Diagnostic Investigation of Liftoff Time of Solid Propellant Rockets

Vignesh Rangaraj, Jerin John, N. Naveen, M. Karuppasamy Pandian, P. Sathyan, and V. R. Sanal Kumar

Abstract—In this paper parametric analytical studies have been carried out to examine the intrinsic flow physics pertaining to the liftoff time of solid propellant rockets. Idealized inert simulators of solid rockets are selected for numerical studies to examining the pre-ignition chamber dynamics. Detailed diagnostic investigations have been carried out using an unsteady two-dimensional k-omega turbulence model. We conjectured from the numerical results that the altered variations of the igniter jet impingement angle, turbulence level, time and location of the first ignition, flame spread characteristics, the overall chamber dynamics including the boundary layer growth history are having bearing on the time for nozzle flow chocking for establishing the required thrust for the rocket liftoff. We concluded that the altered flow choking time of strap-on motors with the pre-determined identical ignition time at the lift off phase will lead to the malfunctioning of the rocket. We also concluded that, in the light of the space debris, an error in predicting the liftoff time can lead to an unfavorable launch window amounts the satellite injection errors and/or the mission failures.

Keywords—Liftoff, Nozzle Choking, Solid Rocket, Takeoff.

I. INTRODUCTION

ALTHOUGH technology in the solid propellant rocket field has advanced significantly over the past seven decades, there are many unresolved problems of academic interest. In an attempt to resolve some of these problems and in the light of new findings of Sanal Kumar et al. [1]-[3], a substantial revision of the existing ideas may be necessary. One such problem of interest is the liftoff time prediction prompted by the recent experiences with solid propellant rocket motors having non-uniform port configurations. The liftoff phase is defined in this paper as the time interval between the application of the ignition signal and the instant at which the aerospace vehicle goes from the ground to flying in the air.

The two primary concerns during the liftoff phase are the time taken for the nozzle flow chocking condition and the overall time for establishing the required thrust for the rocket

Vignesh Rangaraj, Jerin John, N. Naveen, Karuppasamy Pandian. M. are undergraduate students at the department of aeronautical engineering, Kumaraguru College of Technology, Coimbatore –641049, Tamil Nadu, India (e-mail: rangaraj.vignesh@gmail.com, jerinjohn17@gmail.com).

Sathyan P. is the graduate student at the department of aerospace engineering Pusan National University, Busan 609-735, South Korea (e-mail: sathyanpadmanabhan@hotmail.com).

V.R. Sanal Kumar is Professor and Aerospace Scientist, is with the Department of Aeronautical Engineering, Kumaraguru College of Technology, Coimbatore – 641 049, Tamil Nadu, India; (corresponding author; phone: +91 - 9150891021 / +91-9388679565, e-mail: vr_sanalkumar @yahoo.co.in).

smooth liftoff. The overall time, that is, the delay in development of full thrust must be kept within some limit and must be reproducible. Most of the high-performance solid rocket motors (SRMs) require greater accuracy in prediction and control of the liftoff phase; i.e., more precise prediction of nozzle flow choking and the thrust transient [1]. A detailed knowledge of thrust transient is also required for the critical guidance, control and collision avoidance of the aerospace vehicles with the space debris. The quantitative prediction and knowledge of the takeoff time and the thrust transient allow and justify the use of a well suited lucrative launch window in addition to the control and guidance requirements of the vehicle.

Literature review on orbital debris reveals that the Inter Agency Space Debris Coordination Committee (IADC), an international body, has catalogued some 20,000 space debris objects including spent rocket parts in low-earth orbits, posing a risk to space missions. Many studies have been carried out on space debris tracking and elimination but these studies reports are not capable to give a practical solution for the space debris mitigation and collision avoidance [4]-[9]. Naveen [8] reveals that, of late, by delaying the liftoff by two minutes at the end of the 51-hour countdown, the Indian Space Research Organisation (ISRO) could evade the collision of their satellite with the catalogued space debris and avoided a probable mission failure. The author further revealed that there were at least six "considerably big" pieces of space debris that posed a danger to the satellites, had the ISRO rocket been launched at the designated time of 9.51am on September 9, 2012. Literature review further reveals that many studies have been carried on boost phase of aerospace vehicles. But the causes of the uncertainties associated with the rocket liftoff time prediction have not been much discussed and admittedly, which is still an elusive problem.

Crosson, et al., [10] reported that characterizing the boost phase of a rocket's flight is challenging when only metric radar data (range and angles) are used. Incorporating the range-acceleration measurements result in superior tracking performance and much improved characterization of boost-phase flight. This paper demonstrates how to estimate range acceleration reliably from the amplitude and phase of the target-reflected radar signal. Their techniques have been used successfully on actual rocket-launch data to improve the post-mission tracking and object identification performance. This paper was exclusively for post–mission and not aimed for pre-mission chamber dynamics evaluation for reducing the uncertainties associated with the liftoff time prediction of the

aerospace vehicles. Danis [11] reported the influence of *a priori* uncertainties in launch time and trajectory fly-out profiles, along with the sensor angle measurement errors, on the estimation of the vehicle launch location and heading angle. An error model is developed to compute the statistics of the estimation errors using a single pair of angle measurements, one from each of two satellites, or both from the same satellite platform. The measurements and estimation methods are described, followed by a derivation of the estimation errors for the hypothetical case of perfect knowledge of trajectory and launch time. In fact this paper was focused on the effect of the uncertainties in launch time and not the cause of it, which is the main theme of this paper.

Real time data of Ariane 5 rocket is presented by Bussiere and Mora [12], where the relative roughness (ε/D) of the booster nozzle is increased from perfect surface finish to 0.012 during a flight that initiates with launch and ends at the orbit. Park et al. [13] carried out the evaluation of critical pressure ratios of sonic nozzles at low Reynolds numbers. The authors reported that the critical pressure ratio is highly dependent on the Reynolds number rather than area ratio especially in the cases with low flow velocity. Alon Gany et al. [14] presented an analysis of swirling flows in choked nozzles. The one-dimensional compressible flow theory was extended to isentropic axisymmetric swirling flows. The most significant accomplishment of this model was its ability to treat a general swirl type of any tangential velocity distribution. A choking criterion similar to that of one-dimensional non-swirling flows was introduced. But in these papers [12]-[14] also no attempts have been made by the authors for predicting the flow choking time.

It is well known that the critical design parameters on the performance prediction of converging nozzles are the geometric features and the operating conditions, which include the stagnant properties at the inlet, frictional and heat transfer behaviors on the nozzle wall; where the latter two are hard to handle together in compressible high-speed flows. It may be noted that the accurate prediction of transient unsteady compressible reacting flows through CD nozzle is still a challenging task for the rocket nozzle designers when the nozzle performance at the liftoff phase is significantly influenced by geometry, inlet conditions, upstream chamber dynamics and sources of non-isentropic character. Many experimental and numerical works reported information on nozzle performance, flow and heat transfer characteristics with various inlet-boundary conditions and flow geometries. But these studies are not focused on the nozzle flow choking time prediction, which is a critical input for the overall takeoff time prediction.

Instabilities in the propulsion of rockets, due to pressure and temperature fluctuations at the upstream of rocket nozzle and the flow geometry, were numerically considered by Assovskii and Rashkovskii [15]. Alper Ozalp [16] investigated the effects of surface roughness and heat flux conditions on compressible converging - nozzle flows for various flow geometries and with different inlet and boundary conditions using a validated numerical model. Using this in-house model the author demonstrated that both surface roughness and heat flux produce lower Mach numbers, moreover the effect of heat flux on the Mach number pattern is more apparent with lower nozzle convergence half angles. The author also demonstrated that the discharge coefficients increase with higher inlet stagnation pressures and with lower convergence half angles, surface roughness and heat flux conditions, where the effect of roughness is more remarkable in lower inlet stagnation pressure cases. The numerical results of Ozalp [16] further reveal that as the effect of surface heat flux in terms of Nusselt number is more apparent in un-choked flows and the role of surface roughness becomes more significant in the choked flow with lower convergence half angles. This author also not reported the flow choking time, which is of topical interest.

Swirling flow in nozzles occurs in a number of important propulsion applications, including turbofans and turbojet engines, spin-stabilized rockets, and integral rocket/ramjets. In certain cases fuel will be injected coaxially along centerline at the nozzle throat. Ahmed Abdelhafez [17] reported that the effect of imparting swirl to under-expanded supersonic nozzle airflow on the choking criteria, shock structure, and mixing. The author reported that that the throat velocity itself (not any of its components) is choked in a swirling flow field. Therefore, the limiting tangential Mach number is unity. Moreover, the application of swirl always results in a reduction in axial Mach number component. The mass flow rate through nozzle is found to be primarily a function of throat static pressure and axial Mach number. The reduction in the latter with swirl explains the observed reduction in mass flow. Greater reservoir pressures, on the other hand, result in higher throat static pressures, which compensates for the reduced axial Mach number, and the mass flow rate can be kept constant at its non-swirling value. It is also found that the distribution of subsonic Mach number in a non-swirling flow is almost not affected with the application of swirl, i.e., non-swirling and swirling flows have the same subsonic Mach number profile. In terms of thrust and specific impulse, the application of swirl at matched nozzle reservoir pressure results in the expected reductions in discharge coefficient, thrust, and specific impulse. At matched mass flow, however, the application of swirl results in the enhancement of both thrust and specific impulse. Though this author also not carried out any estimation of flow choking time, the contents of this dissertation throws light for further modeling effort for an accurate prediction of the flow choking time and /or the liftoff time of aerospace vehicles for various propulsion applications.

Karuppasamy Pandian et al. [18] reported qualitatively in a previous connected paper that when the igniter turbulent intensity is relatively low, the vehicle could liftoff early due to the early flow choking of the rocket nozzle. But the authors did not make any attempt for reporting the detailed parametric analytical studies pertaining to the transient unsteady rocket motor chamber dynamics and its bearing on the nozzle flow choking time. Therefore in this connected paper the authors made an attempt to explore the cause(s) of the uncertainties associated with the rocket liftoff time prediction, after the

initiation of the igniter signal, through the diagnostic investigations of pre-ignition chamber dynamics features of two idealized solid propellant rocket motors.

II. NUMERICAL METHODOLOGY

Numerical simulations have been carried out with the help of a two-dimensional standard k–omega model. This turbulence model is an empirical model based on model transport equations for the turbulence kinetic energy and a specific dissipation rate [19]. This code solves standard k–omega turbulence equations with shear flow corrections using a coupled second-order-implicit unsteady formulation. In the numerical study, a fully implicit finite volume scheme of the compressible, Reynolds-averaged, Navier–Stokes equations is employed. Compared to other available models, this model could well predict the turbulence transition in duct flows and has been validated through benchmark solutions.

Compressibility effects are encountered in gas flows at high velocity and/or in which there are large pressure variations. When the flow velocity approaches or exceeds the speed of sound or when the pressure change in the system is large, the variation of the gas density with pressure has a significant impact on the flow velocity, pressure, and temperature. Compressible flows create a unique set of flow physics for which one must be aware of the special input requirements and appropriate solution techniques. Compressible flows can be characterized by the value of the Mach number. As the Mach number approaches 1.0, compressibility effects become important. When the Mach number exceeds 1.0, the flow is termed supersonic and may contain shocks and expansion that can impact the flow pattern significantly. Compressible flows are typically characterized by the total pressure P_0 and total temperature T_0 of the flow. The following relationships describe the variation of the static pressure and temperature in this flow model as the velocity/Mach number, M changes under isentropic conditions:

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma - 1} \tag{1}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{2}$$

$$\rho = \frac{p_{op} + p}{\frac{R}{M_w} T} \tag{3}$$

Equation (1) predicts a choked flow ($M = 1.0$) at an isentropic pressure ratio for a given specific heat ratio, γ. Note that in this model the choked flow condition will be established at the point of minimum flow area. In the subsequent area expansion, the flow may either accelerate to a supersonic flow in which the pressure will continue to drop or return to subsonic flow conditions, decelerating with a pressure rise. If a supersonic flow is exposed to an imposed pressure increase, a shock will occur, with a sudden pressure rise and deceleration accomplished across the shock. In this model, the compressible flows are described by the standard

continuity and momentum equations with the inclusion of the compressible treatment of the density. For compressible flows, the ideal gas law is written in the form (see (3)) for density, ρ. The temperature, T, can be computed from the energy equation, P is the pressure, P_{op} is the operating pressure, M_w is the molecular weight, R is the gas constant. The energy equation solved by the code will incorporate the coupling between the flow velocity and the static temperature.

The viscosity, μ is determined from the Sutherland formula, given by

$$\mu = \frac{(1 + S)T^{3/2}}{T + S} \tag{4}$$

where $S = 110{:}4/288.15$.

The standard k–*omega* model is an empirical model based on model transport equations for the turbulence kinetic energy (k) and a specific dissipation rate (ω). This code solves standard *k*–omega turbulence equations with shear flow corrections using a coupled second-order implicit unsteady formulation. The turbulence kinetic energy and specific dissipation rate are obtained from the two transport equations [19]. Initial wall temperature, inlet total pressure, and temperature are specified. In the present compressible flow calculations, isentropic relations for an ideal gas are applied to relate total pressure, static pressure, and velocity at a pressure inlet boundary. The specified input of the total pressure, P_t, at the inlet and the static pressure, P_s, in the adjacent fluid cell are thus related as

$$\frac{P_t + P_{op}}{P_s + P_{op}} = \left(1 + \frac{(\gamma - 1)v^2}{2\gamma R T_s}\right)^{\gamma/\gamma - 1} \tag{5}$$

Note that operating pressure appears in the preceding equation because the given boundary condition inputs are in terms of pressure relative to the operating pressure. The preceding equation was used to compute the velocity magnitude of the fluid at the inlet plane with the given inlet total pressure and total temperature. The static temperature at the inlet, T_s, is computed from the given input of total temperature using the following relation:

$$\frac{T_t}{T_s} = 1 + \frac{\gamma - 1}{2} M^2 \tag{6}$$

The SRMs considered here are inert (i.e., no mass addition) and hence at the solid walls a no-slip boundary condition is imposed. The pre-designed CD nozzle exit boundary condition is prescribed. The Courant–Friedrichs–Lewy number is suitably chosen owing to the fact that the time step must be less than a certain time in many explicit time-marching computer simulations; otherwise the simulation will produce incorrect results. Ideal gas is selected as the working fluid. The transient mass addition due to propellant burning is deliberately suppressed in this model to examine the intrinsic

pre-ignition flow features discretely in SRMs. Figs. 1 (a), (b) shows the idealized physical models of SRMs with uniform and non-uniform port geometry with head-end igniter having five small openings for the hot jet to the combustion chamber. One jet is parallel to the axis, two jets are inclined with a jet impingement angle of 30° and the remaining two jets are perpendicular to the axis of the SRMs with divergent port.
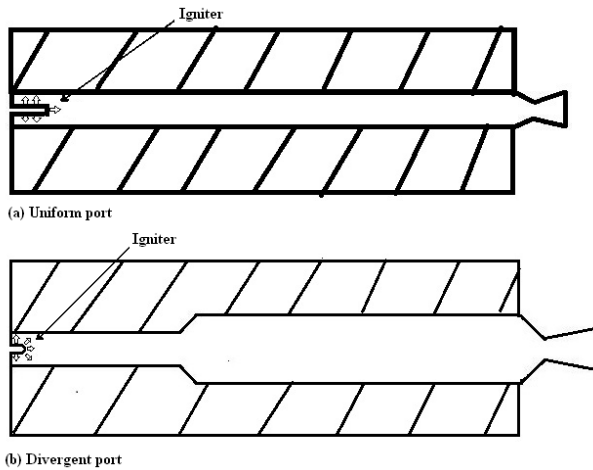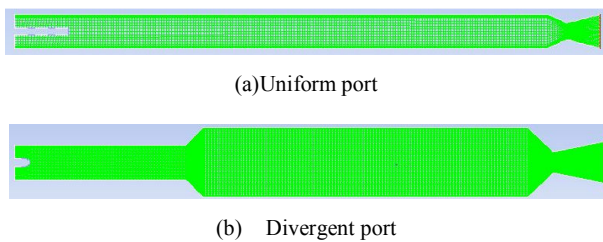

Fig. 1 (a), (b) Idealized physical models of SRMs

(a)Uniform port

(b)   Divergent port

Fig. 2 (a), (b) Grid systems in the computational domain of SRMs

An algebraic grid-generation technique is employed to discretize the computational domain. A typical grid system in the computational region is selected after the detailed grid refinement exercises. Figs. 2 (a), (b) shows the grid systems in the computational domain of SRMs. The grids are clustered near the solid walls using suitable stretching functions. The motor geometric variables and material properties are known *a priori*. In this study hot flow simulations have been carried out, with different igniter characteristics, in inert simulators of SRMs with two different port geometries, viz., uniform, and divergent grain geometry. In all the cases igniter location was fixed at the head-end. At the motor exit pre-designed shock free convergent-divergent (CD) nozzle is fixed for examining the choked flow conditions using an unsteady two dimensional *k-omega* turbulence model.

III.   RESULTS AND DISCUSSION

In the first phase of this study, the hot flow features are examined in SRMs at steady state conditions with two different port geometries and different igniter characteristics.

Figs. 3 (a)-(d) and 4 (a)-(d) are demonstrating the SRMs flow field at the steady state conditions. In both cases, inlet total pressure and total temperature are given as the input to the code. In the uniform port case, the port geometry (L/d = 16) is selected based on a typical SRM. The initial total pressure and temperature are specified as input to the code and a pressure is imposed at the exit. Except for the igniter characteristics, all other parameters are kept constant in the parametric studies. Fig. 5 shows the comparison of the non-dimensional wall temperature distribution along the lower wall of the SRMs with divergent port (l/D=7.5, L/d = 15, x/L = 0.33, divergent location) at the time of nozzle flow choking. It is evident from Fig. 5 that the ignition sequence will not be continuous in both cases due to the flow recirculation and reattachment due to the igniter jet inflow conditions and the motor port geometry. Fig. 7 is demonstrating the radial variations of Mach number during the flow choking time at seven different axial locations of an SRM with divergent port showing the reduction in axial Mach number component, possibly due to the swirling of flow.
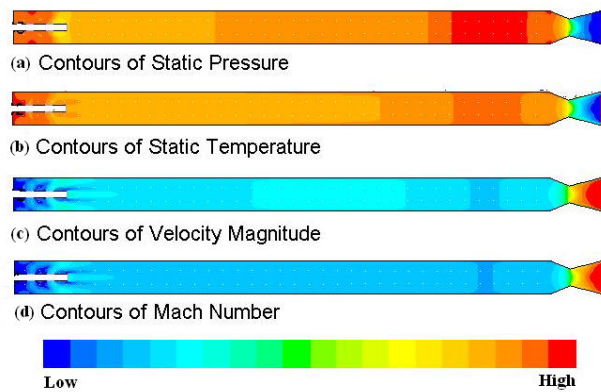

(a) Contours of Static Pressure

(b) Contours of Static Temperature

(c) Contours of Velocity Magnitude

(d) Contours of Mach Number

Low   High

Fig. 3 (a)-(d) Flow features of SRMs with uniform port

(a)   Contours of Static Pressure

(b)   Contours of Static Temperature

(c)   Contours of Velocity Magnitude

(d)   Contours of Mach Number

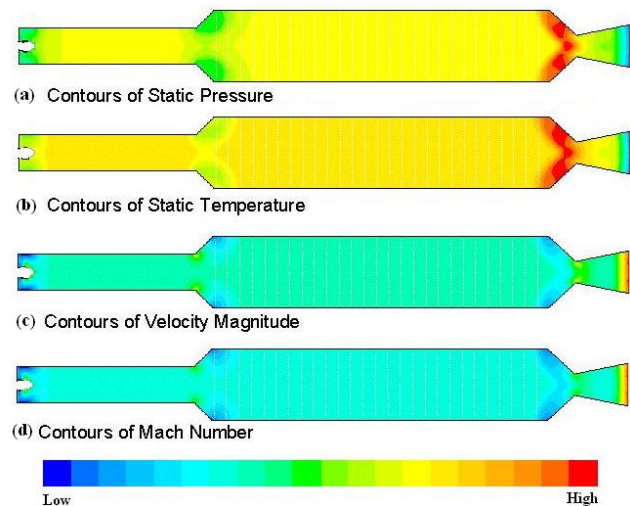Low   High

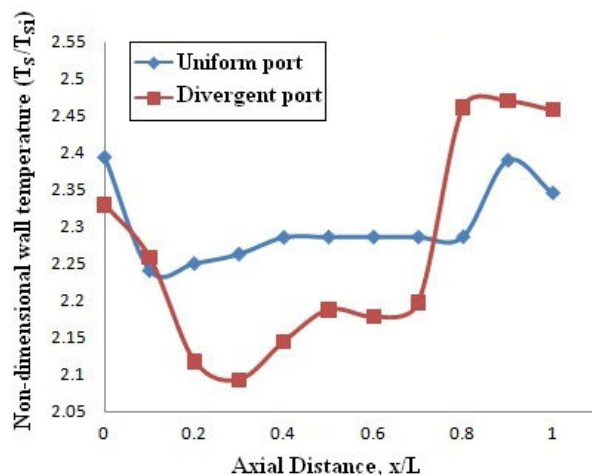Fig. 4 (a)-(d) Flow features of SRMs with divergent port

Fig. 5 Non-dimensional wall temperature distribution along the lower walls of the SRMs at the time of nozzle flow choking
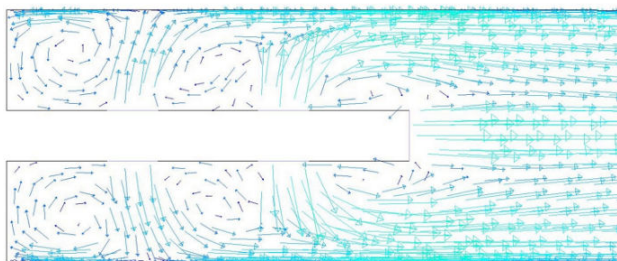

Fig. 6 Typical velocity vectors during the igniter jet flow to SRM with uniform port (enlarged view at the head-end)
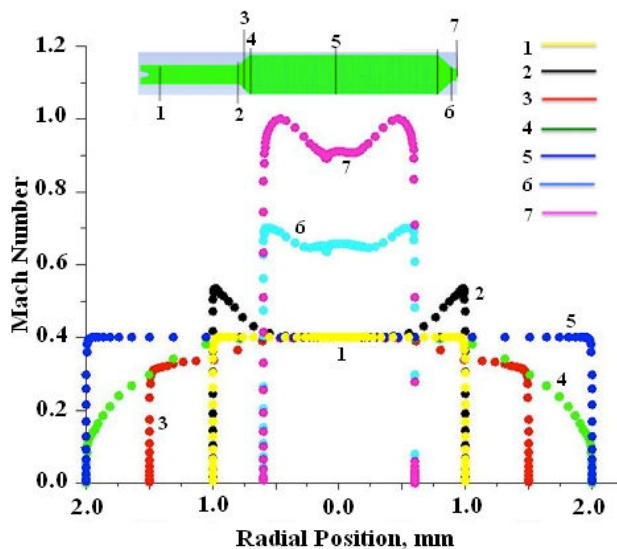

Fig. 7 Demonstrating the radial variations of Mach number during the flow choking time at seven different axial locations of an SRM with divergent port (see inset) showing the reduction in axial Mach number component

Fig. 8 shows that early choking of nozzle will take place at high turbulence intensity of the igniter jet flow for SRM with divergent port. Nevertheless, as reported earlier qualitatively [18], in the case of SRM with uniform port cases the delayed nozzle flow choking is observed at high turbulence intensities. This difference is obviously due to the different transient flow features of SRMs with uniform and non-uniform port geometries. Therefore, designer should not be under the impression that the low turbulence intensities will create a situation of early flow choking and thereby the development of full thrust and takeoff will be early.
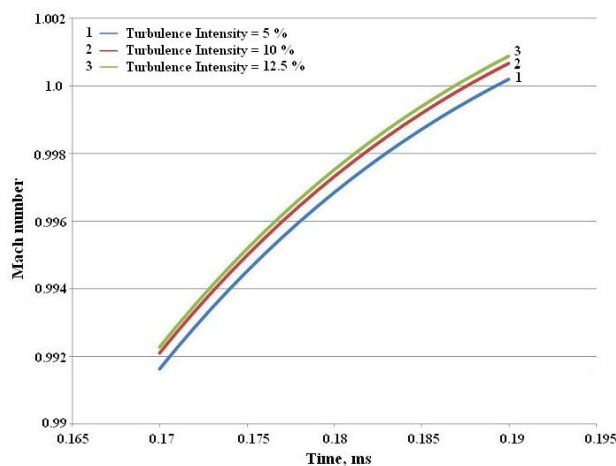

Fig. 8 Demonstrating the variations of nozzle flow choking time of SRM with divergent port at three different igniter jet turbulence intensities

Fig. 9 is demonstrating the variations of nozzle flow choking time of SRM with divergent port at two different igniter jet velocities (Vi=193m/s & 207m/s). It shows that at high igniter jet velocity early nozzle choking will be the result. In all the above cases igniter jet flow was selected as subsonic and choked flow condition was observed a few millimeters downstream of the geometrical throat of the nozzle due to the throat boundary layer effect. It may be noted that the above flow features will be altered in the case of reacting flow with swirl. As reported by the earlier investigator swirl is found to enhance supersonic mixing significantly, where swirl-induced vortices stir up and mix different regions of flow field [17], which is beyond the scope of this paper.
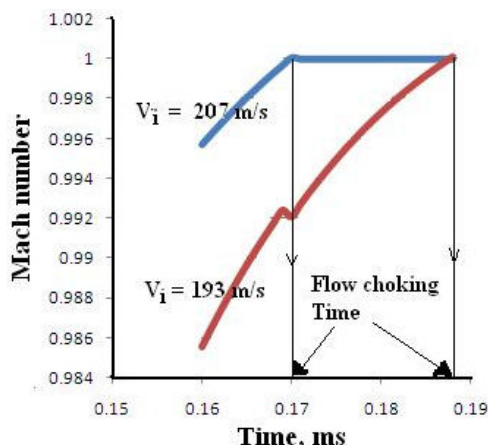
Fig. 9 Demonstrating the variations of nozzle flow choking time of SRM with divergent port at two different igniter jet velocities

We conjectured from the numerical results that the altered variations of the igniter jet impingement angle, turbulence level, time and location of the first ignition, flame spread characteristics, the overall chamber dynamics including the boundary layer growth history are having bearing on the time for nozzle flow chocking for establishing the required thrust for the rocket liftoff. We concluded that, in the light of the space debris, an error in predicting the liftoff time can lead to an unfavorable launch window amounts the satellite injection errors and/or the mission failures.

IV. Concluding Remarks

The action of the rocket engine's combustion chambers and expansion nozzles on a high pressure fluid is able to accelerate the fluid to extremely high speed, and conversely this exerts a large reactive thrust on the rocket, which propels the rocket forwards. In this paper parametric analytical studies have been carried out to examine the intrinsic flow physics pertaining to the nozzle flow choking time and also to the liftoff time of solid propellant rockets. We concluded from the numerical results that the altered variations of the igniter jet impingement angle, turbulence level, time and location of the first ignition, flame spread characteristics, the overall chamber dynamics including the boundary layer growth history are having bearing on the time for nozzle flow chocking for establishing the required thrust for the rocket liftoff. We further concluded that, if there is no efficient control and guidance system, a minor altered flow choking time of the strap-on motors with the pre-determined identical ignition time at the lift off phase will lead to the malfunctioning of the rocket. We also concluded that, in the light of the moving space debris, an error in predicting the liftoff time can lead to an unfavorable launch window amounts the satellite injection errors and/or the mission failures.

Acknowledgment

References

[1] V.R. Sanal Kumar and B. N. Raghunandan, Ignition Transient of Dual-Thrust Solid Propellant Rocket Motors - A Review, 48th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, Atlanta, USA, 30 July1-Aug.'12, Paper No. AIAA 2012-4043.

[2] V. R. Sanal Kumar, B. N. Raghunandan, T. Kawakami, H. D. Kim, T. Setoguchi, and S. Raghunathan, "Studies on Boundary Layer Blockage and Internal Flow Choking in Dual-thrust Motors," AIAA Journal of Propulsion and Power, Vol.24, No.2, March-April 2008.

[3] V. R .Sanal Kumar, B. N. Raghunandan, H. D. Kim, A. Sameen, T. Setoguchi, and S. Raghunathan, "Studies on Internal Flow Choking in Dual-thrust Motors," AIAA Journal of Spacecraft and Rockets, Vol.43, No.5, Sep-Oct 2006, pp.1139-1143.

[4] D. Rex, P. Eichler, U. Soppa, J. Zuschlag, A. Bade, Space debris — Origin, evolution and collision mechanics, Acta Astronautica, Vol. 20, 1989, pp. 209–216.

[5] R. Walker, R. Crowther, G.G. Swinerd, The long-term implications of operating satellite constellations in the low earth orbit debris environment, Advances in Space Research, Vol.19, Issue 2, 1997, pp. 355–358.

[6] R. Janovsky, I. Kalninsch, A. Stichtemath, The meteoroid and debris protection system for the ATV-spacecraft, Acta Astronautica, Vol. 47, Issues 2–9, July–November 2000, pp. 281–287.

[7] M. Arunkumar. M., and V. R. Sanal Kumar, "Studies on space debris tracking and elimination" 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, USA, July 2010, Paper No. AIAA 2010-7008.

[8] N. Naveen., Studies on Space Debris Tracking and Elimination, National Conference on Aerospace Sciences and Technology, Karunya University, Coimbatore, India, 14 September 2012.

[9] Kang, Jin S, Orbital debris mitigation using high density plasma, United States Patent, Pub. No.: US 2013/ 0001365 A, Jan. 3, 2013.

[10] E. L. Crosson, J. B. Romine, D. Willner, S. J. Kusiak, Boost-phase acceleration estimation, Radar Conference, 2000. The Record of the IEEE 2000 International 2000, pp. 210 – 214.

[11] N. J. Danis, Space-based tactical ballistic missile launch parameter estimation, Aerospace and Electronic Systems, IEEE Transactions on Aerospace and Electronics Systems, vol. 29, issue 2, 1993, pp. 412 – 424.

[12] M. Bussiere, and B. Mora, Ariane 5 booster nozzle: components description and dimensioning, Acta Astronautica, vol.34, pp.83-89, 1994.

[13] P. A. Park, Y. M. Choi, H. M. Choi, T. S. Cha, and B. H. Yoon, The evaluation of critical pressure ratios of sonic nozzles at low Reynolds numbers, Flow Measurement and Instrumentation, vol.12, pp.37-41, 2001

[14] Gany Alon, Mor Marat, and Goldman Claudio, Analysis and characteristics of choked swirling nozzle flows, AIAA journal, vol. 43, no. 10, pp. 2177-2181, 2005.

[15] I. G. Assovskii, and S. A. Rashkovskii, Low-frequency instability of solid rocket motors: influence of the Mach effect and charge geometry, Combustion, Explosion and Shock Waves, vol.37, 2001, pp.321-330.

[16] A. Alper OZALP, A computational study to predict the combined effect of surface roughness and heat flux conditions on converging nozzle flows, Transactions of the CSME/de la SCGM, Vol. 29, No.1, 2005.

[17] Ahmed Abdelhafez, Effect of swirl on the choking criteria, shock structure, and mixing in underexpanded supersonic nozzle airflows, Ph.D dissertation, Department of Mechanical Engineering University of Maryland, USA, 2009.

[18] M. Karuppasamy Pandian, K. Krishna Raj. K. Sabarinath, G. Sandeep, V. R. Sanal Kumar, "Influence of Port Geometry on Thrust Transient of Solid Propellant Rockets at Liftoff" International Conference on Aerospace, Mechanical, Automotive and Materials Engineering [ICAMAME 2012], Venice, Italy, November 14-16, 2012, World Academy of Science, Engineering and Technology, vol.71, 2012.

[19] D. C. Wilcox, Turbulence Modeling for CFD, DCW Industries, Inc., La Canada, CA, 1998.