Analysis of Heart Beat Dynamics through Singularity Spectrum

Harish Kumar, Hussein Yahia, Oriol Pont, Michel Haissaguerre, Nicolas Derval and Meleze Hocini

Abstract—The analysis to detect arrhythmias and life-threatening conditions are highly essential in today world and this analysis can be accomplished by advanced non-linear processing methods for accurate analysis of the complex signals of heartbeat dynamics. In this perspective, recent developments in the field of multiscale information content have lead to the Microcanonical Multiscale Formalism (MMF). We show that such framework provides several signal analysis techniques that are especially adapted to the study of heartbeat dynamics. In this paper, we just show first hand results of whether the considered heartbeat dynamics signals have the multiscale properties by computing local preticability exponents (LPEs) and the Unpredictable Points Manifold (UPM), and thereby computing the singularity spectrum.

Keywords—Microcanonical Multiscale Formalism (MMF), Unpredictable Points Manifold (UPM), Heartbeat Dynamics.

I. INTRODUCTION

THE human heartbeat is governed by the autonomic nervous system, and as a consequence, heart rate and heartbeat dynamics extracted from the ECG are important quantitative markers of cardiovascular control [1]. The complex synchronization process between pacemaker cells results in cardiac rhythm and, as a consequence, the heartrate exhibits small chaotic fluctuations. Typically, the amplitude of such fluctuations is much smaller than the average interbeat interval, something that makes the healthy (sinus rhythm) heartbeat appear as mainly periodic. Nevertheless, the fluctuations around this main period are not an unstructured random noise but follow a complex dynamics. Even more, the characterization of these fluctuations is vital for determining whether the heart is healthy or it is indicating signs of a transition to an arrhythmia, despite still appearing regular [2, 3, 4].

Klabunde [5], has emphasized that the human heart is structurally complex and, as a consequence, the electrical activity in it is also complex. In the past decade, many analyses [3, 6-11] have been carried out to characterize the statistical features of human heartbeat dynamics. In these studies, possible different statistical features of Heart beat dynamics in different physiological states have been reported. In particular, an intriguing finding is the multiscale properties in healthy heartbeat and the loss of this multiscale character in pathological hearbeat in patients with congestive heart failure [2]. Such multiscale complexity in healthy Heart beat was further shown to be related to the intrinsic properties of the

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control mechanisms in human heartbeat dynamics and is not simply due to changes in external stimulation and the degree of physical activity [8].

Ivanov [2], in this study of multiscale properties in healthy heart beat is based solely on the Legendre spectrum, and so it corresponds to a canonical formalism, not microcanonical, from a thermodynamic point of view. The multiscale structure observed in heartbeat is a result of a synchronization process in a hierarchic complex network made of cardiac pacemaker cells [12]. As a consequence, the Microcanonical Multiscale Formalism (MMF) [13, 14] is especially appropriate for analyzing this dynamical structure. In particular, an analysis based on the local predictability exponents (LPEs) [15, 16] and the optimal wavelet [17] of heartbeat time series allows directly accessing the geometric features that characterize their multiscale behavior.

The results obtained in [2] are based on a canonical analysis, meaning that the behavior of statistical averages is used to indirectly retrieve the geometric features: scaling exponents of partition functions estimate a curve that can be used to obtain the so-called singularity spectrum by means of a numerically- estimated Legendre transform. This methodology is known to give less accurate estimation on the tails of the singularity spectrum for which a microcanonical analysis has been found to be much more robust and accurate [18]. Having such estimation has a capital importance for anticipating as much as possible when heartbeat dynamics starts drifting from the healthy behavior. Given the quickness with which heart failure can be fatal or leave irreversible after-effects, the precise estimation provided by the MMF has a strong potential in helping to save lives and improve the health of people with cardiac diseases.

The paper is structured as follows: in the next section we introduce the empirical data to be analyzed. In section 3, we introduce the basics of the Microcanonical Multiscale Formalism (MMF) and the methods to accurately retrieve the empirical local predictability exponents (LPEs) from a signal. In section 4 we discuss the experimental results for the considered heartbeat data and consequently discuss the singularity spectrum of these heartbeat data. Finally, in section 5 we draw the conclusions of our work.

II. EMPIRICAL DATA

We have processed electrocardiogram signals together with endocardial potential measured through electrodes in catheters introduced in the heart of a patient and for a regime: sinus rhythm, which contains 21 channels of data: 4 of these are

from electrocardiogram electrodes measuring the potential on the skin (I, II, III and V1) and the other 17 are measured through three catheters (two from the radio-frequency catheter, a catheter of 5 electrodes and another catheter of 10 electrodes). For the sake of simplicity in this paper, the signals corresponding to electrocardiogram electrodes are named as ecg. The three catheters are named as c1, c2 and c3 respectively. Hence the ecg has 4 channels (I, II, III and V1), c1 has 2 channels, c2 has 5 channels and c3 has 10 channels. All of the measures are electric potential differences, and all of them are bipolar except for the V1 which is unipolar. These are sampled at a rate of 1 kHz.

Patient details : 50 year old man with expanded heart and persistent fibrillation (recurrent episodes that last more than 7 days). Measurements are done in left and right atrial appendages and left superior pulmonary vein.

III. The Microcanonical Multiscale Formalism (MMF)

The MMF is a theoretical and methodological framework for the analysis of multiscale signals. Its basic element of description is by means of LPEs of a signal under analysis, which are the exponents describing the local regular/singular behavior of the signal around each point.

A. local predictability exponents (LPEs)

Local predictability exponents have different mathematical definitions depending on the context they are used. The usual notion in complex-signal analysis is related to the Hölder or Hurst exponents, including their respective generalizations. Although different definitions are possible, the conceptual goal is always the same: to describe how the function evolves around a given point by converging to a value (regular) or diverging (singular).

In the most general case, given a signal s that is defined on $\mathbb{R}^d$ domain, the Hölder exponent $h(x)$ of point x is the exponent satisfying the following limit, when it exists [19]:

$$\|s(x+r)-s(x)\| = \alpha(x)\|r\|^{h(x)}+o(\|r\|^{h(x)}) \quad (r \to 0) \quad (1)$$

This means that in the proximity of x the signal follows a power law of exponent $h(x)$. An alternative definition that analytically is slightly more restrictive is usually called the Hurst exponent [20, 21] and defined as $s(x + r) - s(x) = \langle\alpha(x)|r\rangle r^{h(x)-1} + o(r^{h(x)})$ where $\alpha(x)$ is a continuous $(1, 1)$ tensor. For the purpose of this article, analysis of 1D signals of 1 component the definitions actually coincide.

The concept of LPE can be interpreted also in terms of differentiability. A function that is strictly $n-$derivable at point x has a LPE $h(x) = n$. So that in this sense the LPE can be related to non-integer differentiability. In a similar way, as we will see below, it is also related to the content of information.

Nevertheless, Hölder or Hurst exponents defined this way have very specific applicability (e.g., in the case of multiaffine functions) and cannot be directly found in real-world signals. The main reason is that the basic power-law behaviour is masked by the presence of long-range correlations, noisy fluctuations, discretization and finite-size effects. All these

make that the analytical limit described is not practically attainable [22, 14], and a generalized definition of LPE is needed. To achieve this, the objective is to find a certain measure μ for which we could take a similar limit:

$$\mu(B_r(x)) = \alpha(x)r^{d+h(x)} + o(r^{d+h(x)}) \quad (r \to 0) \quad (2)$$

where d is the dimension of the domain, i.e., $d = 1$ in the 1D case, and $B_r(x)$ is a ball centered around x having a radius r for a certain norm (choice to be done for multi-dimensional cases; they all coincide in 1D).

The actual definition of LPE that we will be using in the present article works well in practice and is little affected by the artifacts mentioned above. For it, we will work on the gradient-modulus measure of the signal [22]. This measure is defined from its density:

$$d\mu(x) = \|\nabla s\|(x)dx \quad (3)$$

a definition that is absolutely continuous with respect to the Lebesgue measure. Hence, the measure of any Borelian A is given by:

$$\mu(x) = \int_A dx\|\nabla s\|(x) \quad (4)$$

The gradient-modulus measure characterizes the local singularity of any point. A signal that has a Hölder exponent $h(x)+1$ according to equation (1) will fulfil also equation (2), with this $+1$ shift. Practical calculations of equation (2) can benefit from using wavelet-projected interpolations, this way effectively avoiding some of the discretization effects [23]. The wavelet projection of the measure at point x and scale r is expressed as $\tau_\Psi\mu(x,r) = \int_{\mathbb{R}^d} d\mu(x')r^{-d}\Psi(\frac{(x-x')}{r})$ with Ψ being a predetermined function known as the mother wavelet. As we can see, the operator τ_Ψ is a map from the set M of $\sigma-$finite measures on $\mathbb{R}^d$ to the set of functions $\mathbb{R}^d\times\mathbb{R}^+ \to \mathbb{R}$. A signal that has a LPE at the point x according to equation (2) exhibits this same exponent when wavelet-projected [3, 22], i.e,

$$\tau_\Psi\mu(x,r) = \alpha_\Psi(x)r^{h(x)} + o(r^{h(x)}) \quad (r \to 0) \quad (5)$$

It is worth mentioning that wavelet projections expressed in this way treat the wavelet function as a kernel for the measure and no additional restriction is imposed. This way, we are not limited to use only admissible wavelets (i.e., wavelets can reconstruct the signal). In particular, we can use always positive kernels that do not have zero-crossings. High-order wavelets that exhibit several zero-crossings have a significant loss in spatial resolution [18], but positive kernels minimize spatial spread and can normally reach the original resolution, that is, one sample in the original signal.

B. General Conditions for UPM-measure

The basic requirement to define a singular positive UPM-measure is that it is concerned with the local singular behaviour of functions. The best way to define UPM-measures is as vectorial wavelet projections of standard gradient measures. So, the UPM-measure is a carefully designed vectorial wavelet

projection of the gradient measure so that it penalizes unpredictability. In our method, in contrast with standard singularity analysis, we will not perform many wavelet projections of the UPM measure in order to extract the LPE by means of a log-log regression. Wavelet projecting the measure at several scales is costly in computer time and only serves to enhance the resolution of less singular structures at the cost of coarsening most singular ones [18]. But as we are mainly interested in the most singular structures, it is hence harmful to our interests to project across multiple scales. Instead, we will make use of point estimates [18, 13, 27] of the LPEs, namely:

$$h(x) = \frac{\log(\frac{\tau_\Psi \mu(x, r_0)}{\langle \tau_\Psi \mu(., r_0)\rangle})}{\log r_0} + o(\frac{1}{\log r_0}) \qquad (6)$$

Where $\langle \tau_\Psi \mu(., r_0)\rangle$ is the average value of the wavelet projection over the whole signal and serves to diminish the relative amplitude of the $o(\frac{1}{\log r_0})$ correction. When applying equation 6, we will need that r_0 is small enough to neglect this correction. The scale r_0 will be defined as the smallest accessible one, i.e., one sample scale. We conventionally assign a Lebesgue measure of 1 to the whole space domain, so for a N samples per signal, the value of r_0 is fixed to $r_0 = \frac{1}{\sqrt{N}}$, so in general we need that signals are large enough to make the first term in the right hand side of equation 6, a good approximation of the LPE.

C. Singularity spectrum

A multiscale signal is a co-ordinated ensemble of its fractal components $\mathcal{F}_h$. The fractal dimensions $D(h)$ are the characterization of these fractal components [24], and the whole set of these fractal dimensions forms the so-called singularity spectrum. The singularity spectrum plays a central role in the description of the scaling properties of a multifractal system [25], because it is directly linked to the statistical properties of the system through the famous Parisi-Frisch's formula [26]. In fact, the empirical histogram of singularities $\rho(h)$, when the LPEs are evaluated at the resolution scale r_0, has a simple relation with the singularity spectrum [24]:

$$\rho(h) = A_\rho r_0^{d - D(h)} \qquad (7)$$

where d is the dimension of the signal domain. If the signal has total support (i.e., non-fractal, the common case with real signals) then the support of the function $h(x)$ is also total and has dimension d. So there must exist a fractal component of such a dimensionality, i.e., there is a value of LPE h_1 such that $D(h_1) = d$, and this value necessarily corresponds to the mode (the most probable value) of $\rho(h)$ [18,22,25]. Therefore, when the histogram is normalized by its mode, the proportionality constant A_ρ is removed and so we can retrieve the dimensionless, scale-invariant quantity $D(h) - d$ in the way:

$$D(h) - d = -\frac{\log(\frac{\rho(h)}{\rho(h_1)})}{\log r_0} \qquad (8)$$

for any resolution scale r_0 at which the LPEs are evaluated. $D(h) - d$ is called reduced singularity spectrum, and it is independent of the dimensionality d.
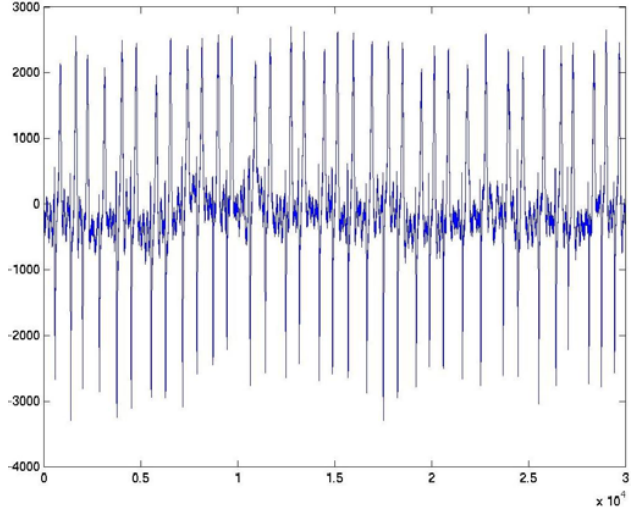

Fig. 1: Channel 1 of ecg signals

D. General Conditions for MMF

So far, the two main elements of MMF (singularity analysis and singularity spectrum) are introduced. The requirements on a real signal in order to accept that MMF holds for it, is as formalize:

(i) For any point x, equation 5 is verified over a large enough range of scales.

(ii) The distribution of LPEs at any valid scale r_0 follows equation 7 with the same curve $D(h)$.

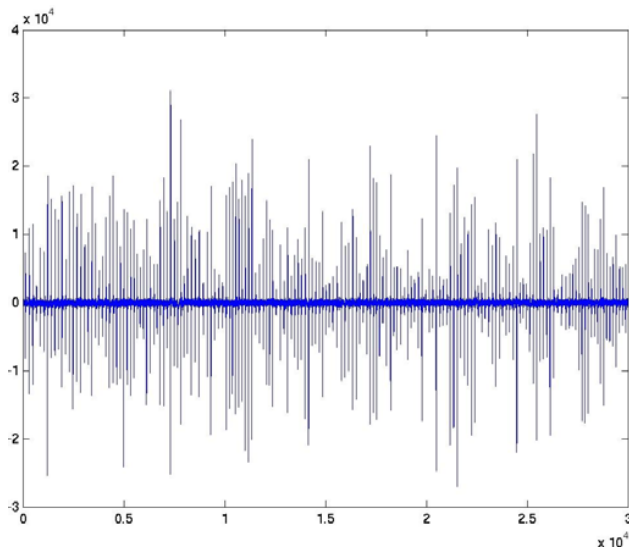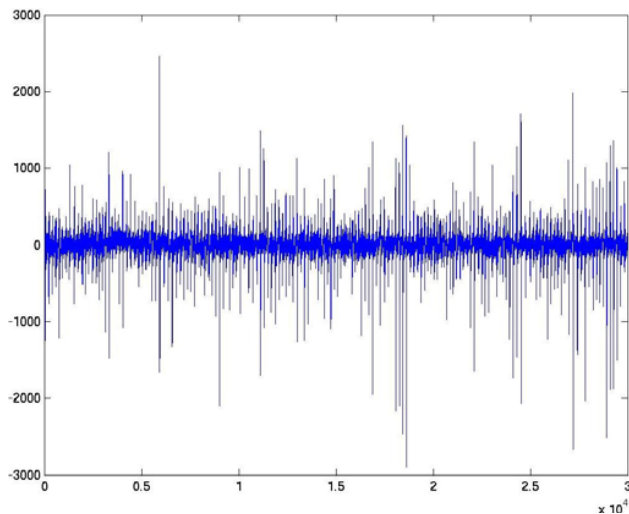(iii) The curve $D(h)$ derived from equation 8 is convex.

These conditions and what they imply are discussed in detail in [13,14]. For the context of this paper, we will just verify the extent of validity of conditions (i), (ii) and (iii) for the heartbeat data. The main virtue of MMF is that it allows an explicit geometrical determination of the fractal components [9]. Hence, the retrieval of the singularity spectrum $D(h)$ by means of equation 8 is more direct than in classical multifractal techniques and less demanding in data size [18].

IV. Experimental Results

The Heartbeat signals considered is 21 channels, as explained before in the section II. Here in this paper, the analysis is depicted only for 4 channels, one channel each from ecg, $c1$, $c2$ and $c3$ respectively. The raw signals recorded from the patient (details in section II), for ecg, $c1$, $c2$ and $c3$ are shown in the figure 1 -4 respectively. The channel 1 is considered of ecg, channel 2 of the catheter $c1$ ($c1$ consists of 2 channels), channel 3 of the catheter $c2$ ($c2$ consists of 5 channels) and channel 4 of the catheter $c3$ ($c3$ consists of 10 channels)
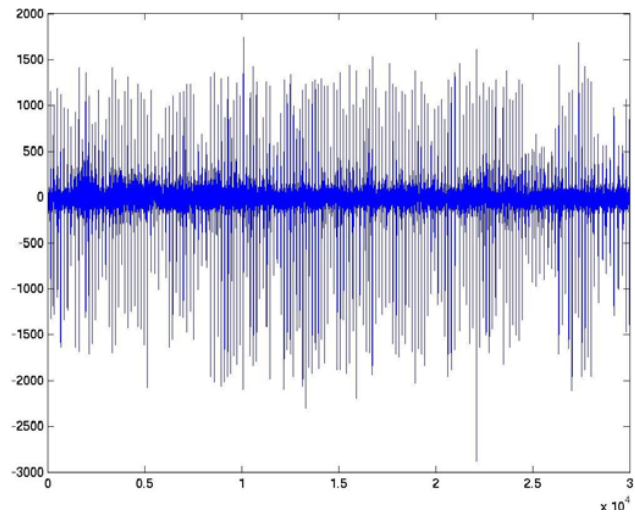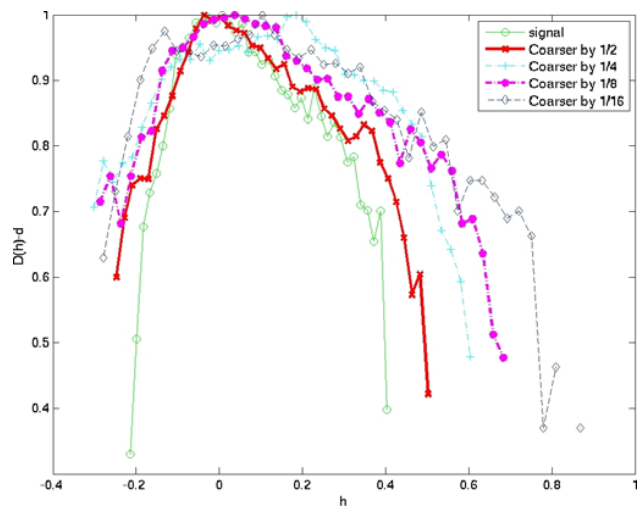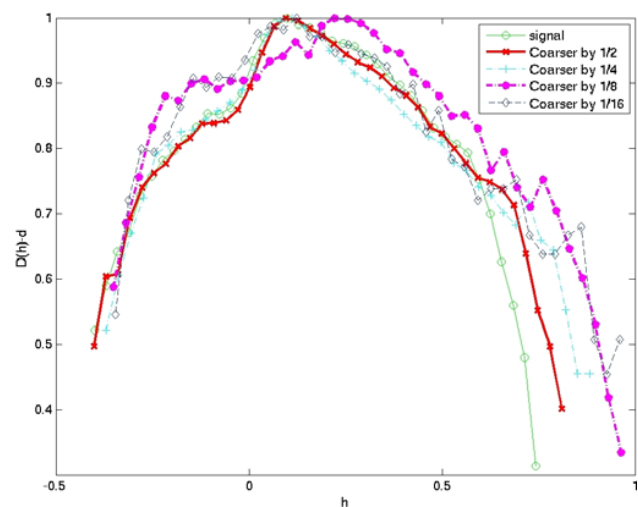
A. Validity of MMF

In order to validate the datasets under study i.e., heartbeat signals, the three requirements presented in sub-section D of the section III must be fulfilled. Regarding condition (i), we must check the range over which equation 5 can be considered

Fig. 2: Channel 2 of $c1$ catheter

Fig. 4: Channel 4 of $c3$ catheter

Fig. 3: Channel 3 of $c2$ catheter

Fig. 5: Singularity Spectrum for channel 1 of ecg signals

Fig. 6: Singularity Spectrum for channel 2 of $c1$ catheter

to fit our data, taking into account the noise, discretization and finite-size effects impose experimental bounds to the validity of this equation. As we are interested in retrieving the LPE, $h(x)$ at each point x over a large enough range of scales, and we have been successful in retrieving this $h(x)$ over 5 different scales, i.e., at the scale of the signal, at $\frac{1}{2}$ the scale of the signal, $\frac{1}{4}$ scale of the signal, $\frac{1}{8}$ scale of the signal, and finally $\frac{1}{16}$ scale of the signal. Thus we conclude that condition (i) is well verified.

Regarding condition (ii), we have obtained the singularity spectra (using equation 7) at five different scales. In Figure 5 - 8, we present the singularity spectra obtained at these five scales, showing that all the spectra coincide to a great extent. Condition (iii) is just requiring that singularity spectra are convex. All the singularity spectra are convex curves, Figures 5 - 8, so condition (iii) is also satisfied. Therefore, we conclude
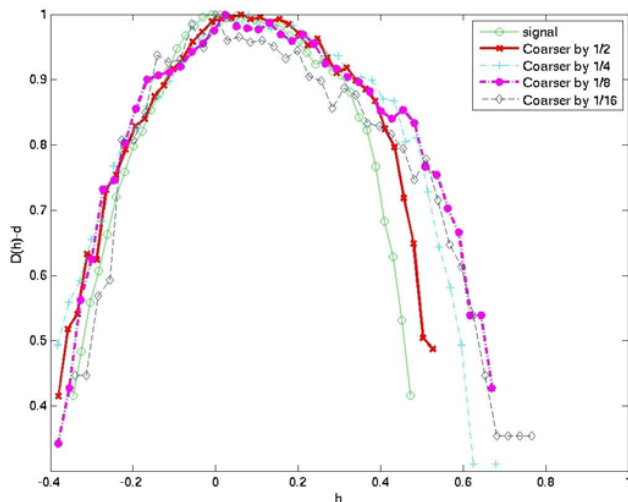
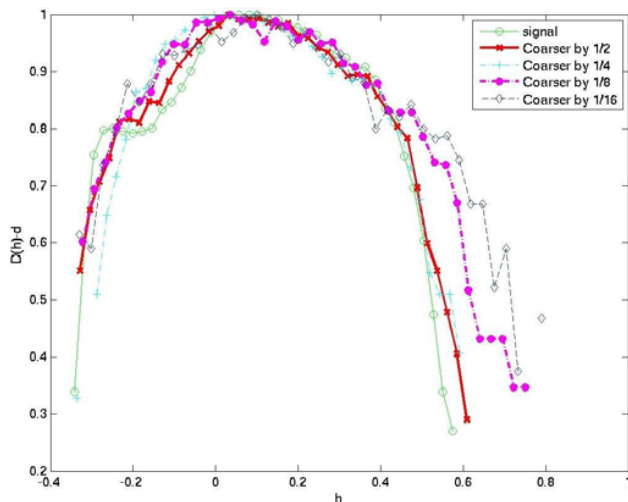Fig. 7: Singularity Spectrum for channel 3 of $c2$ catheter

Fig. 8: Singularity Spectrum for channel 4 of $c3$ catheter

that the heartbeat signals including the electrocardiogram electrodes and catheters signals verify MMF.

V. Conclusions

In this paper, we have shown the application of a novel nonlinear signal-processing framework, the Microcanonical Multiscale Formalism (MMF) to the analysis heartbeat signals. In these signals, there is a multiscale character that is reflected as a definite geometrical structure arranged around manifolds of singularity. This way, the signal can be decomposed into different components depending on their characteristic local predictability exponents (LPEs). The value of the LPE characterizes the power-law behavior under scale changes and directly indicates the information content of the component. As a consequence, the MMF gives a direct access to the geometry of singularity components in a way that characterizes the degree of information contained at each point of the signal. A first observation is that we can reproduce under a micro-

canonical formalism the same type of characterizations about multifractality on heartbeat series that have been reported before in the literature under a canonical framework. Moreover in this paper, the data from catheters (to detect arrhythmias and life-threatening conditions) inside the heart is also considered and we have shown that the heartbeat signals including the catheters and electrocardiogram electrodes signals are multi-scaled in the microcanonical sense of MMF.

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