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Controlled Synchronization of an Array of Nonlinear Systems with Time delays

S.M. Lee, J.H. Koo, J.H. Park, H.Y. Jung, S.C. Won

Abstract—In this paper, we propose synchronization of an array of nonlinear systems with time delays. The array of systems is decomposed into isolated systems to establish appropriate Lyapunov-Krasovskii functional. Using the Lyapunov-Krasovskii functional, a sufficient condition for the synchronization is derived in terms of LMIs(Linear Matrix Inequalities). Delayed feedback control gains are obtained by solving the sufficient condition. Numerical examples are given to show the validity the proposed method.

Keywords—Synchronization, Delay, Lyapunov method, LMI.

I. Introduction

Synchronization phenomena have attracted much attention of researchers in applied physics, biology, social sciences, engineering and interdisciplinary fields. Synchronization is a natural phenomenon, however, in some situations, a control system have to be added to obtain synchronization or a good transient performance. The synchronization obtained by using a control scheme is called controlled synchronization [1].

Many researchers have proposed synchronization techniques of two systems of master-slave scheme [2]-[4]. Recently, synchronization of arrays of coupled systems is of interest. The synchronization of more than two coupled systems has important applications, particularly, in mechanical systems. One important application is a mechanical system which behaves cooperatively to have flexibility and maneuverability, e.g., multifinger robot-hands, multirobot systems and multiactuated platforms.

Rodriguez-Angeles [5] proposed a synchronization technique for arrays of identical mechanical systems with partial measurement. This technique is complex significantly and parameter uncertainties of systems are not considered. To cope with the parameter uncertainties, Dong [6] studied an adaptive control architecture to synchronize two robots with kinematic constraints. Chung and Slotine [7] investigated a technique to synchronize Lagrangian systems which consist of a special network, so called two-way ring. This technique guarantees global exponential convergence, however, no extension to arbitrary network is considered.

J.H. Park is with Department of Electrical Engineering, Yeungnam University, 214-1 Dae-Dong, Kyongsan 712-749, Republic of Korea. Email: jessie@ynu.ac.kr. CORRESPONDING AUTHOR.

S.M. Lee is with Department of Electronic Engineering, Daegu University, Gyungsan 712-714, Republic of Korea. Email: moony@daegu.ac.kr

J.H. Koo and S.C. Won are with Department of Electronic and Electrical Engineering, POSTECH, San 31 Hyoja-Dong, Pohang 790-784, Korea

H.Y. Jung is with Department of Information and Communication Engineering, Yeungnam University, Republic of Korea.

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In this paper, we propose controlled synchronization for arbitrary arrays of nonlinear systems. Since time delay which exists in many applications is a natural phenomenon and often causes instability and poor performance of systems, the effect of time delay is considered. The array of systems is decomposed into isolated systems to establish appropriate Lyapunov-Krasovskii functional. Using the Lyapunov-Krasovskii functional, a sufficient condition for the synchronization is derived in terms of LMI (Linear Matrix Inequality). Delayed feedback control gains are obtained by solving the sufficient condition. Numerical examples are given to show the validity the proposed method.

The paper is organizes as follows. In section 2 we present a configuration of an array and the objective of synchronization. The sufficient condition for stable synchronization derived in LMI terms and delayed feedback gain is given in section 3. Numerical simulations are given in section 4 and the conclusion is presented in section 5.

II. Problem Statement

In this section, the configuration of an array of nonlinear systems is described and the controller is designed. The objective of control is presented based on the configuration of the array and the controller.

A. The array of systems

Consider the connection graph described in Fig. 1. The rectangular is the reference system and circles are nodes. The arrows represent the connection between nodes or between reference and node. All arrows have its directions and the direction of arrows means the flow of information. That is, bidirectional arrows describe nodes are coupled each other where unidirectional arrows describe reference systems only affect the other node. In this paper, it is assumed that the array is a full chain configuration which means there are no isolated nodes.

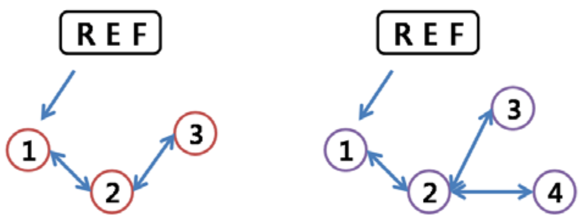

Fig. 1: Examples of an array of systems

B. The Synchronization Objective

Consider the following nonlinear systems consisting of an array with time delays :

$$\begin{aligned} \dot{x}_r(t) &= Ax_r(t) + f(x_r(t),t) + Bu, && (1)\\ \dot{x}_i(t) &= Ax_i(t) + f(x_i(t),t) + B(u+v_i), && (2) \end{aligned}$$

where $i = 1,\ldots,N$, $x_r(t) \in \mathbb{R}^n$ is the state vector of reference, $x_i(t) \in \mathbb{R}^n$ is the state vector of node i, $A \in \mathbb{R}^{n\times n}$ is a constant matrix, $f(\cdot)$ is a vector-valued time varying nonlinear function, $B \in \mathbb{R}^{n\times p}$ is a constant matrix, $u \in \mathbb{R}^p$ is a control input and $v_i \in \mathbb{R}^p$ is a coupling signal.

Assumption 1: The nonlinear function $f(\cdot)$ is bounded and satisfies the Lipschitz condition

$$\|f(x_r(t),t) - f(x_i(t),t)\| \le L_i\|x_r(t) - x_i(t)\| \quad (3)$$

where L_i is a non-negative constant.
The coupling signals which synchronize all nodes are defined as follows :

$$\begin{aligned} v_i &= Kc_{ir}(x_r(t-h) - x_i(t-h)) \\ &+ K\sum_{j=1,j\neq i}^{N} c_{ij}(x_j(t-h) - x_i(t-h)) \\ &= Kc_{ir}e_i(t-h) \\ &+ K\sum_{j=1,j\neq i}^{N} c_{ij}(e_i(t-h) - e_j(t-h)) \\ &= Kc_{ir}e_i(t-h) - K\sum_{j=1}^{N} c_{ij}e_j(t-h) \quad (4) \end{aligned}$$

where h is the time delay, e_i is the error between reference and node i, i.e. $e_i(t) = x_r(t) - x_i(t)$, $C = (c_{ij})_{N\times N}$ is the coupling configuration of the array defined as following : if there is a connection between node i and $j (i \neq j)$, then $c_{ij} = c_{ji} = 1$, otherwise, $c_{ij} = c_{ji} = 0$ and $c_{ii} = -\Sigma_{j=1,j\neq i}^{N} c_{ij}, \quad i = 1,2,\ldots,N.$

Definition 1: The array of systems (1)-(2) is said to achieve asymptotic synchronization if

$$x_r(t) = x_1(t) = x_2(t) = \cdots = x_N(t), t \to +\infty. \quad (5)$$

According to the Definition 1, the synchronization is achieved if $e_i(t) \to 0, \forall i$. By subtracting (2) from (1), the following error dynamics can be obtained :

$$\begin{aligned} \dot{e}_i(t) &= Ae_i(t) + F_i(e_i(t)) - Bv_i \\ &= Ae_i(t) + F_i(e_i(t)) \\ &- BKc_{ir}e_i(t-h) + BK\sum_{j=1}^{N} c_{ij}e_j(t-h) \quad (6) \end{aligned}$$

where $F_i(e_i(t)) = f(e_i(t)+x_i(t)) - f(x_i(t))$. The system (6) can be rewritten in following form :

$$\dot{\mathbf{e}}(t) = A\mathbf{e}(t) + F(\mathbf{e}(t)) + BK\mathbf{e}(t-h)D^T \quad (7)$$

where $\mathbf{e}(t) = \begin{bmatrix} e_1(t) & e_2(t) & \cdots & e_N(t)\end{bmatrix}$, $F(\mathbf{e}(t)) = \begin{bmatrix} F_1(e_1(t)) & F_2(e_2(t)) & \cdots & F_N(e_N(t))\end{bmatrix}$ and the coupling matrix $D = C - diag\{c_{1r}, c_{2r}, \ldots, c_{Nr}\}$. Since the coupling matrix D is symmetric and irreducible, there exists a unitary matrix u such that

$$D = u\Lambda u^T, \quad uu^T = I \quad (8)$$

where $u = \begin{bmatrix} u_1 & u_2 & \cdots & u_N\end{bmatrix} \in \mathbb{R}^{N\times N}$, $\Lambda = diag\{\lambda_1, \ldots, \lambda_N\}$, λ_i is an eigenvalue of matrix D. Applying post-multiplication of u to the system (7) and defining $\mathbf{z}(t) \triangleq \mathbf{e}(t)u$, the system (7) becomes

$$\dot{\mathbf{z}}(t) = A\mathbf{z} + F(\mathbf{e})u + \mathbf{z}(t-h)\Lambda \quad (9)$$

and can be rewritten as

$$\dot{z}_i = Az_i(t) + g_i(t) + \lambda_i z_i(t-h), \quad i = 1,\ldots,N \quad (10)$$

where $g_i(t) \triangleq F(\mathbf{e}(t))u_i$. Thus, the synchronization problem of the array of nonlinear systems is transformed into stabilization problem of the corresponding individual error dynamics. The following lemmas are useful for deriving LMIs conditions of stability of the system (10).

Lemma 1: [8] For any constant matrix $W \in \mathbb{R}^{n\times n} > 0$, a scalar $\tau > 0$ and a vector function $e(t) : \mathbb{R} \to \mathbb{R}^n$ such that the following integration is well defined, then

$$\begin{aligned} &-\tau\int_{-\tau}^{0} \dot{x}^T(t+\xi)W\dot{x}(t+\xi)d\xi \le \\ &\begin{bmatrix} x^T(t) & x^T(t-\tau)\end{bmatrix}\begin{bmatrix} -W & W \\ W & -W\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau)\end{bmatrix}. \quad (11) \end{aligned}$$

III. Main Results

In this section, we derive an LMI condition for delay-dependent stability of error dynamics (10). Consider the assumption 1 to derive bounded condition of the nonlinear term $g_i(t)$. We have

$$\begin{aligned} \|g_i(t)\| &= \left\|\sum_{k=1}^{N}[f(x_r(t),t) - f(x_i(t),t)]u_{ik}\right\| \\ &\le \sum_{k=1}^{N}\|f(x_r(t),t) - f(x_i(t),t)\|\cdot|u_{ik}| \\ &\le \sum_{k=1}^{N} L_k\|x_r(t) - x_i(t)\| = \sum_{k=1}^{N} L_k\|e_k(t)\| \\ &= \sum_{k=1}^{N} L_k\|z(t)u_k^T\| \le \sum_{k=1}^{N} L\|z_k(t)\| \quad (12) \end{aligned}$$

where u_{ik} is k-th element of u_i and $L = \max(L_k)$.
We define augmented vectors for $i = 1,\ldots,N$:

$$\zeta_i(t) = \begin{bmatrix} \dot{z}_i(t) & z_i(t) & z_i(t-h) & g_i(t)\end{bmatrix}. \quad (13)$$

Theorem 1: The error dynamics (10) is asymptotically stable for given time delay h and a scalar ϵ if there exist matrices $P_i > 0, Q_i > 0, R_i > 0, Y \in \mathbb{R}^{n\times n}$, positive constants α_i satisfying the following LMI:

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \star & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ \star & \star & \Phi_{33} & \Phi_{34} \\ \star & \star & \star & \Phi_{44}\end{bmatrix} < 0, \ i = 1,\ldots,N. \quad (14)$$

where

$$
\begin{aligned}
\Phi_{11} &= h\bar{R}_i + \epsilon(\bar{Y} + \bar{Y}^T), \quad \Phi_{12} = \bar{P}_i - \epsilon A\bar{Y} + \bar{Y}^T, \\
\Phi_{13} &= -\epsilon\lambda_i B\bar{K}, \quad \Phi_{14} = -\epsilon Y^T, \\
\Phi_{22} &= \bar{Q}_i - A\bar{Y} - \bar{Y}^T A^T - \frac{1}{h}\bar{R}_i + NL\bar{S}_i, \\
\Phi_{23} &= -\lambda_i B\bar{K} + \frac{1}{h}\bar{R}_i, \quad \Phi_{24} = -Y^T, \\
\Phi_{33} &= -\bar{Q}_i - \frac{1}{h}\bar{R}_i, \quad \Phi_{34} = 0, \Phi_{44} = -\bar{S}_i, \\
\bar{Y} &= Y^{-1}, \quad \bar{P}_i = \bar{Y}^T P_i \bar{Y}, \quad \bar{Q}_i = \bar{Y}^T Q_i \bar{Y}, \\
\bar{R}_i &= \bar{Y}^T R_i \bar{Y}, \quad \bar{S}_i = \alpha_i \bar{Y}^T \bar{Y}, \quad \bar{K} = K\bar{Y}. \qquad (15)
\end{aligned}
$$

Moreover, the delayed feedback control gain is obtained as

$$K = \bar{K}\bar{Y}^{-1}. \qquad (16)$$

Proof. Consider a Lyapunov-Krasovskii functional such as

$$V(z(t)) = \sum_{i=1}^{N} V_i(z_i(t)) \qquad (17)$$

where

$$
\begin{aligned}
V_i(z_i(t)) &= z_i^T(t)P_i z_i(t) + \int_{t-h}^{t} z_i^T(s)Q_i z_i(s)ds \\
&+ \int_{-h}^{0}\int_{t+\theta}^{t} \dot{z}_i^T(s)R_i\dot{z}_i(s)dsd\theta. \qquad (18)
\end{aligned}
$$

The derivative of $V_i(z_i(t))$ is shown as follows :

$$
\begin{aligned}
&\dot{V}_i(z_i(t)) = 2z_i^T(t)P_i\dot{z}_i(t) + z_i^T(t)Q_i z_i(t) \\
&- \; z_i^T(t-h)Q_i z_i(t-h) + h\dot{z}_i^T(t)R_i\dot{z}_i(t) \\
&- \; \int_{t-h}^{t} z_i^T(s)R_i z_i^T(s)ds \\
&\leq \; 2z_i^T(t)P_i\dot{z}_i(t) - z_i^T(t-h)Q_i z_i(t-h) \\
&+ \; z_i^T(t)Q_i z_i(t) + h\dot{z}_i^T(t)R_i\dot{z}_i(t) \\
&+ \; \frac{1}{h}\begin{bmatrix} z_i^T(t) \\ z_i^T(t-h)\end{bmatrix}^T \begin{bmatrix} -R_i & R_i \\ R_i & -R_i\end{bmatrix}\begin{bmatrix} z_i(t) \\ z_i(t-h)\end{bmatrix} \\
&= \; \zeta_i^T(t)\begin{bmatrix} hR_i & P_i & 0 & 0 \\ \star & Q_i - \frac{1}{h}R_i & \frac{1}{h}R_i & 0 \\ \star & \star & -Q_i - \frac{1}{h}R_i & 0 \\ \star & \star & \star & 0\end{bmatrix}\zeta_i(t) \\
&\triangleq \; \zeta_i^T(t)\Omega_{i1}\zeta_i(t). \qquad (19)
\end{aligned}
$$

We can obtain following equality constraint from Eq. (10) [11]:

$$
\begin{aligned}
&2\begin{bmatrix}\dot{z}_i^T(t)\epsilon Y^T & z_i^T(t)Y^T\end{bmatrix} \\
&\quad \times \begin{bmatrix}\dot{z}_i(t) - Az_i(t) - g_i(t) - \lambda_i z_i(t-h)\end{bmatrix} = 0 \qquad (20)
\end{aligned}
$$

for any appropriate dimension matrix Y and any constant scalar ϵ. The equality constraint (20) can be represented by augmented vector $\zeta_i(t)$ as

$$
\begin{aligned}
&\zeta_i^T(t)\begin{bmatrix} \epsilon(Y+Y^T) & -\epsilon Y^T A + Y & -\epsilon\lambda_i Y^T BK & -\epsilon Y^T \\ \star & -Y^T A - A^T Y - \frac{1}{h}R_i & -\lambda_i Y^T BK & -Y^T \\ \star & \star & 0 & 0 \\ \star & \star & \star & 0 \end{bmatrix}\zeta_i(t) \\
&\triangleq \zeta_i^T(t)\Omega_{i2}\zeta_i(t) \qquad (21)
\end{aligned}
$$

Now, to deal with the nonlinearities, the following inequality from (12) has to be considered :

$$
\begin{aligned}
&\sum_{i=1}^{N}\left(\|g_i(t)\| - \sum_{k=1}^{N} L\|z_k(t)\|\right) \\
&\quad = \sum_{i=1}^{N}(\|g_i(t)\| - NL\|z_i(t)\|) < 0 \qquad (22)
\end{aligned}
$$

and this inequality holds if following inequalities are all satisfied :

$$\|g_i(t)\| - NL\|z_i(t)\| < 0, \quad i = 1, \ldots, N. \qquad (23)$$

The inequalities (23) are placed to each Lyapunov-Krasovskii functional $V_i(z_i(t))$ so that

$$\zeta_i^T(t)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -NLI & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}\zeta_i(t) = \zeta_i^T(t)\Omega_{i3}\zeta_i(t) \leq 0, \qquad (24)$$

for $i = 1, \ldots, N$.
By applying S-procedure [9] and utilizing (19), (21) and (24), we have following inequalities :

$$
\begin{aligned}
\dot{V}_i(z_i(t)) &\leq \zeta_i^T(t)(\Omega_{i1} + \Omega_{i2} - \alpha_i\Omega_{i3})\zeta_i(t) \\
&\triangleq \zeta_i^T(t)\Omega_i\zeta_i(t), \quad i = 1, \ldots, N \qquad (25)
\end{aligned}
$$

where α_i is a positive constant. It is obvious that if the matrix Ω_i is negative definite for all i, the error dynamics (10) is asymptotic stable. However, it is not easy to solve the sufficient condition (25) and to find the control gain K because it is not a standard LMI form. To represent the sufficient conditions in LMI terms, pre and post multiply the matrix $diag\{\bar{Y}, \cdots, I\}$ to both side of matrix Ω_i as following :

$$
\begin{aligned}
&\begin{bmatrix} \bar{Y} & & & \\ & \bar{Y} & & \\ & & \bar{Y} & \\ & & & I \end{bmatrix}^T \Omega_i \begin{bmatrix} \bar{Y} & & & \\ & \bar{Y} & & \\ & & \bar{Y} & \\ & & & I \end{bmatrix} = \\
&\begin{bmatrix} h\bar{Y}^T R_i\bar{Y} + \epsilon(\bar{Y} + \bar{Y}^T) & \bar{Y}^T P_i\bar{Y} - \epsilon A\bar{Y} + \bar{Y}^T & \epsilon\lambda_i BK\bar{Y} & -\epsilon I \\ \star & \bar{Y}^T Q_i\bar{Y} - A\bar{Y} - \bar{Y}^T A^T - \frac{1}{h}\bar{Y}^T R_i\bar{Y} + \alpha_i NL\bar{Y}^T\bar{Y} & \lambda_i BK\bar{Y} + \frac{1}{h}\bar{Y}^T R_i\bar{Y} & -I \\ \star & \star & -\bar{Q}_i - \frac{1}{h}\bar{Y}^T R_i\bar{Y} & 0 \\ \star & \star & \star & -\alpha_i I \end{bmatrix}, \qquad (26)
\end{aligned}
$$

then the inequality (26) is equivalent to (14) and guarantees the asymptotical stability. ■

IV. NUMERICAL EXAMPLE

In this section, two examples are used to illustrate the validity of the proposed method given in Theorem 1.

Example 1: [10] Consider an array of 5 linear systems where each node is a three-dimensional linear systems described by

$$\begin{bmatrix}\dot{x}_{i1}\\ \dot{x}_{i2}\\ \dot{x}_{i2}\end{bmatrix}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}\begin{bmatrix}x_{i1}\\ x_{i2}\\ x_{i3}\end{bmatrix}+\begin{bmatrix}1\\1\\1\end{bmatrix}(u+v_i)$$

and

$$\begin{aligned} D &= C - diag\{c_{1r}, c_{2r}, \cdots, c_{Nr}\}\\ &= \begin{bmatrix}-2 & 1 & 0 & 0 & 1\\ 1 & -3 & 1 & 1 & 0\\ 0 & 1 & -2 & 1 & 0\\ 0 & 1 & 1 & -3 & 1\\ 1 & 0 & 0 & 1 & -2\end{bmatrix} - \begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}.\end{aligned}$$

The matrix D is irreducible and its eigenvalues are $\lambda_i = -4.8150, -3.6728, -2.7995, -1.5714, -0.1414$.
Time delay h and a constant ϵ is given by $h = 0.5$, $\epsilon = 0.1$. The initial state values are randomly selected and the control input u is chosen as $u(t) = \sin(t)$. By solving the LMI in Theorem 1, one can obtain the delayed feedback gain $K = [0.0411 \quad 0.0248 \quad 0.0216]$. The synchronization error is shown in Figs. 2-3.

Example 2: Consider an array of 3 nonlinear systems:

$$\begin{bmatrix}\dot{x}_{i1}\\ \dot{x}_{i2}\end{bmatrix}=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}\begin{bmatrix}x_{i1}\\ x_{i2}\end{bmatrix}+f(x)+\begin{bmatrix}0\\1\end{bmatrix}(u+v_i)$$

and

$$\begin{aligned} D &= C - diag\{c_{1r}, c_{2r}, \cdots, c_{Nr}\}\\ &= \begin{bmatrix}-1 & 1 & 0\\ 1 & -2 & 1\\ 0 & 1 & -1\end{bmatrix} - \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\end{aligned}$$

where $f(x) = [\frac{1}{15}\tanh(x_1) \quad 0]^T$. The Lipschitz constant is $L = \frac{1}{15}$. Time delay h and a constant ϵ is given by $h = 0.5$, $\epsilon = 0.5$. The initial state values are randomly selected and the control input u is chosen as $u(t) = \sin(t)$. By solving LMI, one can obtain the delayed feedback gain $K = [0.0494 \quad 0.2176]$. The synchronization error is shown in Figs. 4-5.

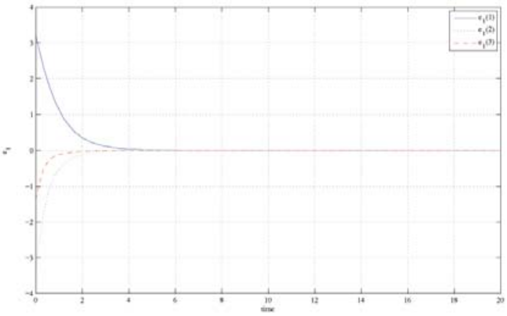

Fig. 2: Synchronization errors between reference and the node $i = 1$

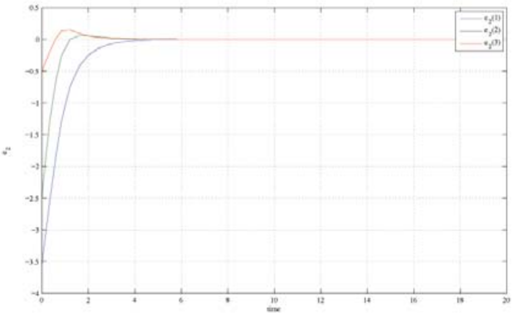

Fig. 3: Synchronization errors between reference and the node $i = 2$

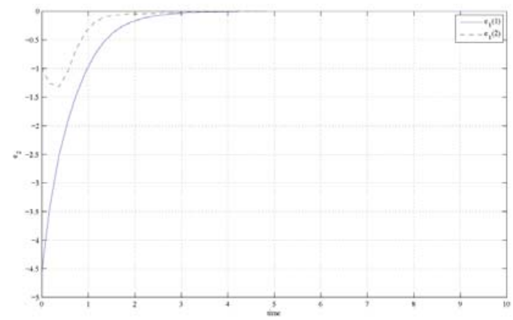

Fig. 4: Synchronization errors between reference and the node $i = 1$

V. CONCLUSIONS

In this paper, we proposed a synchronization technique for an array of nonlinear systems in the presence of time delays. The nonlinearity of the systems is dealt with Lipschitz condition and employed to equality constraints using S-procedure. The sufficient condition in LMI terms and delayed feedback control gain is obtained. Through two numerical examples, we have demonstrated the validity of the proposed methods.

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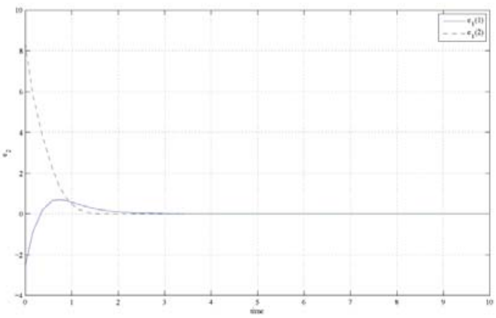

Fig. 5: Synchronization errors between reference and the node $i = 2$

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