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Distribution Centers Reliability Cost in Capacitated Facility Location Problem

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Abstract**—**Recently studies in area of supply chain network (SCN) have focused on the disruption issues in distribution systems. Also this paper extends the previous literature by providing a new bi-objective model for cost minimization of designing a three echelon SCN across normal and failure scenarios with considering multi capacity option for manufacturers and distribution centers. Moreover, in order to solve the problem by means of LINGO software, novel model will be reformulated through a branch of LP-Metric method called Min-Max approach.

Keywords**—**Scenario programming, Distribution, Multi-echelon supply chain design, Reliable facility

I. Introduction

THIS paper seeks to optimize designing a distribution network in terms of locating a number of facilities and adjusting a distribution pattern among manufacturers, distribution centers (DCs) and customers by entering probable expected failure cost of DCs into the extended fixed charge location model. In fact extended fixed charge location model which was presented by Amiri (2006) is a basis for our model [1]. He has changed Pirkul (1998) study by allowing multiple capacities for both suppliers and distribution centers [2].

Furthermore, in order to capture echelons uncertainty, second objective function is inserted to Amiri's model according to the concept presented by Snyder (2005) [3]. He considers a special circumstance in an Un-capacitated Facility Location Problem (UFLP) that some facilities may be failed to service customer demands (due to poor weather, labor actions, sabotage, changes of ownership, or other factors) and by taking into the account this probability, tries to design a distribution pattern which minimizes excessive cost incurred by DCs failure and supply cost concurrently. Shen (2010) addresses, a closely issue to latter study, two different approaches for modeling DCs side uncertainty: 1) a scenario based one, specifying some possible subset of non operational facilities and 2) an individual and independent failure probability inherent in each facility [4].

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Although our model formulation somehow is close to the Snyder's paper concept & Shen's scenario based approach but in terms of applying DCs reliability issue in a capacitated problem is different. According to former papers in an un-capacitated problem when a DCs isn't operable customers which were assigned to it, are planned to be allocated to their nearest open and operable facilities. But for such a situation in our model, mentioned customers can't be surly assigned to the nearest facilities because of capacity constraint. So this problem is more challenging.

However, solving such a bi-objective model with a commercial software involves a strategy for reformulating the correspond model to a single objective one. As a result in this paper a branch of LP-Metric model called Min-Max approach will be adopted.

The rest of this study in order to cover the following discussion is designed as follows. In section 2 the problem structure (Assumptions, Notations and Model) is described then in section 3 the reformulation of model in terms of Min-Max approach is accomplished. Consequently, section 4 provides LINGO result of the output model of pervious section. Finally, conclusions and recommendations for future studies will be explored in section 5.

II. Problem Structure

In this section besides addressing assumptions and the nomenclature, model formulation is presented.

A. Notation

The notations given in nomenclature are required for the purpose of this paper.

1. Indices

I: index set of customers
J: index set of distributor centers
K: index set of suppliers
S: index set of all possible states of distributor centers working and failure except for states of all DCs failure at once. In fact it totally includes $\sum_{z=0}^{J-1}\binom{J}{z}$ number of possible states (suppose at s=1 all DCs work properly).
F_s: Index set of failed DC at state s which is consist of $n(F_s)$ elements
F_s': Complement set of F_s which is consist of $n(F_s')$ elements
R: index set of capacity levels for potential distributor centers
H: index set of capacity levels for potential suppliers

2. Parameters

a_i: Customer demand of zone i
b_j^r: Capacity of DC j at capacity level r

q: Failing probability of each proper DC

p_s: Probability of each state occurrence which is equal to$q^{n(F_s)}\,(1-q)^{n(F_s')}$, but according to the 4th problem assumption in s=1, $p_s = 1$.

F_j^r: Fixed cost of opening DC j at capacity level r

G_k^h: Fixed cost of opening plant k at capacity level h

e_k^h: Capacity of plant k at capacity level h

C_{1ij}: Unit cost of transporting commodities from DC j to customer zone i

C_{2jk}: Unit cost of transporting commodities from plant k to DC j

C_3: Shortage cost (lost-Sales cost)

f_1^*: Optimal value of 1st objective function when the 2nd one is ignored

f_2^*: Optimal value of 2nd objective function when the 1st one is ignored

γ_1: Constant weighting factor of 1st objective function

γ_2: Constant weighting factor of 2nd objective function

3. *Decision Variables*

X_{ij}^s: Percent amount of satisfying customer zone i demand by DC j at state s

Y_{jk}^{rs}: Percent amount of supplying DC j by plant k at state s and capacity level r

$$U_j^r = \begin{cases} 1 & if\ a\ DC\ at\ capacity\ level\ r\ is\ opened\ in\ location\ j \\ 0 & otherwise \end{cases}$$

$$V_k^h = \begin{cases} 1 & if\ a\ plant\ at\ capacity\ level\ h\ is\ opened\ in\ location\ k \\ 0 & otherwise \end{cases}$$

B. Assumption

- The input parameters are deterministic.
- DCs have uniform failure probabilities.
- Multiple capacities are allowed for plants & DCs.
- All DCs work properly at the beginning of running model.

C. Model Formulation

$$\text{Min } f_1 = \sum_{i\in I}\sum_{j\in J} C_{1ij}a_i X_{ij}^1 + \sum_{r\in R}\sum_{j\in J}\sum_{k\in K} C_{2jk} b_j^r Y_{jk}^{r1} + \sum_{j\in J}\sum_{r\in R} F_j^r U_j^r + \sum_{k\in K}\sum_{h\in H} G_k^h V_k^h \quad (1)$$

$$\text{Min } f_2 = \sum_{s\in(S\neq 1)} p_s\Big(\sum_{i\in I}\sum_{j\in J} C_{1ij}a_i X_{ij}^s + \sum_{r\in R}\sum_{j\in J}\sum_{k\in K} C_{2jk} b_j^r\, Y_{jk}^{rs} + C_3\Big(\sum_{i\in I} a_i\Big(\sum_{j\in J} X_{ij}^1 - \sum_{j\in J} X_{ij}^s\Big)\Big)\Big) \quad (2)$$

Subject to:

$$\sum_{j\in J} X_{ij}^1 = 1 \qquad \forall i\in I \quad (3)$$
$$\sum_{j\in J} X_{ij}^s \le 1 \qquad \forall i\in I\ \&\ s\in(S\neq 1) \quad (4)$$
$$\sum_{i\in I} a_i X_{ij}^s \le \sum_{k\in K}\sum_{r\in R} b_j^r Y_{jk}^{rs}\ \ \forall j\in J\ \&\ s\in S \quad (5)$$
$$\sum_{k\in K} Y_{jk}^{rs} \le U_j^r \quad \forall j\in J, r\in R\ \&\ s\in S \quad (6)$$
$$\sum_{k\in K}\sum_{r\in R} b_j^r Y_{jk}^{rs} \le \sum_{h\in H} e_k^h v_k^h \quad \forall k\in K\ \&\ s\in S \quad (7)$$
$$\sum_{r\in R} U_j^r \le 1 \quad \forall j\in J \quad (8)$$
$$\sum_{h\in H} V_k^h \le 1 \quad \forall k\in k \quad (9)$$
$$X_{ij}^s \ge 0 \quad \forall i\in I\ \&\ j\in J\ \&\ s\in S \quad (10)$$
$$U_j^r \in (0,1) \qquad \forall j\in J\ \&\ r\in R \quad (11)$$
$$Y_{jk}^r \ge 0 \quad \forall j\in J\ \&\ k\in K\ \&\ r\in R \quad (12)$$
$$V_k^h \in (0,1) \quad \forall k\in K\ \&\ h\in H \quad (13)$$

Objective function (1) minimizes required total cost of servicing demand zones while ignoring the probability of DCs failure (p_1=1). Equation (2) is also determined to minimizes cost but expected extra cost origins from DCs failure which is made up of two part multiply by probability of occurring each state: first part tries to satisfy customers demand as much as possible by proper DCs and secondly due to the lack of DCs capacity or partial coverage constraint it allocates lost sales cost to the difference amount of satisfied demand between (1) and (2). Constraint (3) ensures that all customers' demands must be satisfied at normal situation (1st state). Constraint (4) prohibits from servicing a customer zone more than its demand in any failure situation (except 1st state). Constraint (5) controls that a DC output doesn't pass its inventory supplied from all potential plants. Constraint (6) imply two fact concurrently: firstly prevents from allocating potential plants to DC j since it hasn't been opening and secondly adjust an opened inventory DC at last as equal as its capacity. Constraint (7) for a plant k blocks extra potential DCs supplying which overflows plant k capacity. Constraint (8) and (9) avoid model in establishing DCs and plants with more than one capacity level in any index set of J and K respectively. Finally, the residual of Constraints determine type of variables.

III. Solution Procedure

According to the Cohon [5], methods of solving multi-objective problems are mainly classified into the Generating and Preference-based methods. Actually Generating methods seek to find as least as possible non-dominant solutions for decision maker without imposing any preferences among objectives. Contrary, Preferences-based methods try to reach the optimal Pareto-front by laying different weighting factors among objective functions.

In this study we adopt LP-Metric method which is located in the category of the Preferences-based one [6]. It could be proved that when L=∞, the formulation of correspond method changed into a kind of Min-Max approach.

$$\text{Min } \alpha \quad (14)$$

Subject to:

$$\alpha \ge \gamma_1\left[\frac{f_1 - f_1^*}{f_1^*}\right] \quad (15)$$
$$\alpha \ge \gamma_2\left[\frac{f_2 - f_2^*}{f_2^*}\right] \quad (16)$$
$$\gamma_1 + \gamma_2 = 1 \quad (17)$$
$$\sum_{j\in J} X_{ij}^1 = 1 \qquad \forall i\in I \quad (18)$$
$$\sum_{j\in J} X_{ij}^s \le 1 \qquad \forall i\in I\ \&\ s\in(S\neq 1) \quad (19)$$
$$\sum_{i\in I} a_i X_{ij}^s \le \sum_{k\in K}\sum_{r\in R} b_j^r Y_{jk}^{rs}\ \ \forall j\in J\ \&\ s\in S \quad (20)$$
$$\sum_{k\in K} Y_{jk}^{rs} \le U_j^r \quad \forall j\in J, r\in R\ \&\ s\in S \quad (21)$$
$$\sum_{k\in K}\sum_{r\in R} b_j^r Y_{jk}^{rs} \le \sum_{h\in H} e_k^h v_k^h \quad \forall k\in K\ \&\ s\in S \quad (22)$$

$$\sum_{r\in R} U_j^r \leq 1 \quad \forall j \in J \tag{23}$$
$$\sum_{h\in H} V_k^h \leq 1 \quad \forall k \in k \tag{24}$$
$$X_{ij}^s \geq 0 \quad \forall i \in I \,\&\, j \in J \,\&\, s \in S \tag{25}$$
$$U_j^r \in (0,1) \quad \forall j \in J \,\&\, r \in R \tag{26}$$
$$Y_{jk}^r \geq 0 \quad \forall j \in J \,\&\, k \in K \,\&\, r \in R \tag{27}$$
$$V_k^h \in (0,1) \quad \forall k \in K \,\&\, h \in H \tag{28}$$

In the following reformulation a new variable (α) and three additional constraints (15)-(17) will be added to our main model. Also it is proven that different values of γ_1 & γ_2 lead to efficient solutions of the main model.

IV. Computational Results

In this section in order to explore different aspects of the proposed model, LINGO results for a random example with 10 potential customer zones, 4 candidate location of DCs and 2 candidate location of manufacturers are presented. In which three capacity levels are considered for both DC & Manufacturer layers. Also computational results involve the analysis of two probably effective parameters such as the failure probability of DCs ($q = 0.02, 0.2$) and shortage cost ($C_3 = 572.09, 1055.27, 1934.66$).

In the Table I through III in terms of three shortage cost and two failure probabilities of DCs the values of two objective functions, the rate of increase and decrease in correspond objectives in comparison to their values in times of ($\gamma_1 = 1$), the statue of opened manufacturers (OM) and DCs (OD) for each solution and local optimum solutions found by LINGO are listed. For instance in TABLE I in terms of ($C_3 = 572.09, q = 0.02, \gamma_1 = 0.95$) just by 2.24% increase in 1st objective function, we can hedge against 152.61% of expected future costs related to 2nd objective function. Also in order to reach this specific solution which is a local optimum based LINGO results, First index set of manufacturers location must be opened at its third capacity level (OM: [3, 0]) and 1st, 2nd and 3rd index set of DCs have to be opened at their maximum capacity level (OD: [3, 3, 3, 0]).

Moreover, as illustrated in the following tables the contrast characteristic of two objective functions could be realized obviously. In fact gradual decreasing in value of γ_1 leads to augment of first objective function and consequently decreasing of the second one which implicitly state the cost of taking into the account the reliability issues along with traditional facility location objectives in stability situation of supplying layers.

Finally, by monitoring data of three tables and parameters analysis at a short glance it could be concluded that with the same increase in 1st objective function, the rate of decrease in the 2nd one has ascending and descending relation with augments of shortage cost and probability of failure respectively.

TABLE I
LINGO Results for $C_3 = 572.09$

Q				Δ	Δ	OM	OD
0.02	1	2253305	249553.8	-	-	[2,0]	[3,3,3,0]
	*0.95	2303875	98790.68	2.24	152.61	[3,0]	[3,3,3,0]
	0.75	2352638	79676.65	4.41	213.21	[2,0]	[3,3,3,2]
	0.5	2368977	77520.73	5.13	221.92	[2,0]	[3,3,3,3]
	0.25	2651528	74789.71	17.67	233.67	[2,2]	[3,3,3,3]
	0	3094803	70370.21	37.35	254.63	[3,3]	[3,3,3,3]
0.2	1	2253305	1892749	-	-	[2,0]	[3,3,3,0]
	0.75	2352638	680322.4	4.41	178.21	[2,0]	[3,3,3,2]
	*0.54	2419547	665317.3	7.38	184.49	[3,0]	[3,3,3,3]
	0.5	2368977	665317.3	5.13	184.49	[2,0]	[3,3,3,3]
	0.25	2587345	645633.9	14.82	193.16	[2,2]	[3,3,3,3]
	0	3229899	613124.5	43.34	208.71	[2,3]	[3,3,3,3]

TABLE II
LINGO Results for $C_3 = 1055.27$

Q				Δ	Δ	OM	OD
0.02	1	2253305	460321.9	-	-	[2,0]	[3,3,3,0]
	*0.98	2303875	136357.2	2.24	237.59	[3,0]	[3,3,3,0]
	0.75	2352638	79982.44	4.41	475.53	[2,0]	[3,3,3,2]
	0.5	2368977	78489.34	5.13	486.48	[2,0]	[3,3,3,3]
	0.25	2594189	75758.33	15.13	507.62	[2,2]	[3,3,3,3]
	0	3088820	71338.83	37.08	545.26	[3,3]	[3,3,3,3]
0.2	1	2253305	3491326	-	-	[2,0]	[3,3,3,0]
	0.75	2352638	803429.6	4.41	334.55	[2,0]	[3,3,3,2]
	0.5	2368977	761779.6	5.13	358.31	[2,0]	[3,3,3,3]
	0.25	2587345	744651	14.82	368.85	[2,2]	[3,3,3,3]
	*0	2853798	709586.9	26.65	392.02	[3,2]	[3,3,3,3]

TABLE III
LINGO Results for $C_3 = 1934.36$

Q				Δ	Δ	OM	OD
0.02	1	2253305	843922.7	-	-	[2,0]	[3,3,3,0]
	*0.99	2303875	204521	2.24	312.63	[3,0]	[3,3,3,0]
	0.75	2352638	81689.7	4.41	933.08	[2,0]	[3,3,3,2]
	0.5	2368977	80252.24	5.13	951.59	[2,0]	[3,3,3,3]
	0.25	2644990	77521.22	17.38	988.63	[2,2]	[3,3,3,3]
	0	3465562	73101.73	53.80	1054.45	[3,3]	[3,3,3,3]
0.2	1	2253305	6400759	-	-	[2,0]	[3,3,3,0]
	0.75	2352638	1020874	4.41	526.99	[2,0]	[3,3,3,2]
	0.5	2368977	937342.5	5.13	582.86	[2,0]	[3,3,3,3]
	0.25	2551893	924247.1	13.25	592.54	[2,1]	[3,3,3,3]
	*0	2905913	885149.7	28.96	623.13	[2,3]	[3,3,3,3]

V. Conclusion

This study provided a framework for extending the previous literature on reliable SCN design by adding manufacturer's layer and facilitating multi capacity levels for both manufacturer and DC echelons. Besides that a reformulation process through a branch of LP-Metric method was carried in order to solve the bi-objective model with LINGO software. Also computational results for a small size problem indicated that by small consideration to the unreliable nature of supplying layers, the system could block the large probable future losses. Additionally, parameters analysis shows that in problems with high penalty cost, utilization of investment on the SCN initialization costs could be more beneficial. However, for future studies it is worthwhile to include some heuristic approaches for solving large scale problems. Moreover, it could be valuable to formulate scenario occurrence probability (changing from one state to another) via Markov process.

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