the critical flow velocity while maintaining the total mass at a constant value. Extensive computer results have been obtained to investigate the functional behavior of the critical flow velocity with the selected design parameters. Additional constraints are added to the optimization model by imposing upper limits on the fundamental eigenvalue to overcome the produced multiplicity near the optimum solution. The proposed optimization model can be regarded as a useful tool in obtaining pipeline designs having enhanced stability and stiffness levels.

II. Mathematical Model

The determination of the critical flow velocity at which static or dynamic instability can be encountered is an important consideration in the design of slender pipelines containing flowing fluid. At sufficiently high flow velocities, the transverse displacement can be too high so that the pipe bends beyond its ultimate strength leading to catastrophic instabilities. The present work is confined to maximization of the critical flow velocity, also termed as the divergence velocity, at which the elastic bending of the pipe increases rapidly to the point of failure. High divergence velocity can be regarded as a major aspect in designing an efficient piping system with enhanced flexural stability. Maximization of the divergence velocity can also have other desirable effects on the overall structural design. It helps in avoiding the occurrence of large displacements, distortions and excessive vibrations, and may also reduce fretting among structural parts, which is a major cause of fatigue failure.

A. Model description

The pipe model under consideration consists of rigidly connected thin-walled tubes, each of which has different material properties, cross-sectional dimensions and length, as shown in Fig. 1. The tube thickness, *h*, is assumed to be very small as compared with the mean diameter, *D*. The pipe conveys an incompressible fluid flowing steadily with an axial velocity U_k through the kth module. The variation in the velocity across the cross section is neglected, and the pipe is assumed to be long and slender so that the classical engineering theory of bending can be applicable. The effects of structural damping, damping of surroundings and gravity are not considered in the present analysis. The inclusion of the surrounding damping can only be significant in cases of buried pipelines [3], which are not treated herein. Practical designs ignoring small damping, which has stabilizing effect on the system motion, are always conservatives. Moreover, the present study does not consider pipes supported vertically, which are subject to additional gravity loads [2]. The model axis in its un-deformed state coincides with the horizontal x-axis, and the free small motion of the pipe takes place in a two dimensional plane with transverse displacement, *w.* Additional simplifying assumptions pertaining to specific derivations are presented in their respective sections.

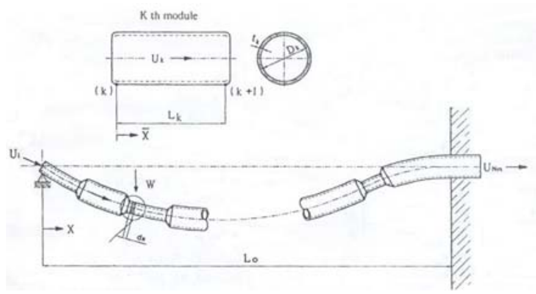

Fig. 1 General configuration of a piecewise axially graded pipe conveying fluid

B. The Eigenvalue problem

The eigenvalue problem associated with divergence instability is described by the differential equation [10]:

$$EI_k w'''' + \rho_f A_k U_k^2 w'' = 0 \tag{1}$$

where the notation ( )′ means differentiation with respect to x. Equation (1) may look like the differential equation, which governs buckling of elastic columns with the term $\rho_f A_k U_k^2$ regarded to be equivalent to the applied compressive force. However, from the mathematical and physical points of view, the problem of determining the critical flow velocity in an elastic pipe is not fully similar to that of the column’s buckling problem. The axial velocity is not, in general, constant lengthwise, as does the axial force in a column, and the distribution of the shearing force is also not the same. New features of buckling optimization of flexible columns have been addressed in [14], [16], where exact analysis was performed for columns having either tubular or solid cross sections. It is convenient to deal with dimensionless quantities so that the analysis can be valid for arbitrary pipe configurations. The various parameters are normalized by their corresponding values of a baseline pipe having the same total mass and length, material and fluid properties, and boundary conditions as well. The baseline pipe has uniform mass and stiffness distributions along its length and is made of

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two different materials denoted by *(A)* and *(B)* with equal volume fractions *(V)*, i.e. $V_A=V_B=50\%$. For the optimized designs, the physical and mechanical properties are allowed to vary lengthwise (i.e. $V_A \neq V_B$), yielding to grading of the material in the direction of the pipe's axis. Assuming no voids are present, the distributions of the mass density (ρ) and modulus of elasticity (E) are determined as follows [18]:

Volume fractions : $V_A(x) + V_B(x) = 1$
Mass density : $\rho(x) = V_A(x)\,\rho_A + V_B(x)\,\rho_B$
Modulus of elasticity: $E(x) = V_A(x)\,E_A + V_B(x)\,E_B$ (2)

Referring to Table 1, it is noticed that the same symbols that define the actual parameters are reused to define their corresponding dimensionless quantities in order to avoid having many subscripts and symbols in the derived equations. Normalizing with respect to the baseline design by dividing (1) by EI_k/L_o^3:

$$w'''' + \lambda_k^2 w'' = 0\,, \tag{3}$$

$$\text{where} \quad \lambda_k = U_k\sqrt{\frac{A_k}{E_k I_k}} = \frac{U\,A_{max}}{\sqrt{A_k E_k I_k}}\,, \qquad k=1,2,\ldots,N_m \tag{4}$$

which is valid over the length of any kth module of the pipe, i.e. $0 \le \bar{x} \le L_k$, where $\bar{x} = x - x_k$ (see Fig. 1).

TABLE I
DEFINITION OF DIMENSIONLESS QUANTITIES

Quantity	Notation	Non-dimensionalization **
Axial coordinate	x	$x \leftarrow x/L_o$
Module's length	L_k	$L_k \leftarrow L_k/L_o$
Wall thickness	h_k	$h_k \leftarrow h_k/h_o$
Mean diameter	D_k	$D_k \leftarrow D_k/D_o$
Cross-sectional area	$A_k\ (=\pi D_k^2/4)$	$A_k \leftarrow A_k/A_o\ \ (= D_k^2)$
2nd Moment of area	$I_k(\approx \pi D_k^3 t_k/8)$	$I_k \leftarrow I_k/I_o\ \ (= D_k^3 t_k)$
Young Modulus	E_k	$E_k \leftarrow E_k/E_o$
Mass density	ρ_k	$\rho_k \leftarrow \rho_k/\rho_o$
Transverse displacement	w	$w \leftarrow w/L_o$
Bending moment	M	$M \leftarrow M^*(L_o/EI_o)$
Shearing force	F	$F \leftarrow F^*(L_o^2/EI_o)$
Axial flow velocity	U_k	$U_k \leftarrow U_k^*(\rho_f A_o L_o^2/EI_o)^{1/2}$
Structural mass	M_s	$M_s \leftarrow M_s/M_o$ $\left(= \sum_{k=1}^{Nm} \rho_k D_k h_k L_k\right)$

** Reference pipe has the following uniform properties:
Area $A_o=\pi D_o^2/4$, $I_o \cong \pi D_o^3 h_o/8$, total mass $M_o=\rho_o \pi D_o h_o L_o$, where ρ_o is the density of the pipe material, L_o is the total length, h_o wall thickness, and E_o is the modulus of elasticity. The notation $x \leftarrow x/L_o$ means that the dimensionless axial distance is equal to its dimensional value divided by the total length of the pipe. $E_o=0.5(E_A+E_B)$, $\rho_o=0.5(\rho_A+\rho_B)$.

In equation (4), U stands for the flow velocity through the pipe module having the maximum cross sectional area A_{max} and N_m is the total number of modules composing the pipeline. It is noted that consideration of the continuity equation provides that $U_k A_k = U A_{max}, k=1,2,\ldots N_m$ Possible boundary conditions at the end supports of the pipeline are stated in the following:

(a) Hinged-Hinged *(H/H)*: $w(0)=w''(0)=0$
$w(1)=w''(1)=0$
(b) Clamped-Hinged *(C/H)*: $w(0)=w''(0)=0$
$w(1)=w'(1)=0$
(c) Clamped-Clamped *(C/C)*: $w(0)=w'(0)=0$
$w(1)=w'(1)=0$

For a cantilevered pipeline, static instability caused by divergence is unlikely to happen. The non-trivial solution of the associated characteristic equation results in a vanishing bending displacement over the entire span of the pipeline. For such pipe configuration, dynamic instability (flutter) may only be considered [2].

III. Solution Procedures

Equation (3) has an exact solution of the form [18]

$$w(\bar{x}) = B_1 + B_2\bar{x} + B_3 \sin\lambda_k\bar{x} + B_4\cos\lambda_k\bar{x} \tag{5}$$

where the B_i's are constants to be determined by applying appropriate boundary conditions. The exact critical flow velocity of a multi-module pipeline model can be best obtained by applying the well-established transmission matrix technique [10] and solving the associated eigenvalue problem. The state vector, $\underline{Z}_k$, at any joint *(k)* within the pipeline is defined as follows

$$\underline{Z}_k^T = [w \quad \varphi \quad M \quad F]_k = [w \ \ -w' \ \ -EIw'' \ \ -EIw''']_k \tag{6}$$

At two successive joints *(k)* and *(k+1)* the state vectors are related to each other by the matrix equation

$$\underline{Z}_{k+1} = [\,T_k\,]\,\underline{Z}_k \tag{7}$$

where $[T_k]$ is a square matrix of order 4x4 known as the transmission or transfer matrix of the kth pipe module. Its individual elements can be obtained by first expressing the coefficients B_i 's of (5) in terms of the state variables at joint *(k)*, and then expressing the state variables at joint *(k+1)* in terms of those at joint *(k)*. The final derived form is

$$[T_k] = \begin{matrix} 1 & -L_k & (C_k-1)/E_k I_k\lambda_k^2 & (S_k/\lambda_k - L_k)/E_k I_k \lambda_k^2 \\ 0 & 1 & S_k/E_k I_k \lambda_k & (1-C_k)/E_k I_k \lambda_k^2 \\ 0 & 0 & C_k & S_k/\lambda_k \\ 0 & 0 & -\lambda_k S_k & C_k \end{matrix} \tag{8}$$

where $C_k = \cos\lambda_k L_k$ and $S_k = \sin\lambda_k L_k$. For a pipeline built from N_m - uniform modules, (7) can be applied at successive joints to obtain

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$$\underline{Z}_{Nm+1} = [\,T\,]\ \underline{Z}_1 \quad (9)$$

where $[T]$ is called the overall transmission matrix formed by taking the products of all the intermediate matrices of the individual modules. Therefore, applying the boundary conditions and considering only the non-trivial solution, the resulting characteristic equation can be solved numerically for the critical flow velocity, U.

IV. Optimization Model Formulation

The optimization problem considered herein is to find a design point $(V_f, h, L)_{k,1,2\,..Nm}$, which provides the highest value of the critical velocity U through a slender composite pipe having a specified total mass. It is cast in the following:

Maximize (U)

Subject to

$$Ms=1.0$$
$$\sum_{k=1}^{Nm} L_k = 1.0$$
$$V_{fmin} \le V_{fk} \le V_{fmax}$$
$$h_{min} \le h_k \le h_{max}$$
$$0.0 \le L_k \le 1.0 \quad (10)$$

where V_{fmin} and V_{fmax} are the lower and upper limits imposed on the fiber volume fraction (e.g. 30% and 70%), and h_{min} and h_{max} are the corresponding values imposed on the wall thickness. The latter may be determined from other strength requirements, which are not considered here. In the case studies treated in the present study, h_{min} and h_{max} are assumed equal to, respectively, 0.5 and 1.25 of the wall thickness of the pipe baseline design. Extensive computer experimentation for obtaining the non-trivial solution of (9), for various pipe configurations, has demonstrated that the critical velocity can be multiple in some zones in the design space. This means that the eigenvalues cross each other, indicating multi-modal solutions (i.e. Bi- Tri- Quadri- modal solutions). Such a multiplicity introduces singularity of the eigenvalue derivatives with respect to the design variables, which does not allow the use of gradient methods [11], [12]. Therefore, it is necessary to formulate the optimization problem with respect to the critical velocity connected with two, three, or four simultaneous divergence modes. The present formulation employs multi-dimensional, non-gradient search techniques to find the required optimum solutions [13], [19]. This formulation requires only simple function evaluations without computing any derivatives for either the objective function or the design constraints. The additional constraints, which ought to be added to the optimization problem described in (10) are:

$$U_1 \le U_j\,, \quad j=2,3\ldots m. \quad (11)$$

where U_1 is the first eigenvalue representing the dimensionless critical flow velocity, $U_j's$ are the subsequent higher eigenvalues, and m is the assumed modality of the final optimum solution. All constraints are augmented with the objective function through penalty multiplier terms, and the number of active constraints at the optimum design point can automatically detect the actual modality of the problem. In the case of single mode optimization, none of the constraints become active at the optimal solution. It is noted that the total mass and length equality constraints can be used to eliminate some of the design variables, which help reducing the dimensionality of the optimization problem. The *MATLAB* optimization toolbox is a powerful tool that includes many routines for different types of optimization encompassing both unconstrained and constrained minimization algorithms [19]. One of its useful routines is named *"fmincon"* which finds the constrained minimum of an objective function of several variables.

V. Results And Discussions

The mathematical model developed in this paper has been applied to obtain the required optimal solutions of *FGM* pipes made of carbon-*AS4* (material *A*) and epoxy-*3501-6* (material *B*), which has favorable characteristics and is highly desirable in several mechanical, civil and aerospace engineering applications [20]. Important properties of the material are given in Table II.

TABLE II
MATERIAL PROPERTIES OF CARBON-AS4/EPOXY-3501-6 COMPOSITE

Property	Carbon Fibers (material A)	Epoxy matrix (material B)
Mass density (g/cm^3)	$\rho_f = 1.81$	$\rho_m = 1.27$
Young's modulus (GPa)	$E_{1f} = 235$	$E_m = 4.3$
Shear modulus (GPa)	$G_{12f} = 27$	$G_m = 1.6$
Poisson's ratio	$\nu_{12f} = 0.2$	$\nu_m = 0.35$

The characteristic equations for calculating the critical flow velocity of the baseline pipe design with various boundary conditions are given in Table III. Considering next a two-module, simply supported pipe with constant diameter and wall-thickness, only four variables denoted by $(V_f, L)_{k=1,2}$ can be considered in the design optimization process. Two variables can be eliminated using the equality constraints imposed on the total length and structural mass.

TABLE III
CRITICAL FLOW VELOCITY OF BASELINE DESIGNS

Type of supports	Velocity equation	$U_{cr,\,i}\ i=1,2,3$
Hinged-Hinged *(H/H)*	$\sin\lambda=0$	$\pi,\ 2\pi,\ 3\pi$
Clamped-Hinged *(C/H)*	$\tan\lambda=\lambda$	$4.493,\ 7.726,\ 10.904$
Clamped-Clamped *(C/C)*	$2\cos\lambda+\lambda\sin\lambda=2$	$2\pi,\ 8.987,\ 4\pi$

Fig. (2) depicts the functional behavior of the dimensionless critical flow velocity, $U_{cr,1}$ augmented with the equality mass constraint, $M_s=1$. It is seen that the function is well behaved and continuous everywhere in the design space $(V_f\text{-}L)_1$, except in the empty region located at the upper right of the whole domain, where the mass equality constraint is violated.

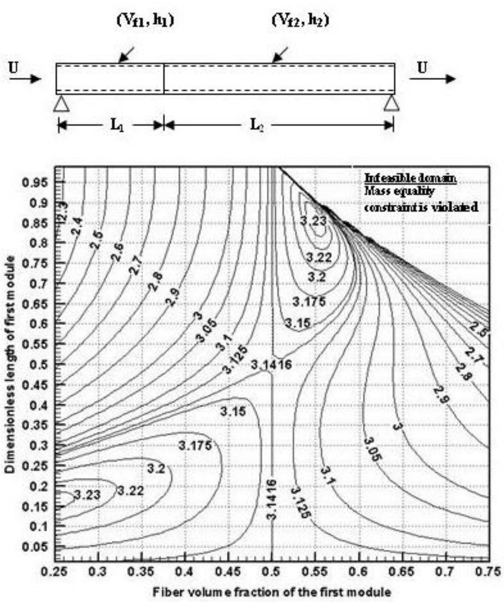

Fig. 2 Effect of material grading on the critical flow velocity for a two-module, *H/H* pipe with constant total mass

The feasible domain is seen to be split by the baseline contours ($U_{cr}=\pi$) into two distinct zones. The one to the right encompasses the constrained global maxima, which is calculated to be $U_{cr}=3.2235$ at the optimal design point $(V_f, L)_{k=1,2}$ =(0.550, 0.80), (0.30, 0.20). Actually, each design point inside the feasible domain corresponds to different material properties as well as different stiffness and mass distributions, while maintaining the total structural mass constant. Fig. (3) shows the developed isodiverts (lines of constant divergence velocity, $U_{cr,1}$) in the (V_{f1}-V_{f2}) design space.

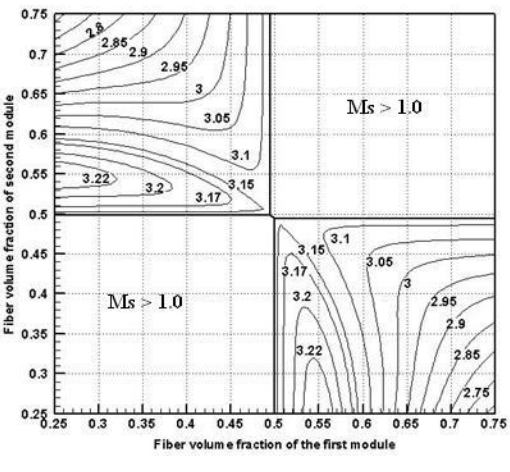

Fig. 3 Isodiverts in the (V_{f1}-V_{f2}) design space for a two-module, *H/H* pipe

The equality mass constraint is violated in the first and third quadrants and the cross lines $V_{f1}=50\%$ and $V_{f2}=50\%$ represent the isodiverts of the baseline value π.Isodiverts for the case of a clamped-hinged, two-module pipe are shown in Fig. 4. In such a case four distinct regions bounded by the baseline value (4.493) can be observed. The global maxima lies in the upper right region with the optimum design point $(V_f, L)_{k=1,2}$ =(0.525, 0.875), (0.325, 0.125) at which $U_{cr,1}$=4.5645. Table IV summarizes the attained optimal solutions for the different types of boundary conditions. Cases of combined material and thickness grading are also included, showing a truly and significant optimization gain for the different pipe configurations. More results indicated that for the case of *H/H* pipelines, good patterns must be symmetrical about the mid-span point. Therefore, it can be easier to cope with symmetrical configurations, which reduce computational efforts significantly, and the total number of variables to half. In this case, the boundary conditions become $w(0)=w''(0)=0$ and $w'(1/2)=w'''(1/2)=0$. For three-module *H/H* pipeline, the attained maximum value of the critical velocity was found to be 3.7955, occurring at the design point $(V_f, h, L)_k$= *(0.625,0.5,0.15625), (0.7,1.1375, 0.6875), (0.625, 0.5, 0.15625)*. This represents about 20.81% optimization gain above the baseline value π.

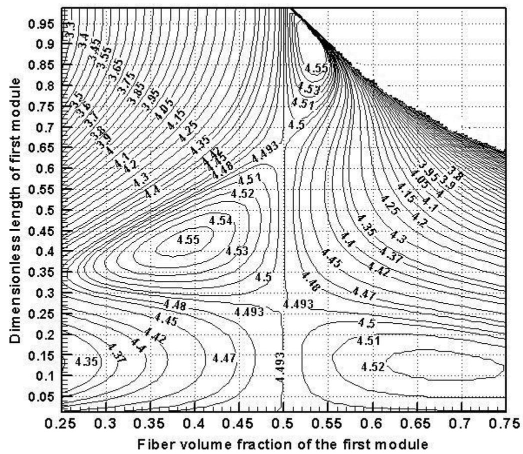

Fig. 4 Isodiverts in the (V_{f1}-L_1) design space for a two-module *C/H* pipe under mass equality constraint

TABLE IV
OPTIMAL SOLUTIONS FOR TWO-MODULE PIPELINES
(CASE OF CONSTANT DIAMETER AND TOTAL MASS)

Support	$(V_f, h, L)_{k=1,2}$	Ucr,max
	Material grading only	
H/H	(0.550, 1.0, 0.800), (0.300, 1.0, 0.200)	3.2235
C/H	(0.525, 1.0, 0.875), (0.325, 1.0, 0.125)	4.5645
C/C	(0.675, 1.0, 0.125), (0.475, 1.0, 0.875)	6.3325
	Combined material & thickness grading	
H/H	(0.70, 1.0, 0.75), (0.65, 0.75, 0.25)	3.6235
C/H	(0.70, 0.95, 0.9), (0.50, 0.85, 0.10)	5.1355
C/C	(0.70, 1.0, 0.60), (0.65, 0.85, 0.40)	7.0965

VI. CONCLUSION

In view of the importance of enhancing the stability and raising the overall (stiffness/mass) level of a *FGM* pipe conveying fluid, an appropriate optimization model has been formulated for a multi-module pipe with discrete distributions of the volume fractions of the selected composite material. The objective function has been measured by maximizing the critical flow velocity at which divergence occurs while maintaining the total structural mass constant. The corresponding optimization gains were calculated based on the initial reference values of the uniform baseline design. Optimization of multi-module pipelines with different support conditions have been thoroughly examined indicating that good patterns of simply-supported pipes should be symmetrical about the mid span of the pipe. The given exact structural analysis leads to the exact flow velocities no matter the number of modules is. It has been confirmed that the module length is most significant design variable in the whole optimization process. Some investigators who apply finite elements have not recognized that the length of each element can be taken as a main design variable in the whole set of optimization variables. It has also been shown that normalization of all terms results in a naturally scaled objective function, constraints and design variables, which is recommended when applying different optimization techniques. The results from the present approach reveals that piecewise grading of the material can be promising producing truly efficient pipeline designs with improved bending stability. In conclusion, a powerful design tool has been obtained by formulating an appropriate objective function and applying mathematical programming techniques to the resulting optimization problem. Other secondary effects such as material and geometrical nonlinearities due to large deformation shall be investigated in future studies.

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