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Selective Harmonic Elimination of PWM AC/AC Voltage Controller Using Hybrid RGA-PS Approach

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***Abstract*—**Selective harmonic elimination-pulse width modulation techniques offer a tight control of the harmonic spectrum of a given voltage waveform generated by a power electronic converter along with a low number of switching transitions. Traditional optimization methods suffer from various drawbacks, such as prolonged and tedious computational steps and convergence to local optima; thus, the more the number of harmonics to be eliminated, the larger the computational complexity and time. This paper presents a novel method for output voltage harmonic elimination and voltage control of PWM AC/AC voltage converters using the principle of hybrid Real-Coded Genetic Algorithm-Pattern Search (RGA-PS) method. RGA is the primary optimizer exploiting its global search capabilities, PS is then employed to fine tune the best solution provided by RGA in each evolution. The proposed method enables linear control of the fundamental component of the output voltage and complete elimination of its harmonic contents up to a specified order. Theoretical studies have been carried out to show the effectiveness and robustness of the proposed method of selective harmonic elimination. Theoretical results are validated through simulation studies using PSIM software package.

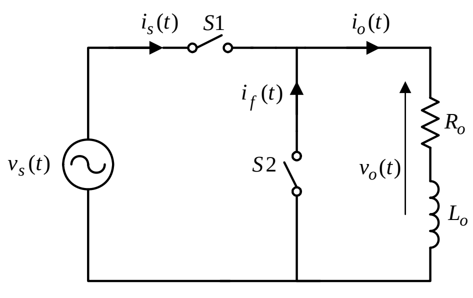
***Keywords*—**PWM, AC/AC voltage converters, selective harmonic elimination, direct search method, pattern search method, Real-coded Genetic algorithms, evolutionary algorithms and optimization.

I. INTRODUCTION

MEDIUM and large power converter in motor drives, AC/AC converters, huge UPS systems, high power flexible alternative current transmission systems (FACTS) and PWM AC-AC voltage controllers, need switching elements which can bear high voltage/current. To overcome limits of semiconductor switches, several new techniques and topologies have been developed [1-3], such as multiple switching elements in one leg of an inverter, multiple rectifiers for unity power factor correction, optimization of motor performance indices such as harmonic current, torque ripple [4-6].

AC/AC line-commutated phase-angle control or integral cycle control with thyristors technology have been widely used. However, these techniques has many drawbacks, the retardation of the firing angle causes a lagging power factor at the input side, plentiful low order harmonics in both of supply voltages/currents and a discontinuity of power flow to the load appears [7].

The Selective harmonic elimination (SHE) PWM based methods can theoretically provide the highest quality output among all the PWM methods. SHE has been a research topic since the early 1960's, first examined in [8] and developed into a mature form in [9-11] during the 1970's. SHE offers several advantages compared to traditional modulation methods [12] including acceptable performance with low switching frequency to fundamental frequency ratios, direct control over output waveform harmonics, and the ability to leave triplen harmonics uncontrolled to take advantage of circuit topology in three phase systems. These key advantages make SHE a viable alternative to other methods of modulation in applications such as variable speed drives [13, 14], or dual-frequency induction heating [15]. This method is sometimes called a programmed PWM technique. However, the drawback of these methods is a heavy computational burden and a complicated hardware [16]. The main challenge of solving the associated nonlinear equations, which are transcendental in nature and therefore have multiple solutions, is the convergence.



(a) Circuit configuration

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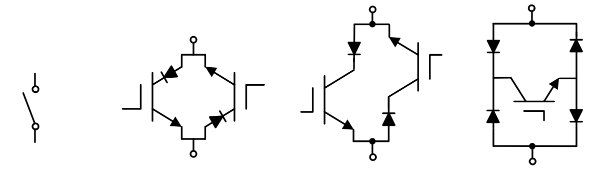
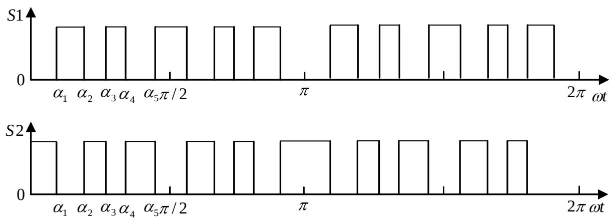


Switch 2 Reverse blocking IGBTs 2 IGBTs + 2 Diode One IGBT + 4 diodes

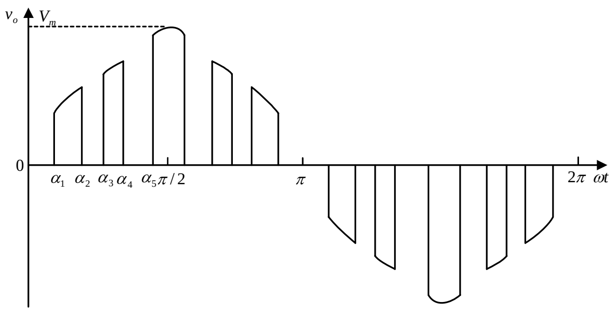
(b) Bi-directional switch realization

Fig. 1 PWM AC/AC voltage converter with bi-directional switches.

II. Principle of Operation and Problem Formulation



(a) Gating signals



(b) Output voltage

Fig. 2 Gating signals and output voltage of PWM AC/AC voltage converter

Fig. 1(a) shows the power circuit configuration of single-phase PWM AC/AC voltage converter, which is composed of two bi-directional power switches, one connected in series and the other in parallel with the load. The series-connected switch S1 regulates the power delivered to the load, and the parallel one S2 provides a freewheeling path to discharge the stored energy when the series one is turned off. Standard AC/AC converters require bi-directional switches. The switches are assumed to have bi-directional voltage blocking ability as well as bi-directional current conduction and turn off capability. At present, the switches must be realized by two reverse blocking insulated gate bi-polar junction transistors (IGBT) or by inverse parallel connection of two conventional IGBTs and two diodes or by a conventional IGBT inside a diode bridge as shown in Fig. 1(b). The present paper uses the last configuration for simplicity. Theoretically, the switching must be instantaneous and simultaneous and alternate current path has to be provided. For practical realizations, the finite switching times and delays in the drive circuits and controlled switches have been taken into account. The firing angles are generated by a microprocessor, which generates supply-synchronized pulses from a look up table.

When a switching function shown in Fig. 2 (a), is applied to the single-phase AC/AC PWM voltage controller, the output voltage appears in the PWM form at the load terminals. S1 is turned on at various switching angles α_1, α_3, $\ldots \alpha_{M-1}$ and turned off at α_2, α_4, $\ldots \alpha_M$ per quarter cycle.

Fig. 2 (b) shows the idealized output voltage waveform of the AC/AC voltage controller, where the quarter-wave symmetry is preserved which will null the even harmonics, as it will be shown. By the proper choice of PWM switching angles, the fundamental component can be controlled and a selected low order harmonics can be eliminated. Owing to the PWM waveform characteristics of odd function symmetry and half wave symmetry, even harmonics are absent and only odd harmonics exist. The Fourier series expressions for the output voltage, which are expressed in terms of the M switching point variables, can be easily express as follows:

$$v_o = \sum_{n=1}^{\infty} A_n \sin(nwt) + B_n \cos(nwt) \qquad (1)$$

where $B_n = 0$ for n=1, 2, 3, …… and thus the above equation reduces to:

$$v_o = \sum_{n=1,3,5,\ldots}^{\infty} A_n \sin(nwt) \qquad (2)$$

The value of A_n is computed as:

$$A_n = \frac{2V_m}{\pi} \sum_{i=1}^{M} (-1)^i \left[\frac{\sin(n-1)\alpha_i}{(n-1)} - \frac{\sin(n+1)\alpha_i}{(n+1)} \right] \qquad (3)$$

where $n = 3,5,\ldots,2M-1$, M is the number of switching angles per quarter cycle, α_i is the ith switching angles and V_m is the maximum value of the input voltage. The fundamental component is given by:

$$A_1 = (1+\frac{2}{\pi})V_m \sum_{n=3,5,\ldots}^{\infty} (-1)^i \left[\alpha_i - \frac{\sin 2\alpha_i}{2} \right] \qquad (4)$$

The problem objective is to find the switching instants such that $A_1 = V_m^*$ and to perform SHE to a specified order where V_m^* is the maximum value of the reference output voltage. In order to proceed with the optimization/minimization, an objective function describing a measure of effectiveness of eliminating selected order of harmonics while maintaining the fundamental at a pre-specified value must be defined. This is converted to an optimisation problem subject to constraints. Let $F(\alpha)$ be the objective function, which it will be minimized and is defined as

$$F(\alpha) = (A_1 - V_m)^2 + A_3^2 + \ldots\ldots\ldots + A_{M-1}^2 \qquad (5)$$

The correct solution must satisfy the condition

$$0 \le \alpha_1 \le \alpha_2 \le .... \alpha_{M-1} \le \alpha_M \le \frac{\pi}{2} \qquad (6)$$

The task is to determine the firing instants such that objective function $F(\alpha)$ subject to the constraint of (6) is minimized. Therefore, the output voltage is regulated ideally over the full range [0, Vm] by changing the modulation index mi which is defined as (A_1/Vm) and has no harmonics within that range, to obtain the switching instants according to Fig. 2(a).

The optimization of objective function (5) subject to the constraints of (6) is usually achieved using conventional optimization techniques as Newton-Raphson method, random-search (RS) method and Rosenbrock's method [21]. Traditional optimization methods suffer from various drawbacks, such as prolonged and tedious computational steps and convergence to local optima; thus, the more the number of harmonics to be eliminated, the larger the computational complexity and time.

The new search pattern proposed in this paper has the features of complete harmonic elimination up to the specified order as well as linear fundamental output voltage control and requires a low order pulse number, as it will be shown later in the next section.

III. Overview of RGA and PS

1) Overview of Genetic Algorithms

GAs are inspired by the study of genetics [17-19]. They are conceptually based on natural evolution mechanisms working on populations of solutions. An interesting feature of GAs is that they do not require any prior knowledge of the solution and they tend to exhibit reliable performance on the majority of the problems [19].

Initially, GAs were designed to operate using binary representations of the problem parameters (or unknowns). In recent studies however, the superiority of higher cardinality alphabet GAs (floating point or integer) has been demonstrated with respect to their applications to various problems. A brief description of a real-coded GA is given in the next section.

A. Real Coded GA

In a Real-Coded GA, all decision variables (unknowns) are expressed as real numbers. Explicit conversion to binary does not take place. A reduction of computational effort is an obvious advantage is of real-coded GA. Another advantage is that an absolute precision is now attainable by making it possible to overcome the crucial decision of how many bits are needed to represent potential solutions.

As in a conventional GA, an initial population of chromosomes (potential solutions) is randomly created. The best size of this population is subject to experimentation with the problem at hand. Having created a population of chromosomes, it is possible to assess the performance, or fitness, of individual members of a population. This is done through an objective function (7) that characterizes an individual's performance in the problem domain. Then a method known as *ranking* [20], is used to rank individuals according to their objective values. Based on that ranking (i.e. fitness) of each chromosome in the initial population, a selection scheme is carried out to pick the best individuals as members of the new generation.

The selection scheme used is known as *Stochastic Universal Sampling* [21]. This scheme, probabilistically selects individuals for reproduction according to their fitness. That is simply implemented by finding the cumulative sum of fitness of each chromosome in the population and generating and equally spaced numbers between 0 and that sum. Therefore, only one random number is generated, all the others used being equally spaced from that point. The index of the chromosome selected is determined by comparing the generated numbers with the cumulative sum. The probability of an individual being selected is then given by

$$F(x_i) = \frac{f(x_i)}{\sum_{i=1}^{N_{ind}} f(x_i)} \qquad (7)$$

where $f(x_i)$ is the fitness of individual x_i and $F(x_i)$ is the probability of that individual being selected. A discrete recombination method (equivalent to crossover) is employed for mating individuals and breeding of offsprings. Discrete recombination exchanges variable values between the individuals. A method known as simple crossover [19, 22] is implemented. Specifically, let's assume that $C_1 = \left(c_1^1..c_n^1\right)$ and $C_2 = \left(c_1^2..c_n^2\right)$ are two chromosomes that are being subjected to crossover. A position $i \in \left(1,2,3,......,n-1\right)$ is randomly assigned. The two new chromosomes are made as the following:

$$C_{1,new} = \left(c_1^1, c_2^1, ..., c_i^1, c_{i+1}^2, ...c_n^2\right) \qquad (7)$$

$$C_{2,new} = \left(c_1^2, c_2^2, ..., c_i^2, c_{i+1}^1, ...c_n^1\right) \qquad (8)$$

Mutation of real-valued population is accomplished with the breeder genetic algorithm in [23]. Each variable is mutated with a probability by addition of small random values (size of the mutation step). The mutation step can be reduced as the algorithm evolves.

The proposed RGA uses a generation gap and fitness-based reinsertion to implement an *elitist* strategy whereby the best individuals always propagate through to successive generations. For example, if G-gap = 90%, then population_size ×G-gap new individuals are produced at each generation. And then population_size ×(G-gap -1) best chromosomes are copied intact from the parent generation to the new generation to complete the population size (i.e. fill the gap). According to [18], a better average fitness is attained with the adoption of elitist strategy.

The RGA algorithm stops when any of the following conditions occurs:

- The number of iterations performed by the algorithm reaches the value of max iteration.

- The total number of objective function evaluations performed by the algorithm reaches the value of Max function evaluations.
- The change in the objective function from one generation to the next successful poll is less than objective function tolerance.

B. Pattern Search Method

The Pattern Search (PS) optimization routine is a derivative free evolutionary technique that is suitable to solve a variety of optimization problems that lie outside the scope of the standard optimization methods. Generally, PS has the advantage of being very simple in concept, and easy to implement and computationally efficient algorithm. Unlike other heuristic algorithms, such as GA [18, 19], PS possesses a flexible and well-balanced operator to enhance and adapt the global and fine tune local search. A historic discussion of direct search methods for unconstrained optimization is presented in reference [24]. The authors gave a modern prospective on the classical family of derivative-free algorithms, focusing on the development of direct search methods.

The Pattern Search (PS), algorithm proceeds by computing a sequence of points that may or may not approaches to the optimal point. The algorithm starts by establishing a set of points called *mesh*, around the given point. This current point could be the initial starting point supplied by the user or it could be computed from the previous step of the algorithm. The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a *pattern*. If a point in the mesh is found to improve the objective function at the current point, the new point becomes the current point at the next iteration.

This maybe better explained by the following:

First: The Pattern search begins at the initial point X_0 that is given as a starting point by the user. At the first iteration, with a scalar =1 called *mesh size,* the pattern vectors are constructed as $[0\ \ 1]$, $[1\ \ 0]$, $[-1\ \ 0]$ and $[0\ \ -1]$, they may be called direction vectors. Then the Pattern search algorithm adds the direction vectors to the initial point X_0 to compute the following mesh points:

$$X_0 + [1\ \ 0]$$

$$X_0 + [0\ \ 1]$$

$$X_0 + [-1\ \ 0]$$

$$X_0 + [0\ \ -1]$$

Fig. 3 illustrates the formation of the mesh and pattern vectors. The algorithm computes the objective function at the mesh points in the order shown.

The algorithm polls the mesh points by computing their objective function values until it finds one whose value is smaller than the objective function value of X_0. If there is such point, then the poll is successful and the algorithm sets this point equal to X_1.

After a successful poll, the algorithm steps to iteration 2 and multiplies the current mesh size by 2, (this is called the *expansion factor* and has a default value of 2). The mesh at iteration 2 contains the following points: 2*[1 0] $+X_1$, 2*[0 1] $+X_1$, 2*[-1 0] + X_1 and 2*[0 -1] $+X_1$. The algorithm polls the mesh points until it finds one whose value is smaller than the objective function value of X_1. The first such point it finds is called X_2, and the poll is successful. Because the poll is successful, the algorithm multiplies the current mesh size by 2 to get a mesh size of 4 at the third iteration because the expansion factor =2.

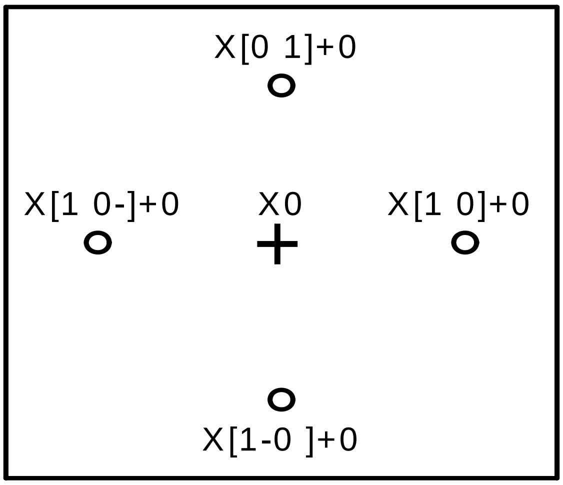


Fig. 3 PS Mesh points and the Pattern

Second: Now if iteration 3, (mesh size= 4), ends up being unsuccessful poll, i.e. none of the mesh points has a smaller objective function value than the value at X_2, so the poll is called an unsuccessful poll. In this case, the algorithm does not change the current point at the next iteration. That is, $X_3 = X_2$. At the next iteration, the algorithm multiplies the current mesh size by 0.5, a contraction factor, so that the mesh size at the next iteration is smaller. The algorithm then polls with a smaller mesh size.

The Pattern search optimization algorithm will repeat the illustrated steps until it finds the optimal solution for the minimization of the objective function. The PS algorithm stops when any of the following conditions occurs:

- The mesh size is less than mesh tolerance.
- The number of iterations performed by the algorithm reaches the value of max iteration.
- The total number of objective function evaluations performed by the algorithm reaches the value of Max function evaluations.
- The distance between the point found at one successful poll and the point found at the next successful poll is less than X tolerance.
- The change in the objective function from one

successful poll to the next successful poll is less than the objective function tolerance.

All the parameters involved in the PS optimization algorithm can be pre-defined subject to the nature of the problem being solved.

IV. Solution Methodology

The solution of the SHE problem by the proposed hybrid RGA-PS can be summarized as in the following Pseudo code:

Step 1: Formulate the SHE problem.
Step 2: RGA proceeds by randomly generating a population of potential solutions.
Step 3: Do
- i. Assesses the population fitness is using the objective function (i.e. eq. 7).
- ii. Ranking carried out.
- iii. Selection is employed to pick the best individuals as members.
- iv. Create offsprings based on discrete recombination (crossover and mutation).
- v. Elitism is employed and a new generation is created.
- vi. Identify the best individuals in new generation (i.e. $\mathbf{RGA_{best}}$) using objective function.

Step 4: Solve the SHE problem using PS and $\mathbf{RGA_{best}}$ is used as a starting point.
Step 5: The solution provided by PS is injected in the generation formed in step v.
Step 6: While (none of the RGA stopping criteria is not met)

V. Numerical Results and Simulation Results

A set of Matlab files implementing the proposed hybrid RGA and PS method have been used in to optimize the objective function of (7) subject to the constraint of (8) for SHE of AC/AC PWM converters.

Initially, several runs have been carried out with different values of the key parameters of RGA and PS. The parameters used in the implementation of RGA and PS are listed in Table I. As for the stopping criteria, all tolerances were set to 10^{-6} and the maximum number of iterations and function evaluations are set to 1000.

TABLE I
RGA and PS Parameters

RGA		PS	
Population Size	**200**	**mesh size**	**1**
Mutation	**0.02**	**mesh expansion factor**	**2**
Crossover	**0.8**	**mesh contraction factor**	**0.5**
Generation Gap	**0.9**		

The program is executed for different values of number of switching instants per quarter cycle (M) and for different modulation index (mi). The calculated switching angles are simulated using the software package PSIM for verification purposes. The results are presented in this section.

1) Eliminating the 3rd and 5th Harmonics

Three switching instants per quarter cycle (*M=3*) are chosen that is aimed to eliminate (*M-1*) harmonics. Fig. 4 shows the calculated switching angles profile for different values of modulation index $(0.0 \leq m_i \leq 1.0)$ with the elimination of 3rd and 5th order components.

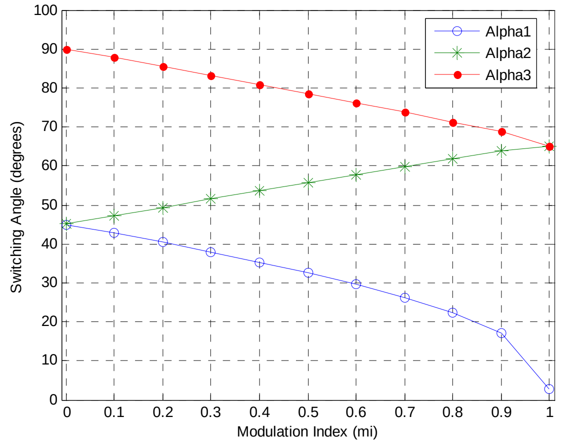


Fig. 4 Switching angles profile versus m_i for *M=3* with eliminating 3rd and 5th order harmonics.

As an illustration of effectiveness of the proposed method, Fig. 5 depicts a comparison of the convergence characteristics of hybrid RGA-PS and RGA methods, where variation of the objective function value at different iterative steps is plotted for each method. It is obvious from the convergence that a near optimal solution was achieved by RGA-PS in about 21 iterations and another 30 iterations to refine the solution to ultimately converge to an optimal solution point in 51 iterations. The CPU execution time of RGA-PS is 1.830523 sec. On the other hand, RGA by it self is evidently much slower and requires more evolutions to converge to the same solution given by RGA-PS. Table II shows a comparison of RGA-PS and GA results for the entire modulation indexes after RGA-PS exit (convergence). It is apparent that RGA-PS has been extremely successful in completely eliminating the selected harmonics and efficiently obtaining the desired output fundamental voltage, whereas the outcome of RGA, when it is stopped at the same time RGA-PS exits, appears to be quite far off and it defiantly needs to evolve more which in turn requires additional computational effort and time to eliminate the selected harmonics.

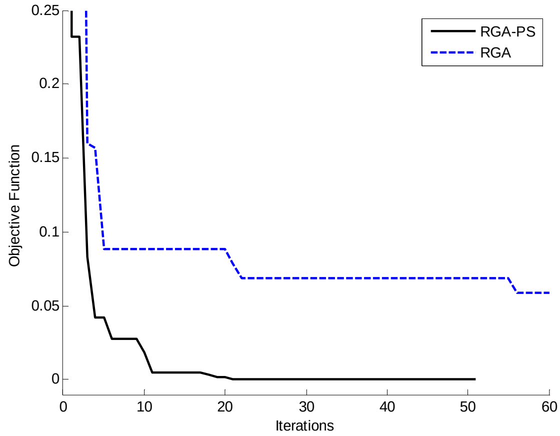


Fig. 5 Value of the objective function with GA and GA-PS methods for M=3 with eliminating 3rd and 5th order harmonics

As an illustration of effectiveness of the proposed method, Fig. 5 depicts a comparison of the convergence characteristics of hybrid RGA-PS and RGA methods, where variation of the objective function value at different iterative steps is plotted for each method. It is obvious from the convergence that a near optimal solution was achieved by RGA-PS in about 21 iterations and another 30 iterations to refine the solution to ultimately converge to an optimal solution point in 51 iterations. The CPU execution time of RGA-PS is 1.830523 sec. On the other hand, RGA by it self is evidently much slower and requires more evolutions to converge to the same solution given by RGA-PS.

TABLE II
COMPARISON OF GA AND GA-PS RESULTS FOR M=3 WITH ELIMINATING 3RD AND 5TH ORDER HARMONICS

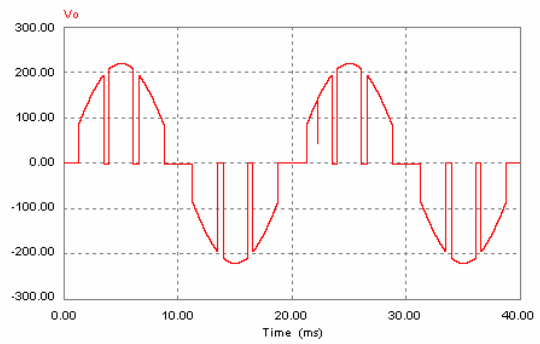
m_i	Hybrid RGA-PS			RGA		
	A_1	B_3	B_5	A_1	B_3	B_5
0.1	0.1000	0.0000	0.0000	0.2781	0.0483	0.1965
0.2	0.2000	0.0000	0.0000	0.1970	0.0072	0.0050
0.3	0.3000	0.0000	0.0000	0.3025	0.0014	0.0008
0.4	0.4000	0.0000	0.0000	0.6903	0.2043	0.0491
0.5	0.5000	0.0000	0.0000	0.5091	0.0069	0.0021
0.6	0.6000	0.0000	0.0000	0.5911	0.0052	0.0044
0.7	0.7000	0.0000	0.0000	0.6440	0.0059	0.0547
0.8	0.8000	0.0000	0.0000	0.7996	0.0008	0.0010
0.9	0.9000	0.0000	0.0000	0.8989	0.0020	0.0003
1.0	1.0000	0.0000	0.0000	0.9940	0.0001	0.0012

Table II shows a comparison of RGA-PS and GA results for the entire modulation indexes after RGA-PS exit (convergence). It is apparent that RGA-PS has been extremely successful in completely eliminating the selected harmonics and efficiently obtaining the desired output fundamental voltage, whereas the outcome of RGA, when it is stopped at the same time RGA-PS exits, appears to be quite far off and it defiantly needs to evolve more which in turn requires additional computational effort and time to eliminate the selected harmonics.

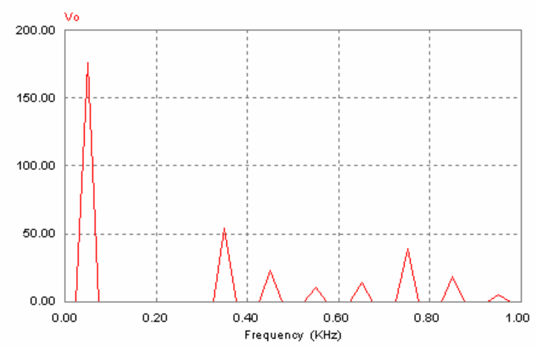
To verify the validity of the proposed method, a PWM AC/AC voltage controller was simulated using the PSIM software simulator using the firing instants obtained in Fig. 4. The simulated circuit parameters are listed in Table III. Time-domain waveforms of output voltage and current of PWM AC/AC voltage converters while maintaining the fundamental component at 0.80 p.u. and eliminating of 3rd and 5th order components are presented in Fig. 6. Frequency spectrum of the output voltage and current for the same simulation conditions is illustrated in Fig. 7. The simulation results are in full agreement with theoretical results. From Fig. 7, it is evident that GA-PS method works successfully in the present problem by eliminating all the desired harmonics and also maintaining required fundamental output voltage.

TABLE III
SIMULATED CIRCUIT PARAMETERS

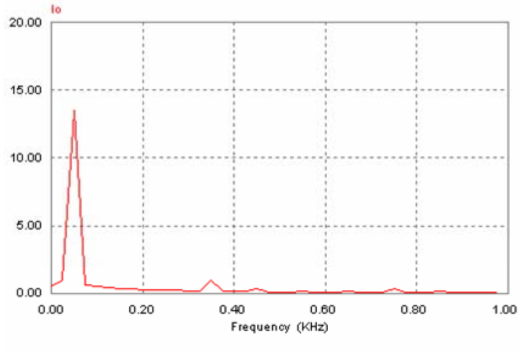
Parameter	Symbol	Value
Maximum supply voltage	V_m	**220 [V]**
Rated power	P	**2.2 [kW]**
Load resistance	R	**10 [Ω]**
Load inductance	L	**6..5 [mH]**
Switching instants per quarter cycle	M	**3, 5**



(a) Output voltage
Fig. 6 Output voltage and current waveforms for M=3 with eliminating 3rd and 5th harmonics



(a) Output voltage



(b) Output current

Fig. 7 Frequency spectra for the results in Fig. 6

VI. Conclusion

Selective harmonic elimination/control has been a widely researched alternative to traditional pulse-width modulation techniques. In this paper, a new selective harmonic elimination/control based on hybrid Genetic Algorithm-Pattern Search method is proposed for PWM AC/AC voltage controllers with forced commutations. The theoretical PWM pattern is found by solving the nonlinear harmonic equations which describe the suggested PWM method. The new pattern proposed in this paper has the features of complete harmonic elimination up to the specified order as well as linear voltage control and requires a low order pulse number. With the proposed hybrid RGA-PS method, complete elimination of desired harmonics is attainable in a relatively fast CPU, while with RGA method, complete elimination of desired harmonics may be possible, but it has to be on the expense time and computational effort; rather this method minimizes all desired harmonics.

The feasibility and effectiveness of the proposed algorithm is evaluated with intensive simulation studies. Further work should focus on practical real-time implementation of the SHE-PWM AC/AC voltage converters.

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