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Application of GM (1, 1) Model Group Based on Recursive Solution in China's Energy Demand Forecasting

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***Abstract*—**To learn about China's future energy demand, this paper first proposed GM(1,1) model group based on recursive solutions of parameters estimation, setting up a general solving-algorithm of the model group. This method avoided the problems occurred on the past researches that remodeling, loss of information and large amount of calculation. This paper established respectively all-data-GM(1,1), metabolic GM(1,1) and new information GM (1,1)model according to the historical data of energy consumption in China in the year 2005-2010 and the added data of 2011, then modeling, simulating and comparison of accuracies we got the optimal models and to predict. Results showed that the total energy demand of China will be 37.2221 billion tons of equivalent coal in 2012 and 39.7973 billion tons of equivalent coal in 2013, which are as the same as the overall planning of energy demand in The 12th Five-Year Plan.

***Keywords*—**energy demands, GM(1, 1) model group, least square estimation, prediction

I. Introduction

IN the era of economic globalization, rapid economic growth and high population growth in size, as well as speeding up the process of urbanization, are the main features of China's current economic and important factors to boost energy consumption growth. However, large rise in the manufacturing sector brought more energy consumption continuing to increase, therefore to forecast energy demand becomes urgent and more meaningful in China. Studies have shown that using traditional forecasting methods cannot reflect the impact brought about by disturbance factors from external system, as well as the uncertainty brought about by the international environmental and domestic policies. Dr Deng Julong proposed grey systems theory to head with the problems which are uncertainty and a small amount of data for the first time in 1982[1]-[3]. Reference[4] has been first to come up with the concept of GM (1, 1) model group, including three subsidiaries: all-data-GM(1,1)built on the entire sequence $X^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$, metabolic GM(1,1) built on the new sequence obtained by inserting $x^{(0)}(n+1)$ and deleting $x^{(0)}(1)$, new information GM(1,1) built by inserting $x^{(0)}(n+1)$ into the sequence $X^{(0)}$. Its application is very extensive [5]-[6].

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Through the study of literature in the past, we can know that applications of GM(1,1) model are required to insert new information and to get the solution by remodeling, which will result in loss of information and increasing the amount of calculation. Faced with this problem, by approaching the organic link between the subsidiaries of GM(1,1) model group, this article is the first to set up the general solving-algorithm of GM(1,1) model group based on the recursive solution of parameter estimation. Articles gave a detailed description for its modeling steps and applied this model group to forecast the energy demand in China.

II. Recursive-Solution of Least Squares Estimation of New Information GM (1,1) Model

If established GM (1, 1) model by using initial sequence of non-negative data $X^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$, then the least square estimate of the sequence of parameters is recorded as

$$\hat{a}(n)=(B_n^T B_n)^{-1}B_n^T Y_n$$

If we record $(B_n^T B_n)^{-1}$ as $P(n)$, thus

$$\hat{a}(n)=P(n)B_n^T Y_n$$

Assuming that we establish a new $(n+1)$- equation that is

$$x^{(0)}(n+1)+az^{(1)}(n+1)=b$$

The following theorem gives the generated based on the added-value of the new information GM(1,1) model's least square estimation of parameters.

*Theorem 2.1.*Assuming initial sequence of non-negative as following

$$X^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n),x^{(0)}(n+1))$$

And G is the once-added-mean generated $(n+1)\times n$ -order matrix of the sequence $X^{(0)}$.

Then a new information GM (1, 1) model is

$$x^{(0)}(k)+az^{(1)}(k)=b\ ,k=2,3,\cdots,n,n+1$$

The model's least square estimation of parameters is

$$\hat{a}(n+1)=(B_{n+1}^T B_{n+1})^{-1}B_{n+1}^T Y_{n+1} \tag{1}$$

Where

$$B_{n+1}=-G^T\cdot M$$

$$Y_{n+1}=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \\ x^{(0)}(n+1) \end{bmatrix}, M=\begin{bmatrix} x^{(0)}(1) & -1 \\ x^{(0)}(2) & 0 \\ \vdots & \vdots \\ x^{(0)}(n) & 0 \\ x^{(0)}(n+1) & 0 \end{bmatrix},$$

$$G=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ \frac{1}{2} & 1 & 1 & \cdots & 1 \\ 0 & \frac{1}{2} & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{2} \end{bmatrix}_{(n+1)\times n}.$$

Proof: From [4] we can know that

$$B_{n+1}=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n+1) & 1 \end{bmatrix}_{n\times 2}=\begin{bmatrix} -\frac{1}{2}(x^{(1)}(1)+x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2)+x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n)+x^{(1)}(n+1)) & 1 \end{bmatrix}_{n\times 2}$$

$$=-\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & \cdots & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \cdots & 0 & 0 \\ & \cdots & & \cdots & & \cdots \\ 0 & 0 & 0 & \cdots & \frac{1}{2} & \frac{1}{2} \end{bmatrix}_{n\times(n+1)} \cdot \begin{bmatrix} x^{(1)}(1) & -1 \\ x^{(1)}(2) & -1 \\ \vdots & \vdots \\ x^{(1)}(n+1) & -1 \end{bmatrix}_{(n+1)\times 2}$$

Because

$$x^{(1)}(k)=\sum_{i=1}^{k} x^{(0)}(i), k=1,2,\cdots,n+1$$

Therefore

$$B_{n+1}=-\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & \cdots & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \cdots & 0 & 0 \\ & \cdots & & \cdots & & \cdots \\ 0 & 0 & 0 & \cdots & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 & 0 \\ & \cdots & & \cdots & & \cdots \\ 1 & 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x^{(0)}(1) & -1 \\ x^{(0)}(2) & 0 \\ \vdots & \vdots \\ x^{(0)}(n) & 0 \\ x^{(0)}(n+1) & 0 \end{bmatrix}$$

$$=-\begin{bmatrix} 1 & \frac{1}{2} & 0 & \cdots & 0 \\ 1 & 1 & \frac{1}{2} & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & \frac{1}{2} \end{bmatrix}_{n\times(n+1)} \cdot \begin{bmatrix} x^{(0)}(1) & -1 \\ x^{(0)}(2) & 0 \\ \vdots & \vdots \\ x^{(0)}(n) & 0 \\ x^{(0)}(n+1) & 0 \end{bmatrix}_{(n+1)\times 2}$$

$$=-G^T\cdot M$$

Finally taking $B_{n+1}=-G^T\cdot M$ into least square estimation of parameters of GM (1, 1) model $x^{(0)}(k)+az^{(1)}(k)=b$, that is $\hat{a}(n+1)=(B_{n+1}^T B_{n+1})^{-1}B_{n+1}^T Y_{n+1}$.Conclusion certificated.

For new information GM(1,1) model inserted the latest information $x^{(0)}(n+1)$, equation (1) is model parameters solution obtained through re-modeling. But solving inverse operation $(B_{n+1}^T B_{n+1})^{-1}$ will take a lot of time. In the study of mode group of GM(1,1),we hope to improve model information of a new information GM(1,1) by using the information of all-data-GM(1,1), that is to preserve existing model information. The following using a recursive algorithm, makes the new GM(1,1) least squares estimation of model parameters are constantly refreshed.

Lemma2.1 If the non-singular square matrices A and A+BD are the same order, then

$$(A+BD)^{-1}=A^{-1}-A^{-1}B(I+DA^{-1}B)^{-1}DA^{-1} \qquad (2)$$

On the basis of all-data-GM(1,1) model, if $x^{(0)}(n+1)$ is the most current information, the n+1 equations of a new information GM(1,1) are:

$$x^{(0)}(2)+az^{(1)}(2)=b$$
$$x^{(0)}(3)+az^{(1)}(3)=b$$
$$\vdots$$
$$x^{(0)}(n+1)+az^{(1)}(n+1)=b$$

That is, the matrix equation of a new GM(1,1) is

$$Y_{n+1}=B_{n+1}\hat{a}$$

Where

$$Y_{n+1}=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \\ x^{(0)}(n+1) \end{bmatrix}=\begin{bmatrix} Y_n \\ \hdashline x^{(0)}(n+1) \end{bmatrix},$$

$$B_{n+1}=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \\ -z^{(1)}(n+1) & 1 \end{bmatrix}=\begin{bmatrix} B_n \\ \hdashline Z_{n+1} \end{bmatrix},$$

$$Z_{n+1}=\begin{bmatrix} -z^{(1)}(n+1) & 1 \end{bmatrix}.$$

According to the block matrix multiplication, we have

$$B_{n+1}{}^T\cdot B_{n+1}=\begin{bmatrix} B_n^T & Z_{n+1}{}^T \end{bmatrix}\cdot\begin{bmatrix} B_n \\ Z_{n+1} \end{bmatrix}=B_n{}^T B_n+Z_{n+1}{}^T Z_{n+1}$$

If we record $P(n+1)=(B_{n+1}^T B_{n+1})^{-1}$, then on the ground of lemma 2.1, the following lemma recursive conclusion is established.

Lemma2.2 Let $\gamma_{n+1}=\left(1+Z_{n+1}P(n)Z_{n+1}^T\right)^{-1}$, then

$$P(n+1)=P(n)-\gamma_{n+1}P(n)Z_{n+1}{}^T Z_{n+1}P(n) \qquad (3)$$

Theorem2.2 The information of GM(1,1) model established by using all data is $P(n) = (B_n^T B_n)^{-1}$, $\hat{a}(n) = (B_n^T B_n)^{-1} B_n^T Y_n$. There is the most current information $x^{(0)}(n+1)$, then the least square parameter estimation sequence of the new information GM(1,1) $x^{(0)}(k) + az^{(1)}(k) = b$, $k = 2,3,\cdots,n,n+1$ is

$$\hat{a}(n+1) = \hat{a}(n) + \omega_{n+1} \cdot \varepsilon(n+1) \tag{4}$$

Where

$$\omega_{n+1} = \gamma_{n+1} P(n) Z_{n+1}{}^T,$$
$$\varepsilon(n+1) = x^{(0)}(n+1) - Z_{n+1}\hat{a}(n).$$

Proof: According to (1), the least square parameter estimation sequence of the new information GM(1,1) is

$$\begin{aligned}\hat{a}(n+1) &= \left(B_{n+1}{}^T \cdot B_{n+1}\right)^{-1} B_{n+1}{}^T Y_{n+1} \\ &= P(n+1)\begin{bmatrix} B_n^T & Z_{n+1}{}^T \end{bmatrix}\begin{bmatrix} Y_n \\ x^{(0)}(n+1) \end{bmatrix} \\ &= \left(P(n) - \gamma_{n+1}P(n)Z_{n+1}{}^T Z_{n+1}P(n)\right)\left(B_n^T Y_n + Z_{n+1}{}^T x^{(0)}(n+1)\right) \\ &= \hat{a}(n) + \gamma_{n+1}P(n)Z_{n+1}{}^T \left(\frac{x^{(0)}(n+1)}{\gamma_{n+1}} - Z_{n+1}P(n) \right. \\ &\qquad \left. \cdot Z_{n+1}{}^T x^{(0)}(n+1) - Z_{n+1}\hat{a}(n)\right)\end{aligned}$$

we have $\gamma_{n+1} = \left(1 + Z_{n+1}P(n)Z_{n+1}{}^T\right)^{-1}$, Equaling to

$$\frac{1}{\gamma_{n+1}} = \left(1 + Z_{n+1}P(n)Z_{n+1}{}^T\right)$$

So

$$\hat{a}(n+1) = \hat{a}(n) + \gamma_{n+1}P(n)Z_{n+1}{}^T \left(x^{(0)}(n+1) - Z_{n+1}\hat{a}(n)\right)$$

From $\omega_{n+1} = \gamma_{n+1}P(n)Z_{n+1}{}^T$, $\varepsilon(n+1) = x^{(0)}(n+1) - Z_{n+1}\hat{a}(n)$, Therefore

$$\hat{a}(n+1) = \hat{a}(n) + \omega_{n+1} \cdot \varepsilon(n+1)$$

Because formula (4) establishes the relationships between parameters estimation of the subsidiaries of GM(1,1) model group, it is not necessary to remodel and solve the least square parameter estimation of the new information GM(1,1) for us. We can get the solution just by refreshing all-data-GM(1,1) model's parameter estimation, that is, adding correction items $\omega_{n+1} \cdot \varepsilon(n+1)$ to the least square parameter estimation sequence of all-data-GM(1,1) $\hat{a}(n)$ which we can get it $\hat{a}(n+1)$. $P(n)$ in $\omega_{n+1} = \gamma_{n+1}P(n)Z_{n+1}{}^T$ can be updated on the ground of recursive formula (3). It's easy to know that γ_{n+1} is a scalar, solving inverse matrix has completely avoided in (3) and (4). Thus computational efficiency for solving a new information GM(1,1) model's parameters is greatly improved. Besides, equation (4) gives a very intuitive forms that $\hat{a}(n+1)$ is proportional to the amended $\varepsilon(n+1) = x^{(0)}(n+1) - Z_{n+1}\hat{a}(n)$ which may indicate the fitting errors between the new information $x^{(0)}(n+1)$ as well Z_{n+1} and estimation of $\hat{a}(n)$ before updating. Correction coefficients $\omega_{n+1} = \gamma_{n+1}P(n)Z_{n+1}{}^T$ decide the weight value on the fitting error when amending $\hat{a}(n)$.

TABLE I
2005-2010 YEAR'S ANNUAL TOTAL ENERGY CONSUMPTION IN CHINA
Unit: millions of tons of equivalent coal

Years	Energy consumption
2005	20.3227
2006	24.627
2007	26.5583
2008	28.5
2009	30.6647
2010	32.4939

Data from the database of China National Statistics

III. GM (1, 1) Model Group's Modeling-steps Based on Recursive Solution of Parameter Estimation

By approaching the organic link between the subsidiaries of GM(1,1) model when modeling, this article has set up a general algorithm in the research of least squares parameters estimation of subsidiaries in GM(1,1) model group. First, we got the parameters values through solving the all-data-GM(1,1) model. Then, we obtained the parameters values of metabolic GM(1,1) by using the added-mean value generated matrix of all-data-GM(1,1) model. Last, using all-data-GM(1,1) model parameter estimated to plus an amendment items (scalar) that are new information GM(1,1) model parameter. The specific modeling-steps of GM(1,1) model group based on recursive solution of parameter estimation are as follow:

1) Solving the data-matrixes *Y*, *G*, *M* (expressions can be given in theorem2.1) determined by the original sequence $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))$;

2) According to (6.6) and (6.7) in [7], we can get $F, C, P(n), \hat{a}(n)$.Where

$$F = X^{(0)} G G^T X^{(0)T}$$

$$C = (n-1)x^{(0)}(1) + \sum_{i=2}^{n}(n - i + \frac{1}{2})x^{(0)}(i)$$

$$P(n) = (B^T B)^{-1} = \frac{1}{(n-1)F - C^2}\begin{bmatrix} n-1 & C \\ C & F \end{bmatrix}$$

$$\hat{a}(n) = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y = P(n) B^T Y$$

So that we can obtain the time response sequence of the all-data-GM (1, 1) model:

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a}; k = 1,2,\cdots,n$$

3) We can obtain a new modeling-sequence by inserting $x^{(0)}(n+1)$ and deleting the old one $x^{(0)}(1)$. Then we can get the time response sequence of metabolic GM(1,1) model following method 2);

4) The data sequence of new information GM(1,1) model can be obtained through adding the new information $x^{(0)}(n+1)$.

According to the (4) $\hat{a}(n+1)=\hat{a}(n)+\omega_{n+1}\cdot\varepsilon(n+1)$ we have $\hat{a}(n+1)$. Therefore, the time response sequence of new information GM(1,1) model can be solved.

IV. Application

In the stage of industrialization and rapid development in China currently, energy demand used to develop economy is much higher than that of developed countries. Economic development almost depends on manufacturing in China so that energy demand intensity more so is expected to continue to rise. According to the historical data of energy consumption in China in the year 2005-2010, the paper will modeling, forecasting as well as comparative analysis of each model's accuracy. The information of 2005-2010 year's annual total energy consumption is as shown in Table I. The total energy consumption is 34.8 millions of tons of equivalent coal in 2011.

Solution: 1) All-data-GM(1,1) model. The original sequence is $(20.3227, 24.627, 26.5583, 28.5, 30.6647, 32.4939)$. According to the above section of modeling steps, we can obtain the time response sequence of the GM(1,1) model.

$$\begin{cases}\hat{x}^{(1)}(k+1)=344.601849e^{0.06933k}-324.27915\\ \hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)\end{cases}$$

2) Metabolic GM(1,1) model. We can obtain the modeling sequence as following by inserting $x^{(0)}(7)=34.8$,The new sequence is $(24.627, 26.5583, 28.5, 30.6647, 32.4939, 34.8)$.

Then following the calculation method of all-data-GM, we can get the GM(1,1) time response sequence is

$$\begin{cases}\hat{x}^{(1)}(k+1)=385.07435e^{0.06689k}-360.4473\\ \hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)\end{cases}$$

3) A new information GM(1,1) model. This sequence is $(20.3227, 24.627, 26.5583, 28.5, 30.6647, 32.4939, 34.8)$.

According to (4) and the results of 1), the GM(1,1) time response sequence is

$$\begin{cases}\hat{x}^{(1)}(k+1)=350.4806e^{0.06831k}-330.158\\ \hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)\end{cases}$$

4) Comparison of accuracies. As shown in the Table II.

In the Table II, we can conclude that metabolic GM(1,1) model is superior to the new information GM(1,1) model

TABLE II
Comparison of Accuracies

	All-data-GM(1,1)		Metabolic GM(1,1)		New information GM(1,1)	
Years	Simulation value	Relative error	Simulation value	Relative error	Simulation value	Relative error
2005	20.32				20.32	
2006	24.74	0.46	24.63		24.78	0.62
2007	26.52	0.16	26.64	0.31	26.53	0.10
2008	28.42	0.28	28.48	0.06	28.41	0.33
2009	30.46	0.67	30.45	0.69	30.42	0.81
2010	32.65	0.47	32.56	0.21	32.57	0.22
2011	34.99	0.55	34.81	0.04	34.87	0.20
Average relative error	0.43		0.26		0.38	

which is superior to all-data-GM(1,1) model. More information on new, higher the simulation precision of metabolic model are. So we can use the metabolic GM(1,1) model to forecast the total energy demand in China in 2012 and 2013. We can predict that the total energy demand of China is 37.2221 billion tons of coal equivalent in 2012 and 39.797 billion tons of coal equivalent in 2013 on the ground of the time response sequence of metabolic model. Faced with continued rapid growth in energy demand, China should strengthen its efforts to promote the transformation of economic growth mode in China. The planning of energy development in The 12th Five-Year Plan of China have put forward to change the mode of energy development, vigorously adjust the energy structure, reasonably control energy consumption as the guiding ideology, promoting the production and use of energy changes. Therefore, at the same time taking appropriate measures to increase the production, the government should encourage energy saving and emission reduction, implement overall development of energy, economy and the environment in The 12th Five-Year Plan.

V. Conclusion

Faced with the problem of loss of information and increasing the amount of calculation resulted from remodeling when applying GM(1,1) model group to predict, this article is the first to set up the general solving-algorithm of GM(1,1) model group based on the recursive solution of parameter estimation by approaching the organic link between the subsidiaries. Not only is using this method able to significantly reduce the amount of calculation, but also strengthen the organic link between subsidiaries and the analysis of the group as a whole to explain. Through comparing accuracies of three models and then selecting optimal model used for forecasting, application displayed the results was the same as the overall planning of energy demand in The 12th Five-Year Plan.

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